A Paradigm for Solving the QCD Zero Mode Problem in DLCQ

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The original motivation for applying the "front-form" approach of Dirac [1] to quantum chromodynamics was that it offered a transparent picture of hadrons in QCD arising from partonic excitations above a simple vacuum. However, it is now emerging that the "instant form" vacuum structure for generating spontaneous symmetry breaking is manifested in the front form in the singular infrared region $k^+ \equiv (k^0 + k^3)/\sqrt{2} \to 0$. Thus confinement-physics may arise from modes of small or vanishing k^+. The subsequent "zero mode problem", how to deal with such modes, has impeded progress. *Discretised* Light Cone Quantisation (DLCQ) defines a field theory on the light-cone such that it is *a priori* infrared regular: "space" is of compact longitudinal (x^-) length L and transverse length $L_\perp$. Bosonic fields are assigned periodic boundary conditions guaranteeing the standard Euler-Lagrange equations. The compact space leads to discrete Fourier momenta so that the zero mode of some generic bosonic field φ, $\overset{\circ}{\varphi} \equiv \langle \varphi \rangle_0 \equiv \int_{-L}^{+L} dx^- \varphi(x^-, x_\perp)/2L$, can be cleanly extracted from the other modes. At the very least this generates an *unambiguous* zero mode problem to be solved.

My early attempts suggest that to tackle the problem in QCD I face two problems from the outset: nonperturbative gauge-fixing and renormalisation. A class of models generated by *dimensional reduction* from higher to 1+1 dimensions avert these two impediments while still containing some of the structure of the original theory. The following analysis of zero modes arising in such theories gives a paradigm of how the "zero mode problem" can be solved in 3+1 QCD. What will emerge is a picture of how, even including zero modes in a consistent field-theoretic way, something like the original partonic picture of the infinite momentum frame can appear.

I begin with pure SU(2) glue in 2+1 dimensions which has the advantage of avoiding complications from the $(\mathbf{F}_{ij})^2$ term in the Lagrangian. Bold face here means a matrix in SU(2) colour space. Using the labels $\alpha(\beta) = +, -$ for light-cone Lorentz indices the Lagrangian density $\mathcal{L} = -(1/2)\mathrm{Tr}(\mathbf{F}^{\mu\nu}\mathbf{F}_{\mu\nu})$ decomposes into $\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\mathbf{F}^{\alpha\beta}\mathbf{F}_{\alpha\beta} + 2\mathbf{F}^{\alpha\perp}\mathbf{F}_{\alpha\perp})$. *Dimensional reduction* means assuming that all the fields of the problem are independent of the one transverse dimension: $\partial_\perp \mathbf{A}^\mu = 0$. In other words, I consider the subsector of zero modes with respect to the transverse coordinate. This is not even unphysical: in light-cone field theory, the *longest* transverse modes could give the largest scale structure in hadron wavefunctions of the *complete* theory. A DLCQ treatment ignoring zero modes has been given by [2]. Marking the distinction to standard two-dimensional QCD is the presence still in problem of the transverse gluon *component*. Identifying it with an adjoint scalar field $\mathbf{\Phi}$ elegantly formulates the problem: Defining the covariant derivative $\mathbf{D}^\alpha \equiv \partial^\alpha \mathbf{1} - ig[\mathbf{A}^\alpha, \cdot]$ the Lagrangian takes the form of 1+1 SU(2) gauge theory coupled to scalar adjoint matter:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\mathbf{F}^{\alpha\beta}\mathbf{F}_{\alpha\beta} - 2\mathbf{D}^\alpha \mathbf{\Phi} \mathbf{D}_\alpha \mathbf{\Phi}). \tag{1}$$

I make the following adjustment of notation: $(\mathbf{A}^+, \mathbf{A}^-) \equiv (\mathbf{V}, \mathbf{A})$. The equations of motion are the following

$$\mathbf{D}_\beta \mathbf{F}^{\beta\alpha} = g\mathbf{J}^\alpha, \text{ with } \mathbf{J}^\alpha = -i[\mathbf{\Phi}, \mathbf{D}^\alpha \mathbf{\Phi}], \text{ and } \mathbf{D}^\alpha \mathbf{D}_\alpha \mathbf{\Phi} = 0. \tag{2}$$

Introducing the matter currents $\mathbf{J}^\alpha$ simplifies the formalism enormously.

It is useful to introduce a colour helicity basis for the SU(2) matrices $\mathbf{A}^\mu = \tau^3 A_3^\mu + \tau^+ A_+^\mu + \tau^- A_-^\mu$, where $\tau^\pm \equiv (\tau^1 \pm i\tau^2)/\sqrt{2}$ and the τ^a are 1/2 the respective Pauli matrices. One can introduce a 'metric' such that these colour indices behave like light-cone Lorentz indices: $\tau^\pm = \tau_\mp$. The off-diagonal components are thus hermitian conjugates of each other. For the adjoint scalar field this decomposition means breaking the problem into a Hermitian scalar field φ_3 and a complex scalar field φ_-, with $\varphi_+ = \varphi_-^\dagger$.

Now it is an old story why the light-cone gauge $\mathbf{A}^+ = 0$ is not permissible, at least in the formulation with periodic boundary conditions [3]. The nearest admissible gauge is $\partial_- \mathbf{A}^+ = \partial_- \mathbf{V} = 0$ which leaves intact the zero mode of $\mathbf{A}^+$. The residual freedom with respect to x^- independent gauge transformations further permits rotation in colour space diagonalising the zero mode of $\mathbf{V}$. Thus $\mathbf{V} = v\tau^3$. This mode v is essentially related to the gauge-invariant Wilson loop around compact x^- space. I define the dimensionless quantity $z \equiv gvL/\pi$. There remains a trivial Gribov gauge fixing ambiguity: large gauge transformations which shift $z \to z+1$. We could choose, for example, to work in the first "fundamental modular region" [4] $0 < z < 1$.

In the language of the Dirac constraint procedure this gauge fixing renders "second class" the corresponding components of the Gauss law constraint equations, meaning one can implement them in the sense of strong quantum operator equations in the subsequent quantum theory. Alternately, these are the off-diagonal projections of the equation of motion, $-(\mathbf{D}_-)^2 \mathbf{A} = g\mathbf{J}^+$, namely

$$-(\partial_-)^2 A_3 = gJ_3^+, \; -(\partial_- \pm igv)^2 A_\pm = gJ_\pm^+. \tag{3}$$

The matter current components in the helicity basis will be given explicitly below. Note that the zero mode of the first of these equations gives a "first class" constraint, and thus is only implementable on physical states $\overset{\circ}{J_3^+} |\mathrm{phys}\rangle = 0$. This will be further discussed below.

Quantisation is canonical. The complex fields $\varphi_\pm$ have conjugate momenta

$$\pi^\pm = (\partial_- \mp igv)\varphi_\mp \tag{4}$$

with the equal x^+ commutators

$$[\varphi_-(x^-), \pi^-(y^-)] = [\varphi_+(x^-), \pi^+(y^-)] = \frac{i}{2}\delta_L(x^- - y^-). \tag{5}$$

In particular, $\overset{\circ}{\varphi}_\pm$ have conjugate momenta indicating they are *dynamical* field variables. In contrast, φ_3 has momentum $\pi^3 = \partial_-\varphi_3$ so $\overset{\circ}{\pi}{}^3 = 0$; $\overset{\circ}{\varphi}_3$ is not an independent field. For the *normal* modes of φ_3 we have the canonical commutator

$$[\varphi_3(x^-), \pi^3(y^-)] = \frac{i}{2}\bar{\delta}_L(x^- - y^-) \tag{6}$$

where the bar over the delta function denotes the absence of its zero mode piece, $1/2L$. The following mode expansions can thus be employed

$$\varphi_3(x^-) = \frac{a_0}{\sqrt{4\pi}} + \frac{1}{\sqrt{4\pi}}\sum_{n=1}^{\infty}(a_n w_n e^{-ik_n x^-} + a_n^\dagger w_n e^{ik_n x^-}) \tag{7}$$

$$\varphi_-(x^-) = \frac{u_0}{\sqrt{4\pi}}b_0 + \frac{1}{\sqrt{4\pi}}\sum_{n=1}^{\infty}(b_n u_n e^{-ik_n x^-} + d_n^\dagger v_n e^{ik_n x^-}) \tag{8}$$

with $k_n = n\pi/L$ the discrete momenta. The commutators $[a_n, a_m^\dagger] = \delta_{nm}$ apply for the hermitian field normal modes and $w_n = 1/\sqrt{n}$. For the complex field components the nonvanishing commutators are: $[b_n, b_m^\dagger] = [d_n, d_m^\dagger] = \delta_{m,n}$, and $[b_0, b_0^\dagger] = 1$. The coefficient functions are $u_n = 1/\sqrt{|n+z|}$ and $v_n = 1/\sqrt{|n-z|}$. The only other dynamical mode in the theory is the mode v, with conjugate momentum $p = 2L\partial_+ v$. Their commutator is $[v, p] = i$. Its analogue in 1+1 dimensional pure glue coupled to external sources was recently explored in [3]. For the moment I regard it, or z, as a background field.

With these mode expansions the condition on physical states becomes (after a finite subtraction of a c-number)

$$(b_0^\dagger b_0 + \sum_{n=1}^{\infty} b_n^\dagger b_n - \sum_{n=1}^{\infty} d_n^\dagger d_n)|\text{phys}\rangle = 0. \tag{9}$$

Thus physical states are those with the same number of "d_n" particles as "b_0" and normal mode "b_n" particles.

Taking the zero mode projection of the diagonal part of $\mathbf{D}^\alpha\mathbf{D}_\alpha\mathbf{\Phi} = 0$ generates a constraint equation of the form

$$\langle \varphi_+ \frac{1}{(\partial_- + igv)} J_-^+ - \varphi_- \frac{1}{(\partial_- - igv)} J_+^+ \rangle_{0,s} = 0. \tag{10}$$

The subscript s indicates that we have to symmetrise operator products due to possibly noncommuting operators, guaranteeing hermiticity. The components of the matter currents are

$$J_3^+ = \frac{1}{i}(\varphi_+\pi_- - \varphi_-\pi_+)_s \text{ and } J_+^+ = \frac{1}{i}(\varphi_3\pi_+ - \varphi_+\pi_3)_s \tag{11}$$

with $J_-^+ = (J_+^+)^\dagger$. When the mode expansions and the expressions for the currents are inserted, the constraint has the final form:

$$\begin{aligned}
&\sum_{n=0}^{\infty} u_n^2 (b_n^\dagger b_n a_0)_s + \sum_{n=1}^{\infty} v_n^2 (d_n^\dagger d_n a_0)_s = \\
&-\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\sum_{p=1}^{\infty} \delta_{n+m}^p (a_n^\dagger b_m^\dagger b_p + a_n b_m b_p^\dagger)[(\frac{w_n}{u_m} - \frac{u_m}{w_n})u_p^3 + (\frac{w_n}{u_p} + \frac{u_p}{w_n})u_m^3] \\
&+\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\sum_{p=1}^{\infty} \delta_{n+m}^p (a_p^\dagger b_m d_n + a_p b_m^\dagger d_n^\dagger)[(\frac{w_p}{v_n} + \frac{v_n}{w_p})u_m^3 + (\frac{w_p}{u_m} + \frac{u_m}{w_p})v_n^3] \\
&-\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty} \delta_{n+m}^p (a_n^\dagger d_m^\dagger d_p + a_n d_m d_p^\dagger)[(\frac{w_n}{v_m} - \frac{v_m}{w_n})v_p^3 + (\frac{w_n}{v_p} + \frac{v_p}{w_n})v_m^3].
\end{aligned} \tag{12}$$

We see that, at the very least, the mode a_0 generates off-diagonal interactions between the true partons of the theory.

The normal mode operators b_n and d_n should give the partonic interpretation to the theory. The rest need to be "integrated" out. How then can we imagine recovering a unique ground state with b_0, a_0 and z modes around?

The b_0 mode is the only true zero Fock mode in the problem. It can and does mix with the Fock vacuum in formally physical states. In the way the theory has been cast, this mode resembles the QED_{1+1} fermion "zero mode" of [5]. Computation of the spectrum there shows it gives a state whose invariant mass rises steeply in the continuum limit as coupling increases. Similar behaviour here would mean this zero mode, though necessary for other rich aspects of the physics, does not impede the recovery of a unique ground state.

The constrained mode a_0 of course does not impair vacuum triviality. "Integrating" it out here means solving the operator valued constraint equation for it and substituting in the Hamiltonian P^-. This will lead to new interactions in the Hamiltonian between the true partons of the theory. The linear nature of the constraint here means no symmetry breaking effects can be tied to this mode *in this particular theory.* How does one go about solving such a constraint equation? The work of [6] on φ_{1+1}^4 essentially has lead the way for this: a Fock space truncation is a successful nonperturbative approximation scheme. Thus the technology exists to solve operator constraints like (12) for the matrix elements of a_0.

The gauge mode z is the real subtlety. One can write the schematically for the complete Hamiltonian $P^- = -d^2/dz^2 + W_{\rm Fock}(z)$, where W is defined by the Fock sector of the theory. A tedious computation of $\langle 0|W_{\rm Fock}(z)|0\rangle$ shows it to have an absolute minimum at $z = 1/2$. This defines the true vacuum of theory above which one can quantise the quantum mode z by perturbing about this minimum. This spectrum itself has a characteristic mass gap proportional to L and is superimposed on the Fock space spectrum. Nonetheless, a unique vacuum is recovered, and it is *not* that given by $z = 0$.

The general conclusion is that DLCQ is a method that cleanly identifies the "zero mode problem" of QCD and enables a solution. Though much concrete computation needs to be done we can foresee, without brutalising canonical QCD, a picture of rich excitations built on top of a single ground state emerging.

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