Homework problems #9

1. Derive the following expression for the Riemann tensor

$$R_{\lambda\mu\nu\kappa} = \frac{1}{2}\left[\frac{\partial^2 g_{\lambda\nu}}{\partial x^\kappa \partial x^\mu} - \frac{\partial^2 g_{\mu\nu}}{\partial x^\kappa \partial x^\lambda} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^\nu \partial x^\mu} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^\nu \partial x^\lambda}\right] + g_{\eta\sigma}\left(\Gamma^\eta_{\nu\lambda}\Gamma^\sigma_{\mu\kappa} - \Gamma^\eta_{\kappa\lambda}\Gamma^\sigma_{\mu\nu}\right)$$

2. Show the following properties of the Riemann tensor:

   - $R_{\lambda\mu\nu\kappa} + R_{\lambda\nu\kappa\mu} + R_{\lambda\kappa\mu\nu} = 0$
   - $R_{\lambda\mu\nu\kappa;\,\eta} + R_{\lambda\mu\kappa\eta;\,\nu} + R_{\lambda\mu\eta\nu;\,\kappa} = 0$