Supersymmetry and hierarchy

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OUTLINE:

- new physics, hierarchy problem
- discovering supersymmetry
- construction and properties of supersymmetric models
- MSSM, radiative EW breaking, tuning
- SUSY: hierarchy vs flavour

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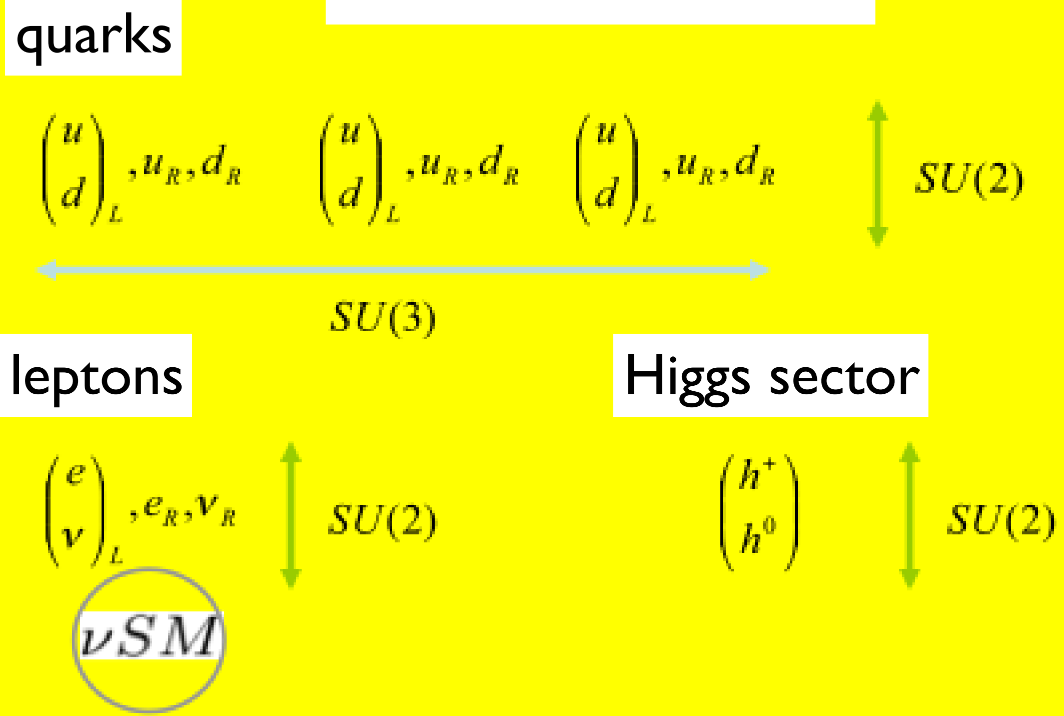
Standard Model



quarks

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R \quad \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R \quad \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R$$

$SU(2)$

$SU(3)$

leptons

$$\begin{pmatrix} e \\ \nu \end{pmatrix}_L, e_R, \nu_R$$

$SU(2)$

νSM

Higgs sector

$$\begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

$SU(2)$

local symmetry: $SU(3)_C \times SU(2)_L \times U(1)$

Facts unexplained by SM

- Observed matter-antimatter asymmetry (nonzero barion number)
- Dark matter and Dark energy
- Inflation

Standard Model cd.

- hierarchy of scales:

$$M_{SM} \approx 100\,\mathrm{GeV}$$

vs

$$M_P \approx 10^{18}\ \mathrm{GeV}$$

$$\delta m_h^2 = O(\lambda, g^2, h^2; \text{ new physics})\, \Lambda^2$$

to keep the Higgs mass low
and to avoid unnatural cancellations

$$\Lambda = \Lambda_{NP} \sim 1 - 10\, \text{TeV}$$

Barnett Newman: Broken Obelisk

Hierarchy problem

Consider a theory with elementary scalar fields, to start with let's consider a single real field

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{1}$$

Parameters in the Lagrangian run according to

$$\begin{aligned} \tfrac{\mu}{Z}\tfrac{dZ}{d\mu} &= \gamma(\lambda) \\ \tfrac{\mu}{m^2}\tfrac{dm^2}{d\mu} &= \gamma_m(\lambda) \\ \mu\tfrac{d\lambda}{d\mu} &= \beta(\lambda) \end{aligned} \tag{2}$$

if Λ is a cut-off, then
$Z(\mu = \Lambda) = Z_0$, $m^2(\mu = \Lambda) = m_0^2$, $\lambda(\mu = \Lambda) = \lambda_0$ are bare parameters. In a theory with a stiff cut-off

$$\mu\frac{dm^2}{d\mu} = +\frac{\hbar\lambda}{16\pi^2}m^2 - \frac{\hbar\lambda}{16\pi^2}\mu^2 + o(\hbar^2) \tag{3}$$

In the cut-off regularization scheme the running parameters are quadratically sensitive to the value of the running energy scale μ^2. Let's solve the above equation (putting $\hbar = 1$)

$$m^2(\mu) \approx m^2(\mu_0) e^{\frac{\lambda}{16\pi^2} \log(\mu/\mu_0)} - \frac{\lambda}{32\pi^2}(\mu^2 - \mu_0^2) \qquad (4)$$

Take $\mu_0 = \Lambda_{cut-off}$

$$m^2(\mu) \approx m^2(\Lambda) e^{\frac{\lambda}{16\pi^2} \log(\mu/\Lambda)} + \frac{\lambda}{32\pi^2}(\Lambda^2 - \mu^2) \qquad (5)$$

This is a classic example of the hierarchy problem: if $\Lambda \gg \mu$, then $m \approx \Lambda$ unless $m^2(\Lambda) e^{\frac{\lambda}{16\pi^2} \log(\mu/\Lambda)} + \frac{\lambda}{32\pi^2}\Lambda^2 \approx 0$. This gives a critical value $m_c^2(\Lambda) \approx \frac{\lambda}{32\pi^2}\Lambda^2$, and to obtain a small value of the physical mass in the IR limit $\mu \to 0$ one needs to tune the initial condition at the scale Λ to the accuracy

$$\left| \frac{m^2(\Lambda) - m_c^2}{m_c^2} \right| \approx \frac{m_{phys}^2}{\frac{\lambda}{32\pi^2}\Lambda^2} \qquad (6)$$

For the physical Higgs particle this is a very small number, unless $\Lambda \approx 1\,\mathrm{TeV}$

In $\overline{MS}$ ($\overline{DR}$) scheme beta-functions depend only on dimensionless couplings and the dependence of running parameters on the renormalizations scale μ is only logarithmic. In this case one can solve formally the RGE for the scalar mass parameter as follows

$$m^2(\mu) = m^2(\mu_0) e^{\int_{\lambda(\mu_0)}^{\lambda(\mu)} \frac{\gamma_m(\lambda')}{\beta(\lambda')} d\lambda'} \tag{7}$$

At one-loop level in a simple scalar field model one finds

$$m^2(\mu) = m^2(\mu_0)(1 + \frac{\lambda(\mu_0)}{16\pi^2} \log(\mu/\mu_0)) \tag{8}$$

To obtain a small physical parameter at a low scale one needs to assume at a high scale $\mu_0 = \Lambda$ a boundary condition $m^2(\mu_0) \ll \Lambda^2$. In general, one would like to set at a high scale f natural boundary conditions $m(f) \sim f$, which, ideally, at low energy μ should give $m(\mu) \sim \mu$ within a few orders of magnitude. However, if there is no physics setting boundary conditions at the high scale, as in the Standard Model, one may impose boundary conditions at a low scale, and consider the high-energy values as a result of the dynamical flow. In such a case the hierarchy problem does not arise.

Discovering supersymmetry

Consider a scalar field ϕ, $\phi = \mathrm{Re}((H - v)/\sqrt{2})$, which has a coupling $L = H\bar{f}f\lambda_f = \frac{\lambda_f}{\sqrt{2}}\phi\bar{f}f + ...$ (like Higgs particle). The contribution to the self-energy coming from this coupling reads

$$\Pi_{\phi\phi}(p = 0) = (-1)2N_f\lambda_f^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2 + m_f^2}{(k^2 - m_f^2)^2}$$
$$= (-1)2N_f\lambda_f^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2}\right), \qquad (9)$$

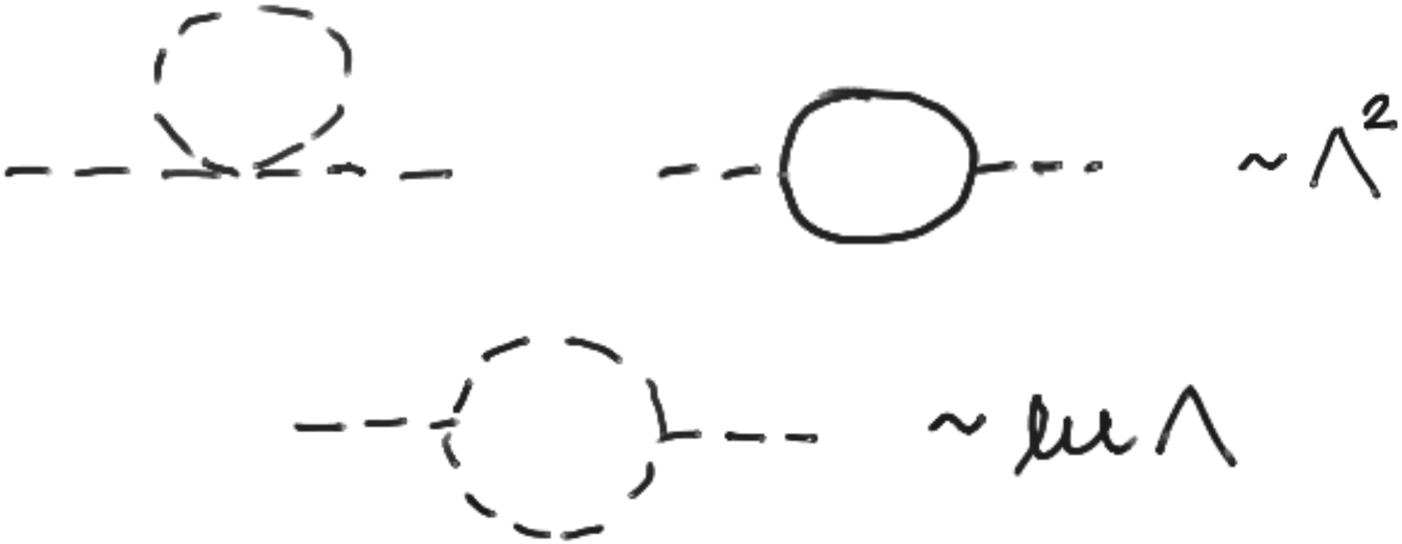
where the first piece in the last line diverges like Λ^2, and is independent of m_f, m_ϕ, while the second goes like $\log \Lambda^2$. The structure of the second term is interesting, since Λ is effectively replaced by m_f, say m_t, up to a logarithmic factor of the order $\log(\Lambda/m_Z)$, which is at most of the order of 30. Such terms appear in RGEs, including the MS or DR schemes. They look much better than quadratic terms, as long as the masses of fermions running in the loop are small. In such a case one could simply neglect quadratic pieces or use MS/DR and neglect the hierarchy problem.

However, one expects new heavy particles in many BSM theories, for instance in GUT models. Thus what we really need is a mechanism, hopefully a symmetry, which would not only cancel quadratic divergencies at all loops, but would also screen low energy physics against corrections coming from heavy particles. This second problem cannot be avoided by simply restricting oneself to MS/DR regularization schemes.

First of all, let us try to get rid of quadratic divergencies. We are going to work in the cut-off regularization, but the same conditions can be obtained in the MS scheme. The point is that in the last case quadratic divergencies are separated from the logarithmic ones at each finite order of perturbation theory, but can be seen as poles in $d-(4-2/L)$ where d is the dimensionality of space-time and L is the number of loops. One should notice, that in the limit $L\to\infty$ quadratic divergencies would reappear as poles in $d-4$, thus cancellation of the poles in $d-(4-2/L)$ in addition to cancellation of poles in $d-4$ for any L makes sense also in MS/DR.

In addition to fermionic loops, also scalar loops lead to quadratic divergencies. Hence, let's consider a Lagrangian with both fermions and scalars coupled to ϕ

$$\begin{aligned} L_{\phi\tilde{f}} = {} & \tfrac{1}{2}\tilde{\lambda}_f \phi^2 (|\tilde{f}_L|^2 + |\tilde{f}_R|^2) \\ & + v\tilde{\lambda}_f \phi (|\tilde{f}_L|^2 + |\tilde{f}_R|^2) \\ & + (\tfrac{\lambda_f}{\sqrt{2}} A_f \phi \tilde{f}_L \tilde{f}_R^* + h.c.) \end{aligned} \tag{10}$$



This gives

$$\Pi^{\tilde{f}}_{\phi\phi}(p=0) = (-1)\tilde{\lambda}_f N_{\tilde{f}} \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 - m^2_{\tilde{f}_L}} + \frac{1}{k^2 - m^2_{\tilde{f}_R}} \right) + \text{terms} \; \sim \log(\Lambda). \tag{11}$$

Quadratic terms cancel if

$$N_f = N_{\tilde{f}_L} = N_{\tilde{f}_R}; \;\; \tilde{\lambda}_f = -\lambda_f^2 \tag{12}$$

It is important to notice, that the above conditions do not touch dimensionfull parameters of the model. Now, let us impose these conditions on the sum of the contributions to the scalar self-energy. Let's use $\overline{MS}/\overline{DR}$ scheme, which means $\int \frac{d^4k}{2i\pi^2} \frac{1}{k^2 - m^2} = m^2(1 - \log(m^2/\mu^2)), \; \int \frac{d^4k}{2i\pi^2} \frac{1}{(k^2 - m^2)^2} = -\log(m^2/\mu^2)$.

Taking for simplicity equal masses of tilded fields one finds

$$\Pi^{\tilde{f}+f}_{\phi\phi}(p=0) = i\frac{\lambda_f^2 N_f}{16\pi^2}\left(-2m_f^2(1-\log\frac{m_f^2}{\mu^2}) + 4m_f^2\log\frac{m_f^2}{\mu^2}\right. \\ \left. + 2m_{\tilde{f}}^2(1-\log\frac{m_{\tilde{f}}^2}{\mu^2}) - 4m_{\tilde{f}}^2\log\frac{m_{\tilde{f}}^2}{\mu^2} - |A_f|^2\log\frac{m_{\tilde{f}}^2}{\mu^2}\right) \qquad (13)$$

where one has denoted $m_f = \lambda_f v/\sqrt{2}$. Now, taking

$$m_f = m_{\tilde{f}};\ A_f = 0 \qquad (14)$$

one finds vanishing total correction. This is a strong hint, that there is a symmetry behind, which becomes exact in the above limit.

Let us violate "softly" this symmetry taking $m_{\tilde{f}}^2 = m_f^2 + \delta^2$ and $\delta, A_f \ll m_f$. In this case

$$\Pi_{\phi\phi}^{\tilde{f}+f}(p=0) = -i\frac{\lambda_f^2 N_f}{16\pi^2}\left(4\delta^2 + (2\delta^2 + |A_f|^2)\log\frac{m_f^2}{\mu^2}\right) \qquad (15)$$

The result is, that the possibly large mass of the fermion, m_f, appears only under the logarithm. In fact, the large cut-off scale Λ has been effectively replaced by a small scale δ, which measures the violation of the symmetry that predicts equal masses of fermions and scalars. These are the crucial observations, which make the supersymmetry useful for description of the low-energy phenomenology. Of course, the other supersymmetry violating term, A_f, also needs to stay small. This means that the central question becomes the question about the origin of the hierarchically small supersymmetry violation, and we shall come back to this point.

Symmetry of the Lagrangian

Let's construct a Lagrangian, where a symmetry imposing required relations on masses and couplings can be realized.

$$S = \int d^4x(L_s + L_f) = \int d^4x(\partial^\mu \phi^* \partial_\mu \phi + i\bar{\psi}\bar{\sigma}^\mu \psi) \tag{16}$$

Consider the following transformation

$$\begin{gathered} \phi \to \phi + \delta\phi,\ \psi \to \psi + \delta\psi \\ \delta\phi = \xi^\alpha \psi_\alpha = \xi^\alpha \epsilon_{\alpha\beta} \psi^\beta,\ \delta\psi_\alpha = -i\sigma^\nu_{\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} \partial_\nu \phi \end{gathered} \tag{17}$$

The variation of the Lagrangia reads

$$\begin{aligned} \delta L_s + \delta L_f = \bar{\xi}\partial^\mu \bar{\psi}\partial_\mu \phi + \partial^\mu \phi^* \xi \partial_\mu \psi - (\xi \sigma^\nu \bar{\sigma}^\mu \partial_\mu \psi)\partial_\nu \phi^* \\ + (\bar{\psi}\bar{\sigma}^\mu \sigma^\nu \bar{\xi})\partial_\mu \partial_\nu \phi = -\partial^\mu(\bar{\psi}\bar{\xi}\partial_\mu \phi) + \partial_\mu(\bar{\psi}\bar{\sigma}^\nu \sigma^\mu \bar{\xi}\partial_\nu \phi) \end{aligned} \tag{18}$$

This is a total derivative, hence

$$\delta S = 0 \tag{19}$$

In the above action the symmetry between bosons and fermions holds only on-shell. Since the Weyl equations has a form of a matrix operator projecting out half of the degrees of freedom, on-shell one has 2 real scalar and 2 real fermionic degrees of freedom. To make the symmetry between fermions and bosons explicit one can introduce an auxiliary complex scalar F

$$\begin{gathered} S = \int d^4x(L_s + L_f + L_{aux}) = \int d^4x(\partial^\mu \phi^* \partial_\mu \phi + i\bar{\psi}\bar{\sigma}^\mu \psi + F^* F) \\ \delta F = -i\bar{\xi}\bar{\sigma}^\mu \partial_\mu \psi \\ \delta\phi = \xi^\alpha \psi_\alpha \ \delta\psi_\alpha = -i\sigma^\nu_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}\partial_\nu \phi + \xi_\alpha F \\ \delta L_s + \delta L_f + \delta L_{aux} = ... + i\partial_\mu(\bar{\psi}\bar{\sigma}^\mu \xi F) \end{gathered} \tag{20}$$

In addition, one can show that a commutator of two transformations on any of the bosonic or fermionic fields is a translation: $(\delta_{\xi_1}\delta_{\xi_2} - \delta_{\xi_2}\delta_{\xi_1})X = -i(\xi_1\sigma^\mu\bar{\xi}_2 - \xi_2\sigma^\mu\bar{\xi}_1)\partial_\mu X$.

What one needs, is a version of the supersymmetric Lagrangian which includes interactions. Let's consider renormalisable interactions described by terms with dim ≤ 4.

$$L_{int} = -\frac{1}{2} W^{jk} \psi_i \psi_j + W^j F_j + h.c. \tag{21}$$

where $W^{jk} \sim \phi$, ϕ^2 and symmetric in flavour indices. A term $U(\phi, \phi^*)$ (no Fs) is forbidden, since its variation does contain neither derivatives nor Fs, while all relevant variations of other terms do. Cancellation of 4-spinor variations implies

$$\frac{W^{jk}}{\partial \phi^n} \text{ totally symmetric, } \frac{W^{jk}}{\partial \phi^{*n}} = 0 \tag{22}$$

Thus one writes

$$W^{jk} = \frac{\partial^2 W}{\partial \phi_j \partial \phi_k} \tag{23}$$

for a function $W = W(\phi)$ known as a holomorphic superpotential.

Cancellation of the 1-derivative variations up to a total divergence implies in turn

$$W^j = \frac{\partial W}{\partial \phi_j} \tag{24}$$

This completes the construction and implies, that renormalizable interactions of chiral multiplets $\Phi = (\phi, \psi, F)$ depend on a single holomorphic function W only! Since SUSY relies only on the holomorphicity of W, the action will be invariant for any W.
Now, one can eliminate the auxiliary field F via its equations of motion

$$\begin{gathered} L_F = F_i F^{*i} + W^i F_i + W_j^* F^{*j} \\ \tfrac{\partial L_F}{\partial F^{*i}} = F_i + W_i^* = 0 \\ F_i = -W_i^* \end{gathered} \tag{25}$$

This gives

$$V = W^j W_j^* \geq 0 \tag{26}$$

Let's take

$$W = \frac{1}{2}M^{jk}\phi_j\phi_k + \frac{1}{3!}y^{jkn}\phi_j\phi_k\phi_n \tag{27}$$

$$\begin{gathered} L_{WZ} = L_{kin} - V(\phi, \phi^*) \\ -\tfrac{1}{2}M^{jk}\psi_j\psi_k + h.c. \\ -\tfrac{1}{2}y^{jkn}\phi_j\psi_k\psi_n + h.c. \end{gathered} \tag{28}$$

The EOMS are

$$\begin{gathered} \partial^\mu\partial_\mu\phi_j = -M^*_{jn}M^{nk}\phi_k + ... \\ i\bar{\sigma}^\mu\partial_\mu\psi_j = M^*_{jn}\bar{\psi}^n \end{gathered} \tag{29}$$

This means

$$M^2_{sc} = M^\dagger_f M_f \tag{30}$$

that is, the same diagonalisation gives eigenstates of the scalar and fermionic mass matrices (alignment holds for exact SUSY).

To have spontaneously broken supersymmetry one needs to find an operator A, composite or elementary, such that $\langle \delta_{susy} A \rangle \neq 0$. Let's have a look at elementary fields in the WZ model. The $\delta\phi = \xi\psi$, but an expetation value for the fermionic field would mean spontaneous breakdown of the Lorentz invariance, which is bad for phenomenology. The δF contains derivatives, the expectation value of which would also break the Lorentz invariance. However

$$\delta\psi_j = \xi F_j + ... \tag{31}$$

and F_j can take an expectation value without violating Lorentz symmetry. This means, that phenomenologically acceptable supersymmetry breaking is signalled by nonzero expectation values of certain F-terms. How to initiate SUSY breakdown? Let's take $W = E\phi$. This gives $F^* = -E \neq 0$ and broken supersymmetry.

Supersymmetry algebra

For a general small continuous transformation one can write

$$\delta_\xi X = \xi \, iTX \tag{32}$$

where ξ is a small transformation "angle" and T is a generator of a transformation. In case of supersymmetry $T \to Q_\alpha$, which is a $(1/2, 0)$ Weyl spinor, since it turns a spin 0 scalar into a left Weyl fermion. Abstract $N = 1$ algebra of generators can be read from transformations of the fields

$$\begin{gathered} \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \\ [P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_\alpha] = 0, \\ \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu. \end{gathered} \tag{33}$$

In addition

$$\begin{gathered} [J_i, Q_\alpha] = \left(\frac{\sigma^i}{2}\right)_\alpha^{\ \beta} Q_\beta \\ [Q_\alpha, T^a] = 0 \end{gathered} \tag{34}$$

This means that we are going to assume that supersymmetry generator commute with generators of all internal gauge symmetries in the models. One consequence of the above is

$$\begin{aligned} H = P_0 &= \tfrac{1}{4}\mathrm{tr}\{Q_\alpha, \bar{Q}_{\dot\alpha}\} \\ \langle \mathrm{any}|H|\mathrm{any}\rangle &= \textstyle\sum_i \left(||Q_i|\mathrm{any}\rangle||^2 + ||\bar{Q}_i|\mathrm{any}\rangle||^2\right) \geq 0 \end{aligned} \quad (35)$$

This result says, that if supersymmetry stays unbroken, the energy of the ground state must be zero. Another consequence is

$$\begin{aligned} \mathrm{tr}[(-1)^{N_f}\{Q_\alpha, \bar{Q}_{\dot\alpha}\}] = 0 &= 2\sigma^\mu_{\alpha\dot\alpha}\mathrm{tr}[(-1)^{N_f}P_\mu] \\ tr(-1)^{N_f} &= 0 \end{aligned} \quad (36)$$

which means that in a given finite dimensional representation one finds equal number bosons and fermions.

Superspace

A useful device which allows to handle general supersymmetric Lagrangians is the superspace - a 4d Minkowski space-time supplemented by 4 anticommuting coordinates

$$\theta_\alpha,\, \bar{\theta}_{\dot{\alpha}};\ \{\theta_\alpha, \bar{\theta}_{\dot{\alpha}}\} = 0,\ \alpha = 1,2 \tag{37}$$

One can define functions of superspace coordinates, which are called superfields. Superfields have finite expansion in anticommuting coordinates, and coefficients of such expnsion are standard fields, on which act supersymmetry transformations. One can define Grassman integrals on superspace, $\int d^2\theta\theta^2 = \int d^2\theta\theta^\alpha\theta_\alpha = 1 = \int d^2\bar{\theta}\bar{\theta}^2$, which act on superfields as projection operators - they project out coefficient of a given power of anticommuting parameters.

Let us define a chiral superfield

$$\begin{aligned}\Phi(y) &= \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \\ &= \phi(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi(x) \\ &\quad + \sqrt{2}\theta\psi(x) + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta^2 F(x) \end{aligned} \qquad (38)$$

where $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$, which includes a scalar field $\phi(x)$, spinor field $\psi(x)_\alpha$ and an auxiliary scalar $F(x)$. The beauty of the superspace formalism lies in the fact, that the Wess-Zumino Lagrangian studied earlier can be written in a imple form

$$\mathcal{L}_0 = \int d^2\theta d^2\bar{\theta}\, \Phi_i\bar{\Phi}_i + \left(\int d^2\theta W(\Phi) + h.c\right) \qquad (39)$$

where $W = \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3!}g_{ijk}\Phi_i\Phi_j\Phi_k$ is the superpotential written in terms of chiral superfields.

Vector superfield in the Wess-Zumino gauge takes the form

$$V^a = \theta\bar{\sigma}^\mu\bar{\theta}v_\mu^a(x) + i\theta^2\bar{\theta}\bar{\lambda}^a(x) - i\bar{\theta}^2\theta\lambda^a(x) + \frac{1}{2}\theta^2\bar{\theta}^2 D^a(x), \quad (40)$$

where $A_\mu^a(x)$ is the real vector field, $\lambda_\alpha^a(x)$ is a Weyl spinor (gaugino) and the auxiliary real fiel is denoted by $D^a(x)$. Consider non-abelian gauge transformations

$$\begin{aligned} \Phi &\to e^{i\Lambda}\Phi \\ V &\to V + i(\Lambda - \Lambda^\dagger) + \dots \\ V &= T_{ij}^a V_a \\ \Lambda &= T_{ij}^a \Lambda_a \end{aligned} \quad (41)$$

where Λ is a chiral superfield (parameter of the gauge transformation) and T^a s are hermitean generators of the gauge transformation. Invariant Lagrangian for gauge fields takes the form

$$\mathcal{L}_V = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2}D^a D^a, \quad (42)$$

where

$$F^a_{\mu\nu} = \partial_\mu v^a_\nu - \partial_\nu v^a_\mu - gf^{abc} v^b_\mu v^c_\nu, \tag{43}$$

and covariant derivatives are

$$\begin{aligned} D_\mu \lambda^a = \partial_\mu \lambda^a - gf^{abc} A^b_\mu \lambda^c \\ D_\mu \phi_i = \partial_\mu \phi_i + igA^a_\mu T^a \phi_i \\ D_\mu \psi_i = \partial_\mu \psi_i + igA^a_\mu T^a \psi_i. \end{aligned} \tag{44}$$

To obtain auge invariant version of the Wess-Zumino model one replaces in the WZ derivatives by covariant derivatives and adds interaction terms supersymmetrizing this replacement

$$\mathcal{L} = \mathcal{L}_{WZ} + \mathcal{L}_V - \sqrt{2} g \left[(\bar{\phi} T^a \psi) \lambda^a + \bar{\lambda}^a (\bar{\psi} T^a \phi) \right] + g(\bar{\phi} T^a \phi) D^a. \tag{45}$$

With the help of EOMs for auxiliary fields

$$\begin{aligned} D^a &= -g\phi^* T^a \phi, \\ Fi &= -\bar{W}_i, \\ \bar{F}_i &= -W_i, \end{aligned} \tag{46}$$

where

$$W_i = \frac{\partial W(\phi)}{\partial \phi^i}, \tag{47}$$

one can finally write down the scalar potential in terms of propagating fields

$$V(\phi, \bar{\phi}) = \bar{F}_i F_i + \frac{1}{2} D^a D^a = \bar{W}_i W_i + \frac{1}{2} g^2 (\bar{\phi} T^a \phi)^2. \tag{48}$$

Note that

$$\delta \lambda^a_\alpha = ... + \frac{1}{\sqrt{2}} \xi_\alpha D^a \tag{49}$$

Hence, if there exists a D^a such that

$$\langle D^a \rangle \neq 0 \tag{50}$$

then supersymmetry becomes spontaneously broken. Actually, this condition is meaningfull only for Abelian factors. For nonabelian gauge group, one can show, that if all F_i vanish, then also all D^as can be made vanish by a suitable field transformation

Goldstino

With exact supersymmetry there exists a conserved supercurrent, which takes the form

$$\begin{aligned} J^\mu_\alpha &= (\sigma^\nu \bar\sigma^\mu \psi_i)_\alpha D_\nu \phi^{*i} - i(\sigma^\mu \bar\psi^i)_\alpha W_i \\ &-\tfrac{1}{2\sqrt{2}}(\sigma^\nu \bar\sigma^\rho \sigma^\mu \bar\lambda^a)_\alpha F^a_{\nu\rho} - \tfrac{i}{\sqrt{2}} g \phi^* T^a \phi (\sigma^\mu \bar\lambda^a)_\alpha \end{aligned} \tag{51}$$

The conservation of this current implies

$$\partial_\mu J^\mu_\alpha = 0 = i\sigma^\mu \partial_\mu \left(F^*_i \bar\psi^i + \frac{D^a}{\sqrt{2}} \bar\lambda^a \right) + \ldots \tag{52}$$

which means that there is in the spectrum a massles Weyl fermion, the Goldstino

$$\bar\Pi = \frac{F^*_i \bar\psi^i + \frac{D^a}{\sqrt{2}} \bar\lambda^a}{\sqrt{|F^i|^2 + D^{a2}/2}} \tag{53}$$

where the normalisation factor $F_\pi = \sqrt{|F^i|^2 + D^{a2}/2}$ is called the Goldstino decay constant.

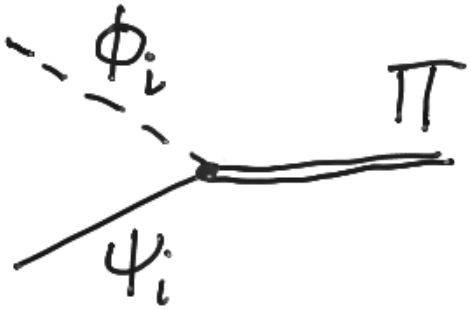
The supercurrent may be written as $J^{\mu}_{\alpha} = iF_{\pi}(\sigma^{\mu}\bar{\Pi})_{\alpha} + j^{\mu}_{\alpha}$, and an effective Lagrangian can be written down, which reproduces the vanishing of the divergence of the supercurrent as the EOM for Goldstino:

$$L_{\Pi} = i\Pi\sigma^{\mu}\partial_{\mu}\bar{\Pi} + \frac{1}{F_{\pi}}\Pi\partial_{\mu}j^{\mu}_{\alpha} + h.c. \tag{54}$$

The current j^{μ}_{α} contains chiral and gauge/gaugino fields. Using EOMs for these fields one finds explict form of the interactions of the Goldstino. In particular, interactions with chiral multiplets take the form

$$L_{\Pi\Phi} = \frac{(m^2_{\psi_i} - m^2_{\phi_i})}{F_{\pi}}\bar{\Pi}\psi_i\phi^{*i} + h.c. \tag{55}$$

This is an interesting relation, which tells one how to search for Goldstinos and superpartners, hence how to test supersymmetry.



Actually, massless Goldstinos are at odds with phenomenology, and one expects that a superHiggs effect takes place. After making supersymmetry a local symmetry, supergravity, there appears in the theory the superpartner of graviton, gravitino, which plays the role of the gauge field. Massless gravitino combines with Goldstino into a massive gravitino. Then the above Lagrangian describes the direct non-gravitational coupling of the helicity $\pm\frac{1}{2}$ components of the gravitino to chiral matter.

Nonrenormalization theorem

General supersymmetric action can be written down with the help of the superspace formalism as follows

$$\mathcal{S} = \int d^4x \int d^4\theta \left[K(\Phi^\dagger, e^V \Phi) + 2\xi V_{U(1)}\right] +$$

$$+ \int d^4x \int d^2\theta \left[Y W(\Phi) + \frac{\tau}{16\pi i} W^\alpha(V) W_\alpha(V) + \sum_{n\geq 2} X_n \left(W^\alpha W_\alpha\right)^n \right] +$$

In this action gauge coupling has bee promoted to a chiral superfield and gauginos are not canonically normalized. The holomorphic coupling constant includes the theta-angle θ_{YM} and is defined as

$$\tau = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2} \qquad (57)$$

This definition is not equivalent to the standard defiition of the "physical" gauge coupling measuring interactions of canonically normalized fields, g_c.

The parameter ξ denotes here the Fayet-Iliopoulos term, which is gauge invarint only for abelian $U(1)$s. The real, gauge invariant, function K is called the *Kähler potential* and describes general kinetic terms for chiral supefields. The Y denotes a spurion, which scales uniformly all couplings in the superpotential, and $W^{a\,\alpha}$ is a chirl superfield which contains field strength of a gauge boson A^a. The tree level action is invariant under a number of symmetries, which must be respected in perturbative calculations

- supersymmetry itself, gauge symmetries
- continuous R-symmetry, which is broken via the fixing of Y
- Peccei-Quin symmetry $\tau \to \tau + r$

In addition, the structure of the action is fixed by taking the limits $\tau \to \infty$, $Y \to 0$.

The nonrenormalization theorem says

- The structure and coefficients of the superpotential are not changed by radiative corrections
- holomorphic coupling is renormalized at 1-loop order only
- coefficients X_n are not renoralized
- Fayet-Illiopoulos terms gets renormalized at 1-loop order only, if anomaly cancellation for the $U(1)$ factor fails
- Kähler potential gets renormalized at all orders

In particular, this means that supersymmetric masses are renoralized only due to wave function renormalizations, that is only logarithmically.

Minimal Supersymmetric Standard Model (MSSM)

Guge group of the SM is $G_{SM} = SU(3) \times SU(2) \times U(1)_Y$. The spectrum contains 3 chiral families of quarks and leptons, gauge bosons - gluons, $W^{\pm}$,B, and a single scalar Higgs doublet

$$V(H, \bar{H}) = \mu^2 \bar{H} H + \frac{\lambda}{2} (\bar{H} H)^2, \tag{58}$$

which implies spontaneous breaking of the electroweak symmetry down to $U(1)_{em}$ of electromagnetism provided that $\mu^2 < 0$ and $\lambda > 0$. One needs to add a superpartner to each known particle, including the Higgs scalars. Higgs particles and chiral fermions fit into chiral superfields, while vector bosons belong to vector superfields together with their superpartners - gauginos.

One needs to add a second Higgs doublet even in the minimal model, since

- adding new charged fermions - higgsinos - spoils anomaly cancellation in SM, which is restored by adding still new fermions from the second doublet,
- superpotential is holomorphic in chiral fields, so one cannot simply conjugate the original doublet, as this would produce an ati-chiral multiplet.

The superpotential in MSSM takes the form

$$W_{MSSM} = U_R \mathbf{h_u} Q H_u + D_R \mathbf{h_d} Q H_d + E_R \mathbf{h_e} L H_d + \mu H_u H_d, \quad (59)$$

where U_R,D_R,Q,E_R,L,H_u,H_d are all chiral superfields (see the Table), Yukawa couplings $\mathbf{h}$ are actually 3×3 matrices in flavour space.

	supermultiplet	fermiony	bozony	$SU(3)$	$SU(2)$	$U(1)_Y$
kwarki	$Q^i = \begin{pmatrix} U^i_L \\ D^i_L \end{pmatrix}$	q^i_L	$\tilde{q}^i_L$	3	2	$\frac{1}{6}$
	U^i_R	u^i_R	$\tilde{u}^i_R$	$\bar{3}$	1	$-\frac{2}{3}$
	D^i_R	d^i_R	$\tilde{d}^i_R$	$\bar{3}$	1	$\frac{1}{3}$
leptony	$L^i = \begin{pmatrix} \nu^i \\ E^i_L \end{pmatrix}$	l^i_L	$\tilde{l}^i_L$	1	2	$-\frac{1}{2}$
	E^i_R	e^i_R	$\tilde{e}^i_R$	1	1	1
bozony Higgsa	$H_u = \begin{pmatrix} H^+_u \\ H^0_u \end{pmatrix}$	$\begin{pmatrix} \tilde{h}^+_u \\ \tilde{h}^0_u \end{pmatrix}$	$\begin{pmatrix} h^+_u \\ h^0_u \end{pmatrix}$	1	2	$\frac{1}{2}$
	$H_d = \begin{pmatrix} H^0_d \\ H^-_d \end{pmatrix}$	$\begin{pmatrix} \tilde{h}^0_d \\ \tilde{h}^-_d \end{pmatrix}$	$\begin{pmatrix} h^0_d \\ h^-_d \end{pmatrix}$	1	2	$-\frac{1}{2}$
gauge bosons	g	$\tilde{g}$	g	8	1	0
	W	W	W	1	3	0
	B	$\tilde{B}$	B	1	1	0

Table 1: Particles in MSSM. Index $i = 1, 2, 3$ labels families.

There are couplings which are allowed by supersymmetry and by gauge symmetries, which should better stay supressed, or absent

$$H_u L, LQD, DDU, LLE. \tag{60}$$

These terms violate barion number or lepton number and may lead to phenomenological disaster. To suppress such terms one invokes R-parity, a discrete symmetry with charges assigned according to the rule

$$P_M = (-1)^F \, (-1)^{3(B-L)}, \tag{61}$$

where B and L are barion and lepton numbers.

None of the superpartners has been observed so far. This means that supersymmetry is broken, and it should be broken spontaneously. However, one can easily convince oneself, that this spontaneous breaking cannot take place in the SM sector. The argument relies on the supersymmetric sum rule

$$\sum_{\text{spin zero}} \text{mass}^2 - 2 \sum_{\text{spin } 1/2} \text{mass}^2 + 3 \sum_{\text{spin } 1} \text{mass}^2 = -2 \sum_{a} \mathrm{D}^a \mathrm{Tr}(\mathrm{T}^a) \tag{62}$$

In the MSSM the RHS is zero, and the vanishing of the LHS must hold in each sector with given unbroken quantum numers (colour and electric charge) since mass matrices must be block diagonal in these labels. For instance, in the colour triplet sector with $Q_e = -1/3$

$$m_d^2 + m_s^2 + m_b^2 = (5\,\mathrm{GeV})^2 \tag{63}$$

If there are no other fermions with this colour and charge this implies that there exist squarks with a mass smaller than 7 GeV, which is ruled out experimentally.

One needs to create a separate sector designed to break supersymmetry. Then, supersymmetry breakdown should be transmitted, via the messenger sector, to the SM particles. The result seen from within the SM are terms that violate supersymmetry explicitly, but without introducing quadratic divergencies. We had already examples of such terms: explicit, nonholomorphic mass terms for scalars and the A-terms. Full set of such terms in the context of MSSM looks as follows:

$$\begin{aligned}\mathcal{L}_{soft} = & - \frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B} + \mathrm{h.c.}\right) \\ & - \left(\tilde{\bar{U}}\mathbf{a_u}\tilde{Q}H_u + \tilde{\bar{D}}\mathbf{a_d}\tilde{Q}H_d + \tilde{\bar{E}}\mathbf{a_e}\tilde{L}H_d + \mathrm{h.c.}\right) \\ & - \tilde{Q}^\dagger\mathbf{m^2_Q}\tilde{Q} - \tilde{\bar{U}}\mathbf{m^2_U}\tilde{\bar{U}}^\dagger - \tilde{\bar{D}}\mathbf{m^2_D}\tilde{\bar{D}}^\dagger - \tilde{L}^\dagger\mathbf{m^2_L}\tilde{L} - \tilde{\bar{E}}\mathbf{m^2_E}\tilde{\bar{E}}^\dagger \\ & - m^2_{H_u}H_u^\dagger H_u - m^2_{H_d}H_d^\dagger H_d + (bH_uH_d + \mathrm{h.c.})\end{aligned} \tag{64}$$

where $\mathbf{m^2_Q}$,$\mathbf{m^2_U}$,$\mathbf{m^2_D}$, $\mathbf{m^2_L}$,$\mathbf{m^2_E}$ are 3×3 matrices in family space, related to sfermion mass matrices, $\mathbf{a_u}$,$\mathbf{a_d}$,$\mathbf{a_e}$, are 3×3 matrices in family space, w przestrzeni rodzin related to the matrices **h** in the superpotential.

Terms containing M_3,M_2,M_1 are gluino, wino and bino mass terms. The terms $m^2_{H_u}$, $m^2_{H_d}$ and b are susy breaking terms in the Higgs sector.
The scalar mass matrix looks as follows

$$\mathcal{L}_{\mathrm{mass}\ f} = -\begin{pmatrix}\tilde{f}^*_L & \tilde{f}^*_R\end{pmatrix} \mathbf{m}^2_f \begin{pmatrix}\tilde{f}_L \\ \tilde{f}_R\end{pmatrix}, \qquad (65)$$

which is non-diagonal and. Because of soft terms $m^2_{Q_3}$,$m^2_{U_3}$,$m^2_{D_3}$,$m^2_{L_3}$,$m^2_{E_3}$, A_t,A_b,A_τ this matrix is also non-aligned with the fermion mass matrix - they cannot be diagonalized simultaneously.

MSSM Higgs sector

Scalar potential in the Higgs sector takes the form

$$\begin{aligned} V &= (\mu^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (\mu^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\ &+ [b(H_u^+ H_d^- - H_u^0 H_d^0) + \mathrm{h.c}] \\ &+ \frac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 \\ &+ \frac{1}{8}(g^2 + g'^2)\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 . \qquad (66) \end{aligned}$$

To find the minimum one notes first, that the $SU(2)$ invariance of the Lagrangian allows one to make $H_u^+ = 0$. Then the condition $\partial V / \partial H_d^- = 0$ gives $H_d^- = 0$. Hence one can assume $H_u^+ = 0\ ,\ H_d^- = 0\ ,\ H_d^0 = H_d\ ,\ H_u^0 = H_u$ which gives a simpler expression for neutral components of the doublets

$$\begin{aligned} V &= (\mu^2 + m_{H_u}^2)|H_u|^2 + (\mu^2 + m_{H_d}^2)|H_d|^2 + (bH_u^0 H_d^0 + \mathrm{h.c}) \\ &+ \frac{1}{8}(g^2 + g'^2)\left(|H_u|^2 - |H_d|^2\right)^2 . \qquad (67) \end{aligned}$$

The mass of the Z can be expressed via the expectation values of the Higgs fields

$$v_u^2 + v_d^2 = v^2 = \frac{4m_Z^2}{g^2 + g'^2}, \tag{68}$$

and the minimization condition $\partial V/\partial H_d = \partial V/\partial H_u = 0$ can be written as

$$\begin{aligned} \mu^2 &= \frac{1}{2}\left[\tan 2\beta \left(m_{H_u}^2 \tan\beta - m_{H_d}^2 \cot\beta\right) - m_Z^2\right] \\ b &= B\mu = \frac{1}{2}\sin 2\beta \left(m_{H_u}^2 + m_{H_d}^2 + 2\mu^2\right), \end{aligned} \tag{69}$$

which allows to express μ and B in terms of soft masses, β angle and the mass of Z. The β angle is defined as follows

$$\tan\beta = \frac{v_u}{v_d} = \frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle}. \tag{70}$$

Two Higgs doublets contain 4 complex fields.
Out of these 3 (G^0 , $G^\pm$) are swallowed by massive bosons Z and $W^\pm$. The remaining ones are a pseudoscalar with the mass

$$m_A^2 = 2\mu^2 + m_{H_u}^2 + m_{H_d}^2, \tag{71}$$

two charged bosons with masses

$$m_{H^\pm}^2 = m_A^2 + m_W^2, \tag{72}$$

and two neutral scalars h i H, whose masses can be found upon diagonalization of the matrix

$$\mathbf{M}_{h,H} = \begin{pmatrix} m_A^2 \sin^2\beta + m_Z^2 \cos^2\beta & -(m_Z^2 + m_A^2)\sin\beta\cos\beta \\ -(m_Z^2 + m_A^2)\sin\beta\cos\beta & m_A^2 \cos^2\beta + m_Z^2 \sin^2\beta \end{pmatrix}, \tag{73}$$

with eigenstates of the form

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H_d \\ H_u \end{pmatrix}. \tag{74}$$

This gives the mass of the light Higgs boson (the little higgs)

$$m_h^2 = \frac{2m_Z^2 m_A^2 \cos^2 2\beta}{m_A^2 + m_Z^2 + \sqrt{\left(m_A^2 + m_Z^2\right)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta}}. \tag{75}$$

This expression gives the upper limit on the tree level mass of the light Higgs

$$m_h^2 \leq m_Z^2 \cos^2 2\beta \leq m_Z^2. \tag{76}$$

Fortunately, this limit can be significantly increased by radiative corrections.

RGE running

A model of supersymmetry breaking gives us an energy scale M_u, below which supersymmetry is broken and the soft terms at this scale. Determination of the parameters at lower scales requires solving the renormalization group equations for the MSSM. In practice, the low scale, M_{EWSB}, is taken to be the geometric mean of the masses of the 3rd generation squarks (called stops)

$$M_{EWSB} = \sqrt{m_{\bar{t}_1}(M_{EWSB}) m_{\bar{t}_2}(M_{EWSB})}. \tag{77}$$

At this scale one minimizes 1-loop corrections to the effective potential. This way one supposedly achieves fast convergence of the loop-series of radiative corrections, which should make 1- and 2-loop corrections dominant. As we already know the form of RGEs depends on the regularization scheme. We shall use the dimensional reduction ($\overline{DR}$) scheme. This procedure respects supersymmetry during the calculations.

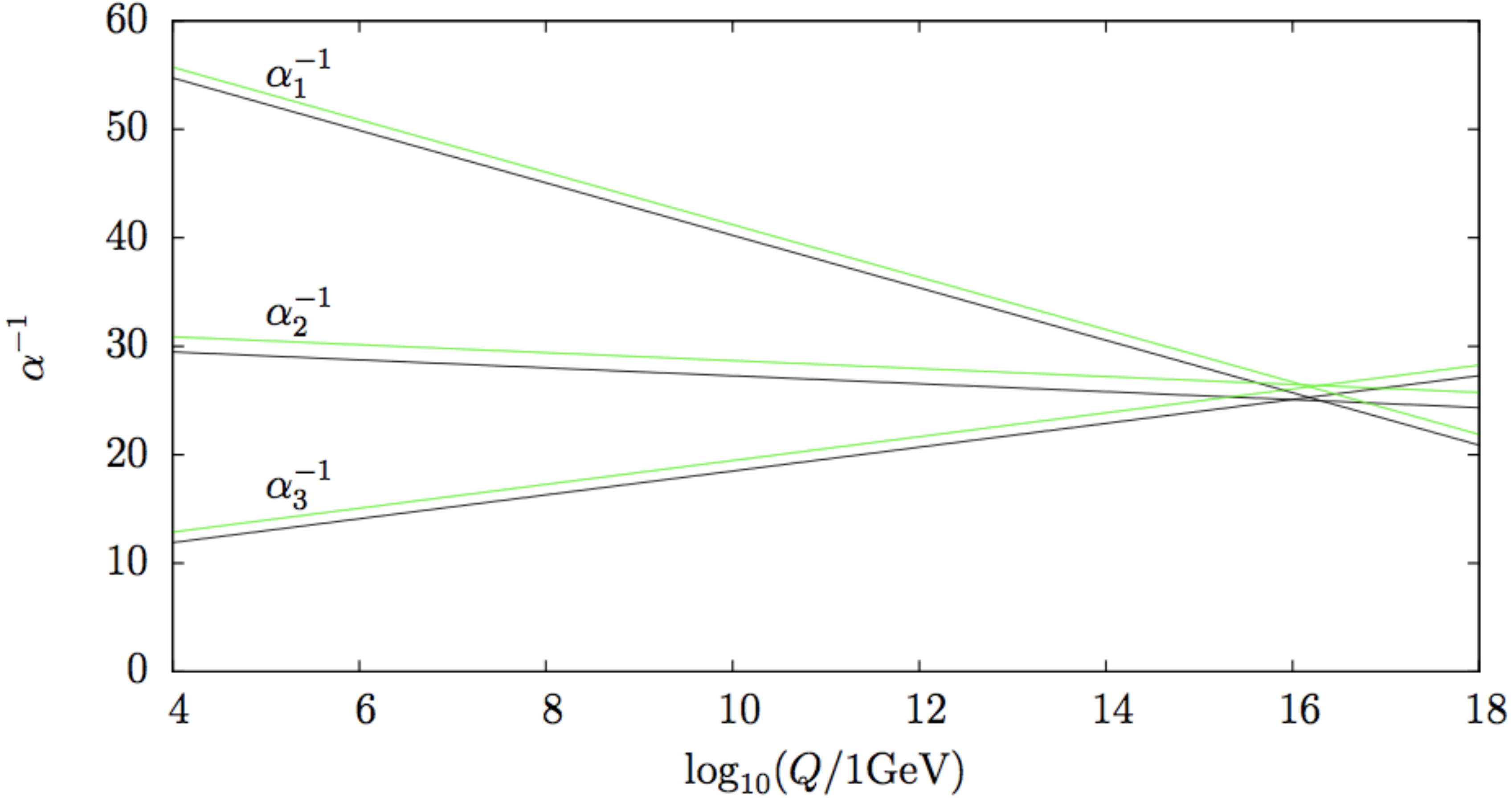
In MSSM RGEs for gauge couplings assume the form

$$\frac{d}{dt}g_i = \frac{1}{16\pi^2} b_{\underline{i}} g_{\underline{i}}^3 \quad , \quad b_i = (\frac{33}{5}, 1, -3), \tag{78}$$

where $t = \ln(Q/Q_0)$, Q is the renormalizations scale and Q_0 is a reference scale. One often uses $\alpha_i = g_i^2/4\pi^2$, for which the equations read

$$\frac{d}{dt}\alpha^{-1} = \frac{b_i}{2\pi}. \tag{79}$$

It is interesting to note, that in MSSM one finds, to an accuracy much higher than in the SM, that the couplings run towards a common value at certain energy scale



60
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40
30
20
10
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
4
6
8
10
12
14
16
18
$\log_{10}(Q/1\text{GeV})$

RGEs for gaugino masses are

$$\frac{d}{dt}M_i = \frac{2}{16\pi^2}M_{\underline{i}}b_{\underline{i}}g_{\underline{i}}^2 \quad , \quad b_i = (\frac{33}{5}, 1, -3). \tag{80}$$

In these equations derivatives are proportional to the masses themselves, which means, that in order to fulfill increasing experimental lower limits at low scales one needs large soft terms at the high scale.
Masses generated for quarks and leptons as a result of the Higgs mechanism depend on matrices **h**. In the leading approximation one can neglect small Yukawa couplings of the 1st and 2nd generations and approximate Yukawa matrices by

$$\mathbf{h_t} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_t \end{pmatrix} \quad , \quad \mathbf{h_b} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_b \end{pmatrix} \quad , \quad \mathbf{h_\tau} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_\tau \end{pmatrix}. \tag{81}$$

RGEs for these couplings read:

$$\begin{aligned}
\frac{d}{dt}h_t &= \frac{h_t}{16\pi^2}\left(6|h_t|^2+|h_b|^2-\frac{16}{3}g_3^2-3g_2^2-\frac{13}{15}g_1^2\right), \\
\frac{d}{dt}h_b &= \frac{h_b}{16\pi^2}\left(6|h_b|^2+|h_t|^2+|h_\tau|^2-\frac{16}{3}g_3^2-3g_2^2-\frac{7}{15}g_1^2\right), \\
\frac{d}{dt}h_\tau &= \frac{h_b}{16\pi^2}\left(4|h_\tau|^2+3|h_b|^2-3g_2^2-\frac{9}{5}g_1^2\right). \qquad (82)
\end{aligned}$$

The assumption about proportionality of soft terms **a** to Yukawa couplings allows us to write down RGEs for A_t , A_b i A_τ

$$\begin{aligned}
8\pi^2\frac{d}{dt}A_t &= 6|h_t|^2A_t+|h_b|^2A_b+\frac{16}{3}g_3^2M_3+3g_2^2M_2+\frac{13}{15}g_1^2M_1, \\
8\pi^2\frac{d}{dt}A_b &= 6|h_b|^2A_b+|h_t|^2A_t+|h_\tau|^2A_\tau+\frac{16}{3}g_3^2M_3+3g_2^2M_2 \\
&\quad +\frac{7}{15}g_1^2M_1, \\
8\pi^2\frac{d}{dt}A_\tau &= 4|h_\tau|^2A_\tau+3|h_b|^2A_b+3g_2^2M_2+\frac{9}{5}g_1^2M_1. \qquad (83)
\end{aligned}$$

Because of positive contributions from gaugino masses the parameters A tend to decrease towards low energy scales
In the RGEs for scalar masses there are contributions proportional to quares of the mass terms and to the respecctive hypercharge

$$X = m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}[\mathbf{m_Q^2} + \mathbf{m_D^2} + \mathbf{m_E^2} - 2\mathbf{m_U^2} - \mathbf{m_L^2}], \tag{84}$$

and contributions related to 3rd family Yukawa couplings

$$\begin{aligned} X_t &= 2|h_t|^2(m_{H_u}^2 + m_{Q_3}^2 + m_{U_3}^2 + A_t^2), \\ X_b &= 2|h_b|^2(m_{H_d}^2 + m_{Q_3}^2 + m_{D_3}^2 + A_b^2), \qquad (85) \\ X_\tau &= 2|h_t|^2(m_{H_d}^2 + m_{L_3}^2 + m_{E_3}^2 + A_\tau^2). \end{aligned}$$

(86)

RGEs for squark and slepton masses of the first two generations take the form

$$16\pi^2 \frac{d}{dt} m_f^2 = -8 \sum_{i=1}^{3} C_i(f) g_i^2 M_i^2 + \frac{6}{5} Y_f g_1^2 X, \tag{87}$$

here Y_f is the hypercharge of the respective field from the table given earlier, and coefficients C are

$$\begin{aligned} C_1(f) &= \frac{3}{5} Y_f^2, \\ C_2(f) &= \begin{cases} \frac{3}{4} & \text{dla} \quad f = Q, L, H_u, H_d \\ 0 & \text{dla} \quad f = U, D, E \end{cases} \\ C_3(f) &= \begin{cases} \frac{4}{3} & \text{dla} \quad f = Q, U, D \\ 0 & \text{dla} \quad f = E, L, H_u, H_d, \end{cases} \end{aligned} \tag{88}$$

For the 3rd generation

$$\begin{aligned}
16\pi^2\frac{d}{dt}m^2_{Q_3} &= -\frac{32}{3}g_3^2M_3^2-3g_2^2M_2^2-\frac{2}{15}g_1^2M_1^2+X_t+X_b+\frac{1}{5}g_1^2X,\\
16\pi^2\frac{d}{dt}m^2_{U_3} &= -\frac{32}{3}g_3^2M_3^2-\frac{32}{15}g_1^2M_1^2+2X_t-\frac{4}{5}g_1^2X,\\
16\pi^2\frac{d}{dt}m^2_{D_3} &= -\frac{32}{3}g_3^2M_3^2-\frac{8}{15}g_1^2M_1^2+2X_b+\frac{2}{5}g_1^2X,\\
16\pi^2\frac{d}{dt}m^2_{L_3} &= -6g_2^2M_2^2-\frac{6}{5}g_1^2M_1^2+X_\tau-\frac{3}{5}g_1^2X,\\
16\pi^2\frac{d}{dt}m^2_{E_3} &= -\frac{24}{5}g_1^2M_1^2+2X_\tau+\frac{6}{5}g_1^2X.
\end{aligned}\tag{89}$$

Negative contributions from gaugino masses allow to obtain large masses at the electroweak scale even with small initial values at the high scale.

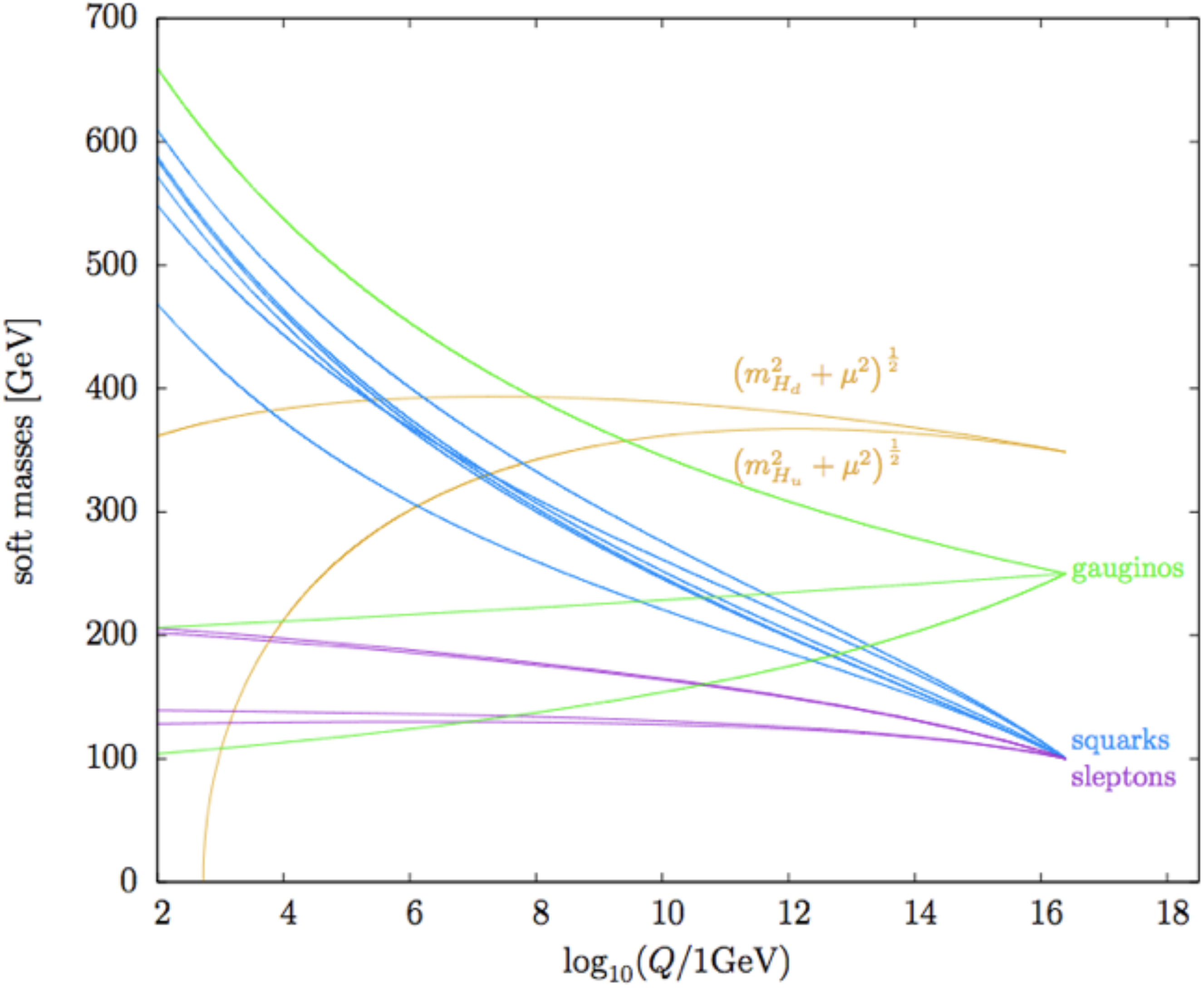
Electroweak breaking

Equatons which describe running in the Higgs sector are

$$\begin{aligned}
16\pi^2 \frac{d}{dt} m_{H_u}^2 &= -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + 3X_t + \frac{3}{5} g_1^2 X, \\
16\pi^2 \frac{d}{dt} m_{H_d}^2 &= -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + 3X_b + X_\tau - \frac{3}{5} g_1^2 X, \\
16\pi^2 \frac{d}{dt} \mu &= \mu \left(3|h_t|^2 + 3|h_b|^2 + |h_\tau|^2 - 3g_2^2 - \frac{3}{5} g_1^2 \right), \qquad (90) \\
16\pi^2 \frac{d}{dt} B &= 6|h_t|^2 A_t + 6|h_b|^2 A_b + 2|h_\tau|^2 A_\tau + 6g_2^2 M_2 + \frac{6}{5} g_1^2 M_1.
\end{aligned}$$

In the above equations the most important contribution to derivatives of scalar masses comes from X_t, since X_b i X_τ are proportional to squares of the respective Yukawa couplings $h_t, h_b < h_t$. To the contrary, X doesn't play a role since soft masses of squarks and sleptons typically are of the same order and their contributions tend to cancel each other in X. It is clear, that the mass which which suffers the fastes decrease with lowering the energy scale is $m_{H_u}^2$.

This is important, since due to the change of the sign of $m_{H_u}^2$ from positive to negative the electroweak symmetry breaks down at low energies, giving masses to the gauge bosons. In the figure one can watch how the renormalized parameter $(m_{H_u}^2 + \mu^2)^{\frac{1}{2}}$ from the Higgs potential evolves with energy scale. The point is that popular theories of soft masses produce positive mass squares at high energies (but there may be negative contributions from D-terms, from anomaly mediation or from Seiberg duality).
Such a phenomenon, known as radiative symmetry breaking, doesn't occur in the SM - the mass parameter from the SM Higgs potential changes by about 4 percent over 16 decades in energy.


700
600
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200
100
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2
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14
16
18
soft masses [GeV]
$\log_{10}(Q/1\mathrm{GeV})$
$(m_{H_d}^2+\mu^2)^{\frac{1}{2}}$
$(m_{H_u}^2+\mu^2)^{\frac{1}{2}}$
gauginos
squarks
sleptons

Fine-tuning in MSSM with 126 GeV Higgs

M. Lewicki and Z.L.
JHEP 1305 (2013) 125,
arXiv:1302.6546 [hep-ph]

Neutral part of the scalar potential in MSSM:

$$\begin{aligned} V &= (\mu^2 + m_{H_u}^2)|H_u|^2 + (\mu^2 + m_{H_d}^2)|H_d|^2 + (bH_uH_d + \mathrm{h.c}) \\ &+ \frac{1}{8}(g^2 + g'^2)\left(|H_u|^2 - |H_d|^2\right)^2 . \end{aligned}$$

which after spontaneous symmetry breaking

$$< H_u >= v_u \quad , \quad < H_d >= v_d$$

$$v^2 = v_u^2 + v_d^2 \quad , \quad \mathrm{tg}\,\beta = \frac{v_u}{v_d} \quad , \quad m_Z^2 = \frac{v^2}{2}(g^2 + g'^2),$$

gives:

- at tree level $m_h < m_Z$
- $m_Z^2 = \mathrm{tg}\,2\beta\left(m_{H_u}^2\,\mathrm{tg}\,\beta - m_{H_d}^2\,\mathrm{ctg}\,\beta\right) - 2\mu^2 \approx -2(m_{H_u}^2 + \mu^2)$

- Pushing light Higgs mass to the observed value of 126GeV requires large radiative corrections, the biggest one comes from top-stop loop

$$\delta m_h^2 = \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12 M_S^2}\right)\right],$$

  where $M_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$ and $X_t = m_t(A_t - \mu \,\mathrm{ctg}\,\beta)$.
- Parameters giving us Z mass also receive top-stop loop corrections

$$\delta m_{H_u}^2|_{stop} = -\frac{3Y_t^2}{8\pi^2}\left(m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2\right)\log\left(\frac{M_u}{\mathrm{TeV}}\right),$$

  where $m_{Q_3}^2$ $m_{U_3}^2$ and A_t are soft terms that predict the stop mass, and M_u is a scale at which soft masses are generated.
- So requiring correct Higgs mass gives large corrections that have to cancel out to give the correct m_Z.

We use the usual definition of fine-tuning with respect to parameter a

$$\Delta_a = \left| \frac{\partial \ln m_Z^2}{\partial \ln a} \right| .$$

which can be rewritten as

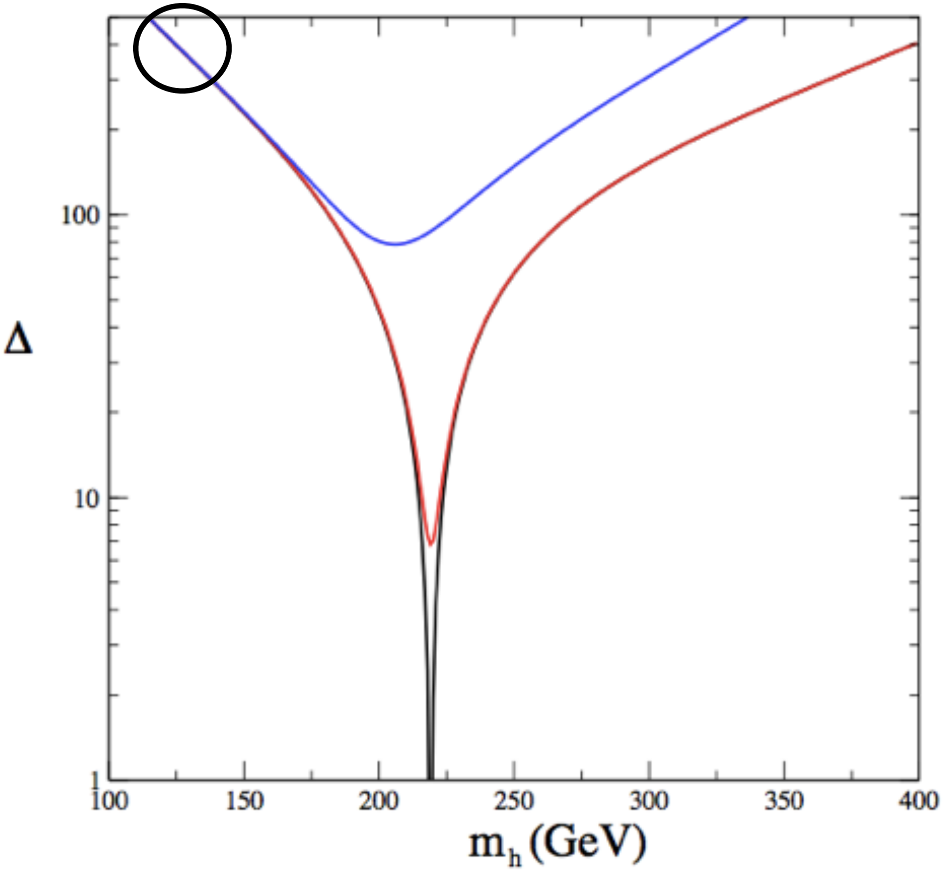
$$\frac{\delta m_Z^2}{m_Z^2} = \Delta_a \frac{\delta a}{a} .$$

And fine-tuning coming from a whole set of parameters a_i

$$\Delta = \max_{a_i} \Delta_{a_i} .$$

FT in SM

$$\frac{\delta m^2}{m^2} = \frac{3}{64\pi^2}\left(3g^2 + g'^2 + 8\lambda - 8h_t^2\right)\frac{\Lambda^2_{cut-off}}{m^2}$$

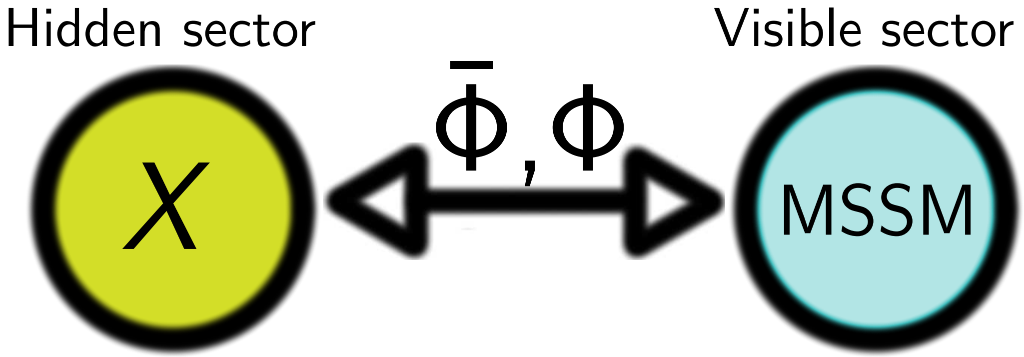


Casas, Espinosa, Hidalgo

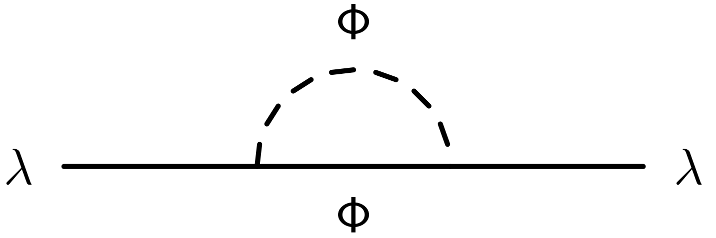
Figure 1: Fine-tuning contours as a function of the Higgs mass in the SM with a common cut-off $\Lambda = 10\,\text{TeV}$. This can be considered as the fine-tuning of the Little Hierarchy problem in the SM. Different curves correspond to progressively more sophisticated definitions of Δ (from black [bottom line] to red to blue [top line], see text for details).

SUSY breaking mediation

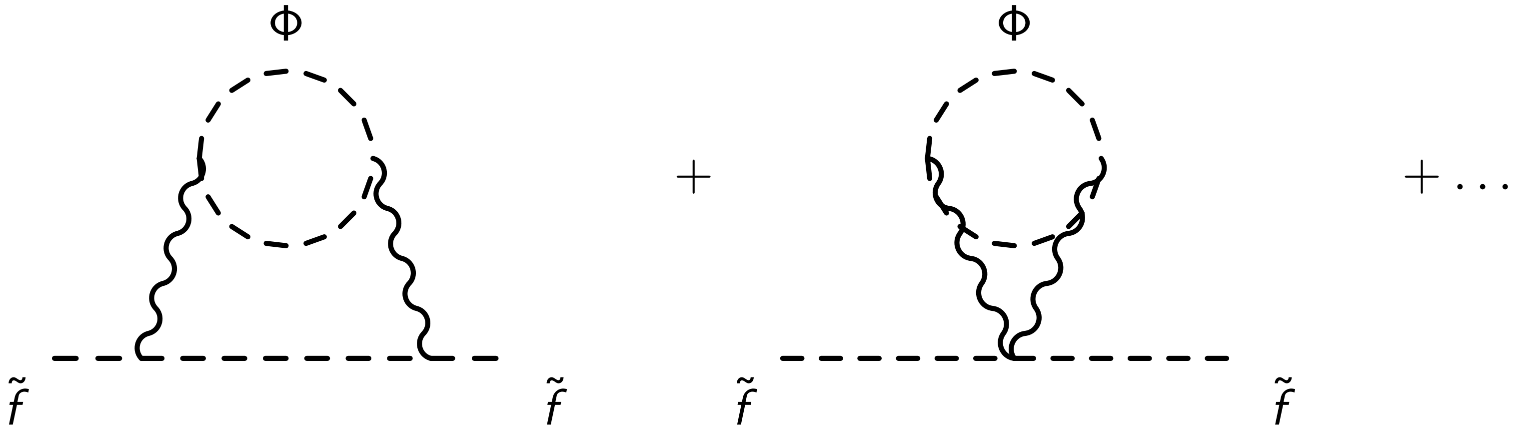
- Supergravity
  - No control over mixing between families → lagre FCNC
- Gauge mediation
  - SUSY is spontaneously broken→ singlet $\langle X \rangle = X + \theta^2 F$
  - breaking is transmitted through messengers $W = \lambda \bar{\Phi} X \Phi$
  - messengers $\bar{\Phi}$,Φ interact with MSSM fields only via gauge interactions



Gauge mediated soft terms



$$\Longrightarrow M_i = \frac{\alpha_i}{4\pi}\frac{F}{X}$$



$$\Longrightarrow m_f^2 = 2\sum_i C_i(f)\left(\frac{\alpha_i}{4\pi}\right)^2\left|\frac{F}{X}\right|^2$$

GGM soft terms

Meade, Shih and Seiberg 0801.3278
Gauge mediated soft terms can be expressed by just six parameters

- Three gaugino masses

$$M_1 = \frac{\alpha_1}{4\pi} m_Y, \quad M_2 = \frac{\alpha_2}{4\pi} m_w, \quad M_3 = \frac{\alpha_3}{4\pi} m_c,$$

- Three parameters determining scalar masses Λ_c^2 , Λ_w^2 , Λ_Y^2 which give

$$m_f^2 = 2\left[C_3(f) \left(\frac{\alpha_3}{4\pi}\right)^2 \Lambda_c^2 + C_2(f) \left(\frac{\alpha_2}{4\pi}\right)^2 \Lambda_w^2 + C_1(f) \left(\frac{\alpha_1}{4\pi}\right)^2 \Lambda_Y^2 \right],$$

- Only negligible A-terms are generated.

Implementations

Two specific models Carpenter et al. 0805.2944

- GGM1

$$W_{GGM1} = X_i(y^i \bar{Q}Q + r^i \bar{U}U + s^i \bar{E}E),$$

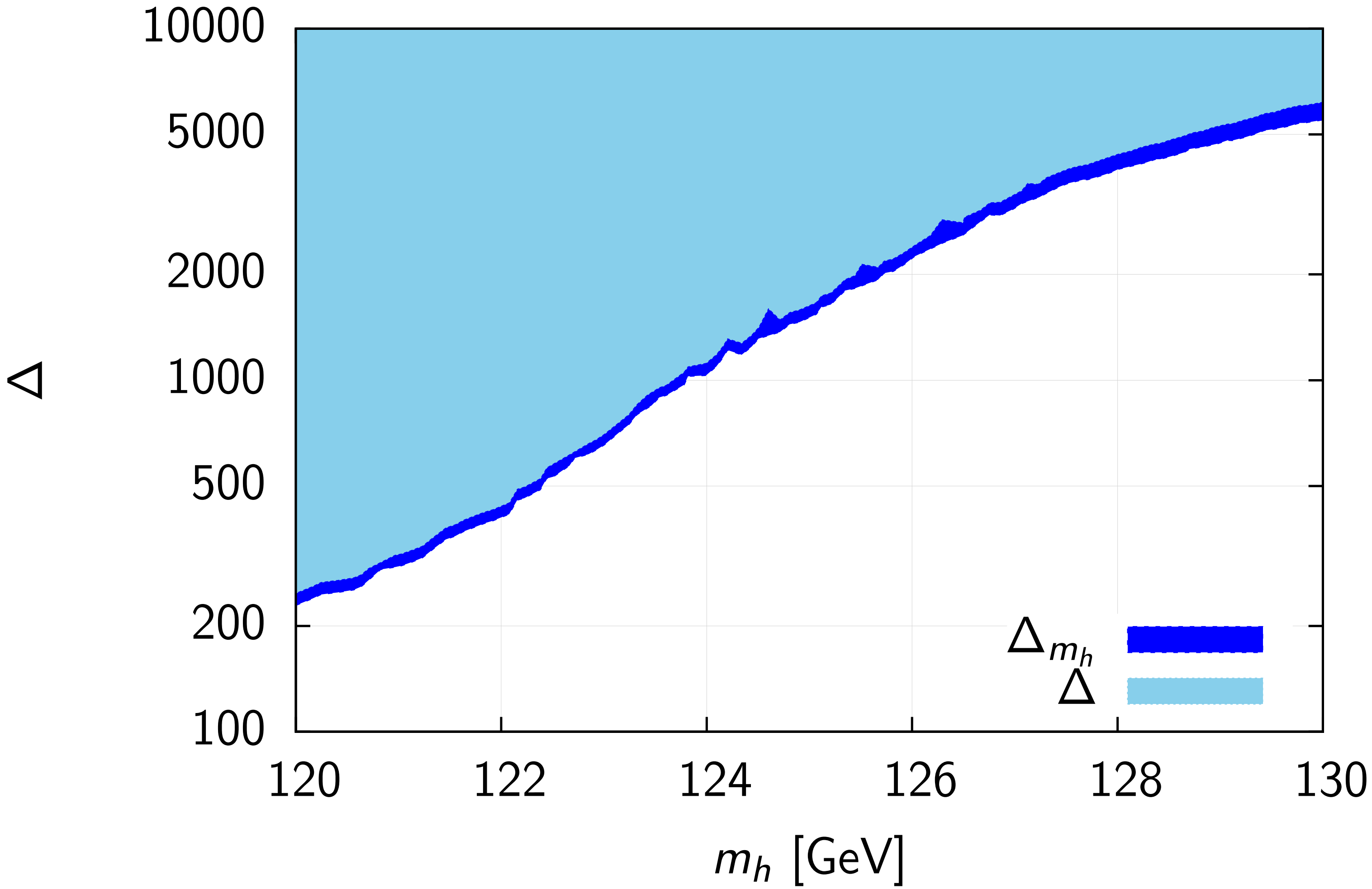
  with three independent parameters Λ_Q,Λ_U,Λ_E

- GGM2

$$W_2 = X_i(y^i \bar{Q}Q + r^i \bar{U}U + s^i \bar{E}E + \lambda^i_q \tilde{q}q + \lambda^i_l \tilde{l}l),$$

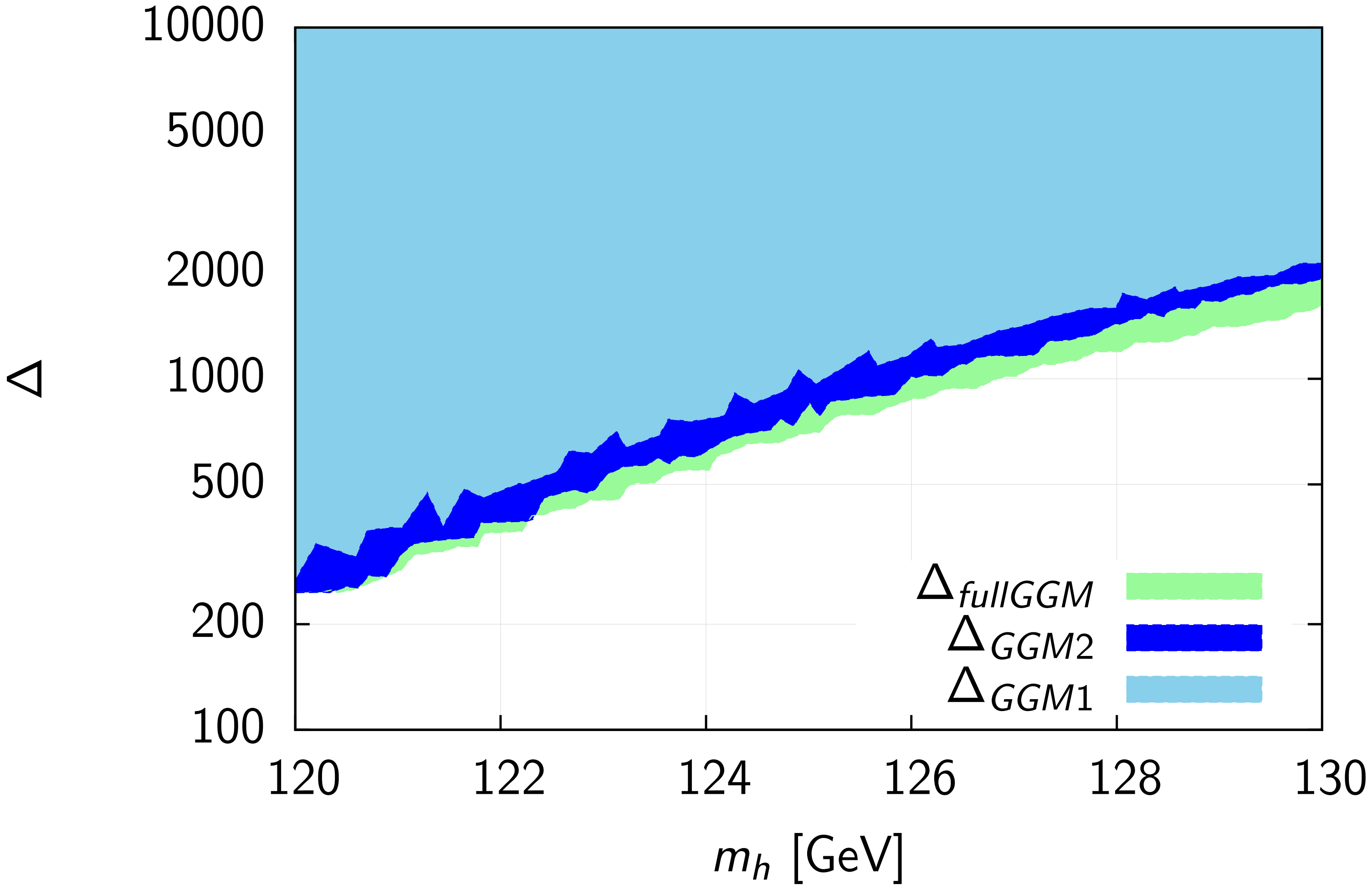
  with five independent parameters Λ_Q,Λ_U,Λ_E,Λ_q,Λ_l

FT in mSUGRA

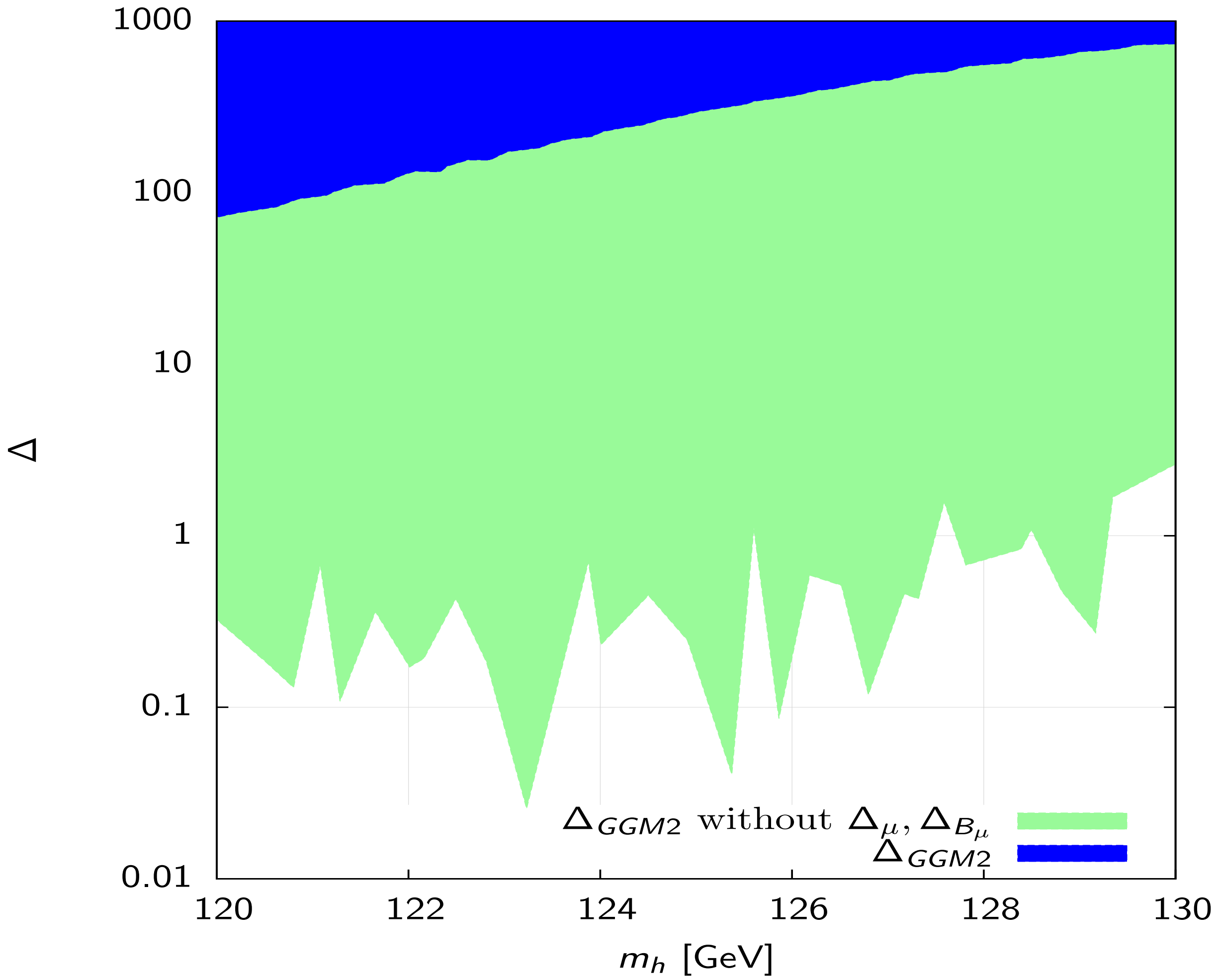


10000
5000
2000
1000
500
200
100
Δ
120
122
124
126
128
130
m_h [GeV]
Δ_{m_h}
Δ

FT in GGM


10000
5000
2000
1000
500
200
100
Δ
120
122
124
126
128
130
m_h [GeV]
$\Delta_{fullGGM}$
Δ_{GGM2}
Δ_{GGM1}

fine-tuning from only gauge mediated soft terms



Brummer and Buchmuller 1201.4338

1. GGM predicts smaller fine-tuning than mSUGRA
2. for $m_h = 126$GeV fine-tuning always larger than 100 unless one includes only gauge mediated soft terms
3. including $g_\mu - 2$ raises fine-tuning about four times, but its still possible to obtain $g_\mu - 2$ within 1σ bound
4. decrease of the Higgs mass down to 123 GeV reduces the fine-tuning by a factor of 2.

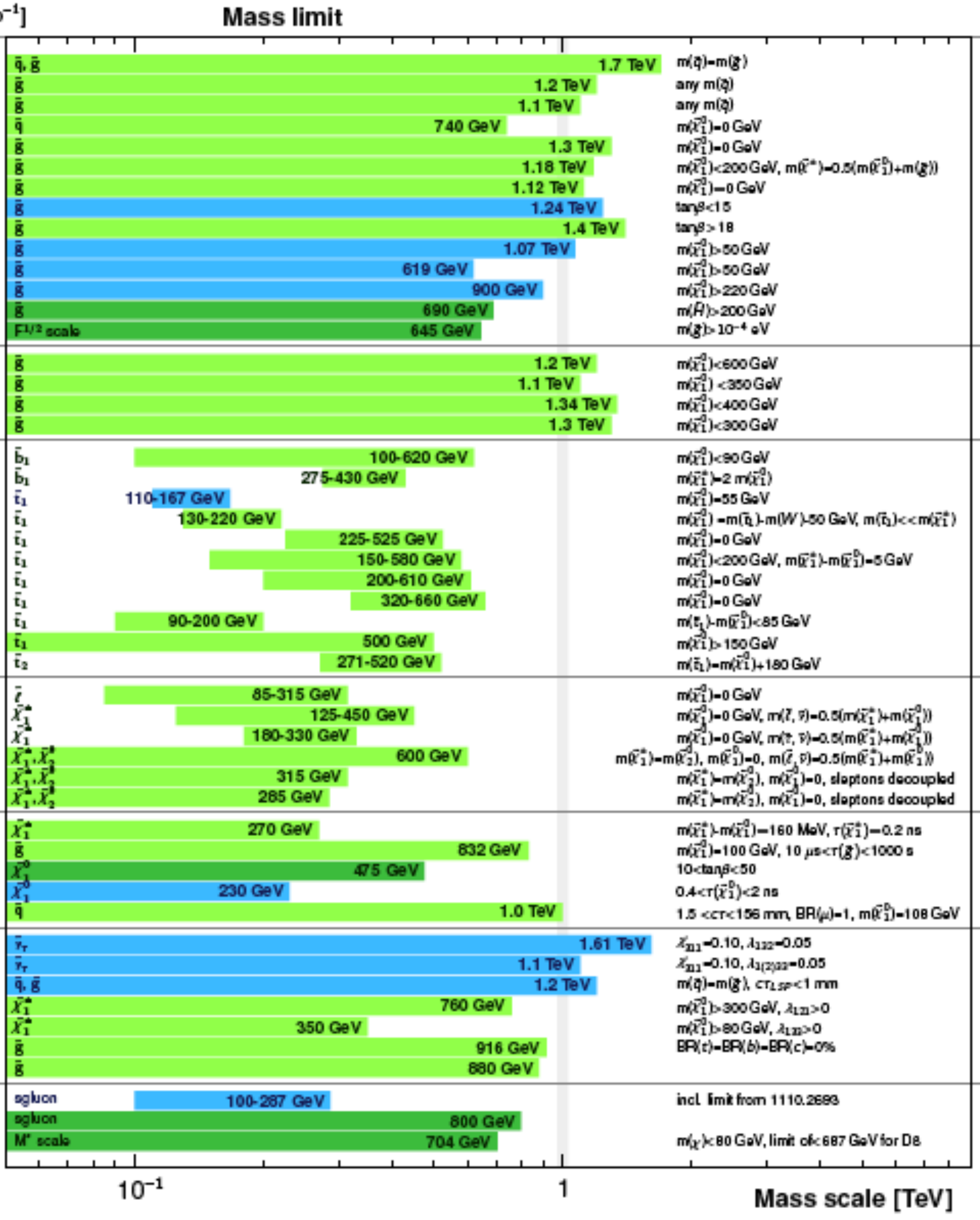
https://twiki.cern.ch/twiki/bin/view/AtlasPublic/SupersymmetryPublicResults

ATLAS SUSY Searches* - 95% CL Lower Limits

Status: SUSY 2013

ATLAS Preliminary

$\int\mathcal{L}\,dt = (4.6 - 22.9)\ \text{fb}^{-1}$ $\sqrt{s} = 7, 8$ TeV



	Model	e, μ, τ, γ	Jets	E_T^{miss}	$\int\mathcal{L}\,dt[\text{fb}^{-1}]$	Mass limit		Reference
Inclusive Searches	MSUGRA/CMSSM	0	2-6 jets	Yes	20.3	$\tilde{q}, \tilde{g}$ 1.7 TeV	$m(\tilde{q})=m(\tilde{g})$	ATLAS-CONF-2013-047
	MSUGRA/CMSSM	1 e,μ	3-6 jets	Yes	20.3	$\tilde{g}$ 1.2 TeV	any $m(\tilde{q})$	ATLAS-CONF-2013-062
	MSUGRA/CMSSM	0	7-10 jets	Yes	20.3	$\tilde{g}$ 1.1 TeV	any $m(\tilde{q})$	1308.1841
	$\tilde{q}\tilde{q}, \tilde{q}\to q\tilde{\chi}_1^0$	0	2-6 jets	Yes	20.3	$\tilde{q}$ 740 GeV	$m(\tilde{\chi}_1^0)=0$ GeV	ATLAS-CONF-2013-047
	$\tilde{g}\tilde{g}, \tilde{g}\to q\bar{q}\tilde{\chi}_1^0$	0	2-6 jets	Yes	20.3	$\tilde{g}$ 1.3 TeV	$m(\tilde{\chi}_1^0)=0$ GeV	ATLAS-CONF-2013-047
	$\tilde{g}\tilde{g}, \tilde{g}\to qq\tilde{\chi}_1^{\pm}\to qqW^{\pm}\tilde{\chi}_1^0$	1 e,μ	3-6 jets	Yes	20.3	$\tilde{g}$ 1.18 TeV	$m(\tilde{\chi}_1^0)<200$ GeV, $m(\tilde{\chi}^{\pm})=0.5(m(\tilde{\chi}_1^0)+m(\tilde{g}))$	ATLAS-CONF-2013-062
	$\tilde{g}\tilde{g}, \tilde{g}\to qq(\ell\ell/\ell\nu/\nu\nu)\tilde{\chi}_1^0$	2 e,μ	0-3 jets	-	20.3	$\tilde{g}$ 1.12 TeV	$m(\tilde{\chi}_1^0)=0$ GeV	ATLAS-CONF-2013-089
	GMSB ($\tilde{\ell}$ NLSP)	2 e,μ	2-4 jets	Yes	4.7	$\tilde{g}$ 1.24 TeV	$\tan\beta<15$	1208.4688
	GMSB ($\tilde{\ell}$ NLSP)	1-2 τ	0-2 jets	Yes	20.7	$\tilde{g}$ 1.4 TeV	$\tan\beta>18$	ATLAS-CONF-2013-026
	GGM (bino NLSP)	2 γ	-	Yes	4.8	$\tilde{g}$ 1.07 TeV	$m(\tilde{\chi}_1^0)>50$ GeV	1209.0753
	GGM (wino NLSP)	1 $e,\mu+\gamma$	-	Yes	4.8	$\tilde{g}$ 619 GeV	$m(\tilde{\chi}_1^0)>50$ GeV	ATLAS-CONF-2012-144
	GGM (higgsino-bino NLSP)	γ	1 b	Yes	4.8	$\tilde{g}$ 900 GeV	$m(\tilde{\chi}_1^0)>220$ GeV	1211.1167
	GGM (higgsino NLSP)	2 e,μ (Z)	0-3 jets	Yes	5.8	$\tilde{g}$ 690 GeV	$m(\tilde{H})>200$ GeV	ATLAS-CONF-2012-152
	Gravitino LSP	0	mono-jet	Yes	10.5	$F^{1/2}$ scale 645 GeV	$m(\tilde{g})>10^{-4}$ eV	ATLAS-CONF-2012-147
3^{rd} gen. $\tilde{g}$ med.	$\tilde{g}\to b\bar{b}\tilde{\chi}_1^0$	0	3 b	Yes	20.1	$\tilde{g}$ 1.2 TeV	$m(\tilde{\chi}_1^0)<600$ GeV	ATLAS-CONF-2013-061
	$\tilde{g}\to t\bar{t}\tilde{\chi}_1^0$	0	7-10 jets	Yes	20.3	$\tilde{g}$ 1.1 TeV	$m(\tilde{\chi}_1^0)<350$ GeV	1308.1841
	$\tilde{g}\to t\bar{t}\tilde{\chi}_1^0$	0-1 e,μ	3 b	Yes	20.1	$\tilde{g}$ 1.34 TeV	$m(\tilde{\chi}_1^0)<400$ GeV	ATLAS-CONF-2013-061
	$\tilde{g}\to b\bar{t}\tilde{\chi}_1^+$	0-1 e,μ	3 b	Yes	20.1	$\tilde{g}$ 1.3 TeV	$m(\tilde{\chi}_1^0)<300$ GeV	ATLAS-CONF-2013-061
3^{rd} gen. squarks direct production	$\tilde{b}_1\tilde{b}_1, \tilde{b}_1\to b\tilde{\chi}_1^0$	0	2 b	Yes	20.1	$\tilde{b}_1$ 100-620 GeV	$m(\tilde{\chi}_1^0)<90$ GeV	1308.2631
	$\tilde{b}_1\tilde{b}_1, \tilde{b}_1\to t\tilde{\chi}_1^{\pm}$	2 e,μ (SS)	0-3 b	Yes	20.7	$\tilde{b}_1$ 275-430 GeV	$m(\tilde{\chi}_1^{\pm})=2\,m(\tilde{\chi}_1^0)$	ATLAS-CONF-2013-007
	$\tilde{t}_1\tilde{t}_1$(light), $\tilde{t}_1\to b\tilde{\chi}_1^{\pm}$	1-2 e,μ	1-2 b	Yes	4.7	$\tilde{t}_1$ 110-167 GeV	$m(\tilde{\chi}_1^0)=55$ GeV	1208.4305, 1209.2102
	$\tilde{t}_1\tilde{t}_1$(light), $\tilde{t}_1\to Wb\tilde{\chi}_1^0$	2 e,μ	0-2 jets	Yes	20.3	$\tilde{t}_1$ 130-220 GeV	$m(\tilde{\chi}_1^0)=m(\tilde{t}_1)-m(W)-50$ GeV, $m(\tilde{t}_1)<<m(\tilde{\chi}_1^{\pm})$	ATLAS-CONF-2013-048
	$\tilde{t}_1\tilde{t}_1$(medium), $\tilde{t}_1\to t\tilde{\chi}_1^0$	2 e,μ	2 jets	Yes	20.3	$\tilde{t}_1$ 225-525 GeV	$m(\tilde{\chi}_1^0)=0$ GeV	ATLAS-CONF-2013-065
	$\tilde{t}_1\tilde{t}_1$(medium), $\tilde{t}_1\to b\tilde{\chi}_1^{\pm}$	0	2 b	Yes	20.1	$\tilde{t}_1$ 150-580 GeV	$m(\tilde{\chi}_1^0)<200$ GeV, $m(\tilde{\chi}_1^{\pm})-m(\tilde{\chi}_1^0)=5$ GeV	1308.2631
	$\tilde{t}_1\tilde{t}_1$(heavy), $\tilde{t}_1\to t\tilde{\chi}_1^0$	1 e,μ	1 b	Yes	20.7	$\tilde{t}_1$ 200-610 GeV	$m(\tilde{\chi}_1^0)=0$ GeV	ATLAS-CONF-2013-037
	$\tilde{t}_1\tilde{t}_1$(heavy), $\tilde{t}_1\to t\tilde{\chi}_1^0$	0	2 b	Yes	20.5	$\tilde{t}_1$ 320-660 GeV	$m(\tilde{\chi}_1^0)=0$ GeV	ATLAS-CONF-2013-024
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1\to c\tilde{\chi}_1^0$	0	mono-jet/c-tag	Yes	20.3	$\tilde{t}_1$ 90-200 GeV	$m(\tilde{t}_1)-m(\tilde{\chi}_1^0)<85$ GeV	ATLAS-CONF-2013-068
	$\tilde{t}_1\tilde{t}_1$(natural GMSB)	2 e,μ (Z)	1 b	Yes	20.7	$\tilde{t}_1$ 500 GeV	$m(\tilde{\chi}_1^0)>150$ GeV	ATLAS-CONF-2013-025
	$\tilde{t}_2\tilde{t}_2, \tilde{t}_2\to\tilde{t}_1+Z$	3 e,μ (Z)	1 b	Yes	20.7	$\tilde{t}_2$ 271-520 GeV	$m(\tilde{t}_1)=m(\tilde{\chi}_1^0)+180$ GeV	ATLAS-CONF-2013-025
EW direct	$\tilde{\ell}_{L,R}\tilde{\ell}_{L,R}, \tilde{\ell}\to\ell\tilde{\chi}_1^0$	2 e,μ	0	Yes	20.3	$\tilde{\ell}$ 85-315 GeV	$m(\tilde{\chi}_1^0)=0$ GeV	ATLAS-CONF-2013-049
	$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to\tilde{\ell}\nu(\ell\tilde{\nu})$	2 e,μ	0	Yes	20.3	$\tilde{\chi}_1^{\pm}$ 125-450 GeV	$m(\tilde{\chi}_1^0)=0$ GeV, $m(\tilde{\ell},\tilde{\nu})=0.5(m(\tilde{\chi}_1^{\pm})+m(\tilde{\chi}_1^0))$	ATLAS-CONF-2013-049
	$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to\tilde{\tau}\nu(\tau\tilde{\nu})$	2 τ	-	Yes	20.7	$\tilde{\chi}_1^{\pm}$ 180-330 GeV	$m(\tilde{\chi}_1^0)=0$ GeV, $m(\tilde{\tau},\tilde{\nu})=0.5(m(\tilde{\chi}_1^{\pm})+m(\tilde{\chi}_1^0))$	ATLAS-CONF-2013-028
	$\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0\to\tilde{\ell}_L\nu\tilde{\ell}_L\ell(\tilde{\nu}\nu), \ell\tilde{\nu}\tilde{\ell}_L\ell(\tilde{\nu}\nu)$	3 e,μ	0	Yes	20.7	$\tilde{\chi}_1^{\pm}, \tilde{\chi}_2^0$ 600 GeV	$m(\tilde{\chi}_1^{\pm})=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)=0$, $m(\tilde{\ell},\tilde{\nu})=0.5(m(\tilde{\chi}_1^{\pm})+m(\tilde{\chi}_1^0))$	ATLAS-CONF-2013-035
	$\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0\to W\tilde{\chi}_1^0 Z\tilde{\chi}_1^0$	3 e,μ	0	Yes	20.7	$\tilde{\chi}_1^{\pm}, \tilde{\chi}_2^0$ 315 GeV	$m(\tilde{\chi}_1^{\pm})=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)=0$, sleptons decoupled	ATLAS-CONF-2013-035
	$\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0\to W\tilde{\chi}_1^0 h\tilde{\chi}_1^0$	1 e,μ	2 b	Yes	20.3	$\tilde{\chi}_1^{\pm}, \tilde{\chi}_2^0$ 285 GeV	$m(\tilde{\chi}_1^{\pm})=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)=0$, sleptons decoupled	ATLAS-CONF-2013-093
Long-lived particles	Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^{\pm}$	Disapp. trk	1 jet	Yes	20.3	$\tilde{\chi}_1^{\pm}$ 270 GeV	$m(\tilde{\chi}_1^{\pm})-m(\tilde{\chi}_1^0)=160$ MeV, $\tau(\tilde{\chi}_1^{\pm})=0.2$ ns	ATLAS-CONF-2013-069
	Stable, stopped $\tilde{g}$ R-hadron	0	1-5 jets	Yes	22.9	$\tilde{g}$ 832 GeV	$m(\tilde{\chi}_1^0)=100$ GeV, 10 $\mu s<\tau(\tilde{g})<1000$ s	ATLAS-CONF-2013-057
	GMSB, stable $\tilde{\tau}, \tilde{\chi}_1^0\to\tilde{\tau}(\tilde{e},\tilde{\mu})+\tau(e,\mu)$	1-2 μ	-	-	15.9	$\tilde{\chi}_1^0$ 475 GeV	$10<\tan\beta<50$	ATLAS-CONF-2013-058
	GMSB, $\tilde{\chi}_1^0\to\gamma\tilde{G}$, long-lived $\tilde{\chi}_1^0$	2 γ	-	Yes	4.7	$\tilde{\chi}_1^0$ 230 GeV	$0.4<\tau(\tilde{\chi}_1^0)<2$ ns	1304.6310
	$\tilde{q}\tilde{q}, \tilde{\chi}_1^0\to qq\mu$ (RPV)	1 μ, displ. vtx	-	-	20.3	$\tilde{q}$ 1.0 TeV	$1.5<c\tau<156$ mm, BR$(\mu)=1$, $m(\tilde{\chi}_1^0)=108$ GeV	ATLAS-CONF-2013-092
RPV	LFV $pp\to\tilde{\nu}_\tau+X, \tilde{\nu}_\tau\to e+\mu$	2 e,μ	-	-	4.6	$\tilde{\nu}_\tau$ 1.61 TeV	$\lambda'_{311}=0.10$, $\lambda_{132}=0.05$	1212.1272
	LFV $pp\to\tilde{\nu}_\tau+X, \tilde{\nu}_\tau\to e(\mu)+\tau$	1 $e,\mu+\tau$	-	-	4.6	$\tilde{\nu}_\tau$ 1.1 TeV	$\lambda'_{311}=0.10$, $\lambda_{1(2)33}=0.05$	1212.1272
	Bilinear RPV CMSSM	1 e,μ	7 jets	Yes	4.7	$\tilde{q}, \tilde{g}$ 1.2 TeV	$m(\tilde{q})=m(\tilde{g})$, $c\tau_{LSP}<1$ mm	ATLAS-CONF-2012-140
	$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to W\tilde{\chi}_1^0, \tilde{\chi}_1^0\to ee\tilde{\nu}_\mu, e\mu\tilde{\nu}_e$	4 e,μ	-	Yes	20.7	$\tilde{\chi}_1^{\pm}$ 760 GeV	$m(\tilde{\chi}_1^0)>300$ GeV, $\lambda_{121}>0$	ATLAS-CONF-2013-036
	$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to W\tilde{\chi}_1^0, \tilde{\chi}_1^0\to\tau\tau\tilde{\nu}_e, e\tau\tilde{\nu}_\tau$	3 $e,\mu+\tau$	-	Yes	20.7	$\tilde{\chi}_1^{\pm}$ 350 GeV	$m(\tilde{\chi}_1^0)>80$ GeV, $\lambda_{133}>0$	ATLAS-CONF-2013-036
	$\tilde{g}\to qqq$	0	6-7 jets	-	20.3	$\tilde{g}$ 916 GeV	BR(t)=BR(b)=BR(c)=0%	ATLAS-CONF-2013-091
	$\tilde{g}\to\tilde{t}_1 t, \tilde{t}_1\to bs$	2 e,μ (SS)	0-3 b	Yes	20.7	$\tilde{g}$ 880 GeV		ATLAS-CONF-2013-007
Other	Scalar gluon pair, sgluon$\to q\bar{q}$	0	4 jets	-	4.6	sgluon 100-287 GeV	incl. limit from 1110.2693	1210.4826
	Scalar gluon pair, sgluon$\to t\bar{t}$	2 e,μ (SS)	1 b	Yes	14.3	sgluon 800 GeV		ATLAS-CONF-2013-051
	WIMP interaction (D5, Dirac χ)	0	mono-jet	Yes	10.5	M* scale 704 GeV	$m(\chi)<80$ GeV, limit of<687 GeV for D8	ATLAS-CONF-2012-147

$\sqrt{s} = 7$ TeV full data

$\sqrt{s} = 8$ TeV partial data

$\sqrt{s} = 8$ TeV full data

CURRENT LHC BOUNDS ON NEW PHYSICS PROVIDING Λ_{NP}

Extra dimensions: ADD, RS,
TC, Z', W'

$$\Lambda_{NP} > 1 - 3\,\mathrm{TeV}$$

Supersymmetry:

$$\Lambda_{NP} \sim 1\ \mathrm{TeV}$$

mass splitting
in the smultiplets

with $M_{\tilde{g}} > 1.2\,\mathrm{TeV} - 1.5\,\mathrm{TeV}$

But:

$$\tilde{t}_L,\, \tilde{t}_R, \tilde{b}_L, \tilde{H} \text{ light} \sim 600 - 700 \text{ GeV}$$

$$\tilde{B},\, \tilde{W},\, \tilde{b}_R, \tilde{Q}_{L\ 12},\, \tilde{U}_{R\ 12}, \tilde{D}_{R\ 12},\, \tilde{L}_i,\, \tilde{e}_i \text{ decoupled} > 1.5 \text{ TeV}$$

SUMMARY

- Superpartners heavy, and one may need to invoke next-next-to-minimal models

- But: supersymmetry is an unique theory, which offers consistent perturbative extension of the Standard Model
  to very-very high energy scales
  (assuming such an extension is needed - that is, if there exists any BSM physics)

- Fine-tuning in minimal susy models is large, but smaller than in the cut-off SM

- It is important to continue the search for supersymmetry, to see whether hierarchy and naturalness are really good guides in searches for new/deeper physics

- New avenues - flavour?

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