Homework problems #8

1. Prove that there is no independent tensor that can be constructed out of first derivatives of $g_{\mu\nu}$.
2. Show that the Riemann tensor transforms as a tensor.
3. Show that
$$\frac{D\,T^{\mu}{}_{\nu}}{D\,\tau} = \frac{d\,T^{\mu}{}_{\nu}}{d\,\tau} + \Gamma^{\mu}_{\lambda\rho}\frac{d\,x^{\lambda}}{d\,\tau}T^{\rho}{}_{\nu} - \Gamma^{\sigma}_{\lambda\nu}\frac{d\,x^{\lambda}}{d\,\tau}T^{\mu}{}_{\sigma}$$
transforms as a tensor.
4. Show that
$$\oint x^{\rho}\,dx^{\nu} = \delta a^{\rho}\delta b^{\nu} - \delta b^{\rho}\delta a^{\nu}$$
for the integral along a parallelogram spanned by δa^{μ} and δb^{μ}.
5. Show that the metric
$$g_{tt} = 1, \quad g_{rr} = -1, \quad g_{\theta\theta} = -r^2, \quad g_{\varphi\varphi} = -r^2\sin^2\theta$$
is equivalent to the Minkowski metric.