High-energy scattering with flavour exchange and properties of reggeon interactions in QCD

Roland Kirschner
DESY - Institut für Hochenergiephysik
Platanenallee 6, D-15735 Zeuthen, Germany

Abstract

We describe the properties of the interaction between reggeized gluons and quarks in QCD.

1. Introduction

In the case of vacuum quantum numbers in the t-channel gluon exchange dominates the exchange of quarks in the perturbative Regge asymptotics. In particular the small x behaviour of the structure functions is dominated by gluons. HERA experiments [1] started to explore the x-region down to $x \sim 10^{-4}$ and the results confirm the predictions [2] of the perturbative leading logarithmic calculations. Now one has to improve the theoretical results by going essentially beyond the leading logarithmic approximation in order to get a quantitative understanding of the saturation of the steep rise of the structure functions, which is expected from simple unitarity arguments. In the talks by J. Bartels and L.N. Lipatov several aspects of this question and recent results have been discussed. For a review see also Ref. 3. The essential problem to solve is to evaluate the contributions from the exchange of more than two reggeized gluons.

In the case of flavour quantum numbers in the t-channel, for example in the flavour non-singlet (valence) part of the structure functions, the perturbative contributions involve the exchange of an quark and an antiquark of different flavours. An experimental test of the predictions of the perturbative QCD Regge asymptotics is more involved in this case. On the other hand the Regge singularities obtained in the perturbative calculations should be related to the meson Regge trajectories at relatively large $|t|$. At present the methods do not allow to investigate the truely non-perturbative Regge behaviour and to obtain informations about the mesons on the trajectories. But we expect that the perturbative results are not too far from the truth.

In this sense the physical questions of the Regge asymptotics and of the light-front approach to hadrons are closely related. From the talk by L. Susskind and from the discussions we learned that the wee (low x) partons are essential in the light-front approach as well as here.

There are also similarities in the methods, which make the exchange of experience worthwile. Since many years it is standard to use the Sudakov representation for momenta and other vectors in analysing the leading contribution of graphs. The leading contributions to the Regge asymptotics show a clear separatin of longitudinal and transverse subspaces. The leading $\ln s$ arises from the longitudinal momentum integration. Non-trivial integrals appear in the transverse momenta.

In the previous talk L. Szymanowski has explained how the multi-Regge effective action can be obtained from the original QCD action in the light-cone gauge. As in the light-front approach the idea of integrating out certain modes is crucial here, only here the region of modes is chosen according to the multi-Regge kinamatics.

In the sum over all diagrams the exchanged fermions as well as the exchanged gluons become reggeons. Their trajectories are denoted by $C_2\alpha_F(\kappa)$ and $1 + N\alpha_G(\kappa)$, respectively.

The exchanged reggeons interact pairwise by emitting and absorbing gluons or quarks. The effective vertices describing the emission of these s-channel quarks and gluons from the exchanged reggeons can be taken from the effective action presented in the previous talk. They can be obtained as a sum of a projection of the original triple-gluon or quark-gluon vertex plus the contribution of production by bremsstrahlung. The propagator in the bremsstrahlung contributions correspond to the heavy modes integrated out in the effective action.

2. Reggeon interactions

The contribution of the exchange of r reggeons to the scattering amplitude $AB \to A'B'$ has the structure $\Phi_A^{(r)} \otimes f^{(r)} \otimes \Phi_B^{(r)}$, where $\Phi^{(r)}$ are impact factors describing the coupling of the external scattering particles to the r reggeons and $f^{(r)}$ is the reggeon Green function. We restrict ourselves to a fixed number of reggeons r ; compare the talk by J. Bartels for questions of changing reggeon numbers. We change from the amplitude to the partial wave. Besides of the angular momentum variable ω $f^{(r)}$ depends on $2r$ transverse momenta $\kappa_\ell, \kappa'_\ell$ obeying $q = \sum \kappa_\ell = \sum \kappa'_\ell$, with q being the momentum transfer. $\otimes$ denotes a r fold transverse momentum integral with the corresponding δ-function of momentum conservation.

In the generalized leading logarithmic approximation $f^{(r)}$ is the sum of all contributions with pairwise intertactions of the r reggeons by exchanging s-channel quarks or gluons. The bare interaction kernels are obtained by combining two effective vertices with the appropriate propagators. They have (in most cases) infrared divergencies, which can be attributed to the s-channel gluon propogator at vanishing transverse momentum. On the other hand the exchanged reggeons give rise to a propagator with an angular momentum part of the form $[\omega - \sum \alpha_\ell(\kappa_\ell)]^{-1}$ involving the trajectories, which are infrared divergent as well. In the case of an overall colour singlet state in the t-channel the infared divergencies cancel and it is convenient to combine the $\alpha_\ell(k_\ell)$ with the bare kernels in such a way that the result defines a finite operator. We list the resulting kernels [4] for the interaction of two reggeized gluons (GG), two reggeized fermions of the same helicity (FF) and of opposite helicities (F $\overline{\mathrm{F}}$), one reggeized gluon and one reggeized fermion. In the latter case there are two contributions, one with s-channel gluon exchange (GF) and one with s-channel fermion exchange (G/F).

$$
\begin{aligned}
\mathcal{H}_{GG} &= |\kappa_1-\kappa_1'|^{-2}\left(\frac{\kappa_1\kappa_2^*}{\kappa_1'\kappa_2'^*}+\frac{\kappa_1^*\kappa_2}{\kappa_1'^*\kappa_2'}\right)-(\alpha_G(\kappa_1)+\alpha_G(\kappa_2))\delta(\kappa_1-\kappa_1'),\\
\mathcal{H}^{(\omega)}_{F\overline{F}} &= |\kappa_1-\kappa_1'|^{-2}\left(\left|\frac{\kappa_1}{\kappa_1'}\right|^{\omega}+\left|\frac{\kappa_1'}{\kappa_1}\right|^{\omega}\frac{\kappa_1^*\kappa_2}{\kappa_1'^*\kappa_2'}\right)-(\alpha_F(\kappa_1)+\alpha_F^*(\kappa_2))\delta(\kappa_1-\kappa_1'),\\
\mathcal{H}_{FG} &= |\kappa_1-\kappa_1'|^{-2}\left(\frac{\kappa_1^*\kappa_2}{\kappa_1'^*\kappa_2'}+\frac{\kappa_2^*}{\kappa_2'^*}\right)-(\alpha_F(\kappa_1)+\alpha_G(\kappa_2))\delta(\kappa_1-\kappa_1'),\\
\mathcal{H}_{FF} &= |\kappa_1-\kappa_1'|^{-2}\left(\frac{\kappa_2^*}{\kappa_2'^*}+\frac{\kappa_1^*}{\kappa_1'^*}\right)-(\alpha_F(\kappa_1)+\alpha_F(\kappa_2))\delta(\kappa_1-\kappa_1'),\\
\mathcal{G}_{FG} &= |\kappa_1-\kappa_1'|^{-2}\left(\frac{\kappa_1\kappa_2^*}{\kappa_1'\kappa_2'^*}-\frac{\kappa_2^*}{\kappa_2'^*}\right). \qquad (1)
\end{aligned}
$$

Usually the logitudinal momentum integration is trivial, leading just to a factor $\frac{1}{\omega}$, if the partial wave (Mellin) representation is used. In the case of fermions with opposite helicities (F $\overline{\mathrm{F}}$) more care about the kinematic limits in this integration has to be applied [5]. In this case a further logarithmic contribution arises in the transverse momentum integral from the region where the ratio of transverse momenta of the reggeon at the effective vertex is not of order 1. The result is represented by the ω-dependent factors in the corresponding kernel above.

3. Operators in the impact parameter space

The symmetry properties of the two-reggeon interactions (1) become more transparent in the impact parameter representation. After Fourier transformation the reggeon Green function $f^{(r)}$ depends besides of ω on the two-dimensional impact parameters $x_\ell, x_\ell', \ell = 1, ..., r$. The two-reggeon interaction (e.g. between the reggeons with label 1 and 2) are represented by operators which are composed out of the operators of differentiation with repect to x_1, x_2 and of multiplication by x_1, x_2. The operators can be looked at as two-body light front hamiltonians acting trivially in the longitudinal (light-cone) coordinates. The resulting expressions for the operators can be decomposed into parts involving either holomorphic ($\partial_1, \partial_2, x_1, x_2$) or anti-holomorphic ($\partial_1^*, \partial_2^*, x_1^*, x_2^*$) operators.

$$
\begin{aligned}
\hat{\mathcal{H}}_{GG} = H_G + H_G^*, \quad \hat{\mathcal{H}}^{(\omega)}_{F\overline{F}} &= H_F^{(\omega)} + P_{12}H_F^{(\omega)*}P_{12}, \quad \hat{\mathcal{H}}_{FF} = H_G + H_{\tilde{F}}^*,\\
\hat{\mathcal{H}}_{FG} &= H_G + P_{12}H_F^{(0)*}P_{12}, \quad \hat{\mathcal{G}}_{FG} = P_{12}G^*_{F/G}P_{12}. \qquad (2)
\end{aligned}
$$

P_{12} denotes the operator of permutation of x_1 and x_2.

The holomorphic operators are [4]

$$
\begin{aligned}
H_G &= 2\psi(1) - \partial_1^{-1}\ \ln x_{12}\ \partial_1 - \partial_2^{-1}\ \ln x_{12}\ \partial_2 - \ln\partial_1 \ - \ln\partial_2,\\
H_F^{(\omega)} &= 2\psi(1) - \partial_1^{-1+\omega/2}\ \ln x_{12}\ \partial_1^{1-\omega/2} - \ \ln\partial_1 - \partial_2^{-\omega/2}\ \ln x_{12}\ \partial_2^{\omega/2}\ - \ \ln\partial_2,
\end{aligned}
$$

$$H_{\tilde{F}} = 2\psi(1) - 2\ln x_{12} - \ln\partial_1 - \ln\partial_2, \quad G_{F/G} = (x_{12}\partial_1)^{-1}, \tag{3}$$

Here $x_{12} = x_1 - x_2$. With these impact parameter operators the equation summing all contributions becomes reminicent to the one of r-body quantum mechanics with pair interactions given by the unconventional hamiltonians $-\mathcal{H}_{12}$ (2). In the multi-reggeon case $r > 3$ the gauge group part of the interaction breaks in general the factorization. However in the limit of large N the holomorphic factorization holds. Then the problem is actually reduced to r-body quantum mechanics in one-dimensional space.

By applying the commutation relations between ∂ and x and some operator identities one can represent the holomorphic operators in several forms. In this way one finds simple transformation properties with respect to the conformal inversion $x \to \frac{1}{x}$ from which one learns that the gluonic reggeons correspond to conformal operators of scaling dimension $\delta = 0$ and of conformal spin $s = 0$ whereas the fermionic reggeon operators have scaling dimension $\delta = \frac{1}{2}$ and spin $s = \pm\frac{1}{2}$ in dependence of the fermion helicity. The conformal symmetry becomes explicite in the representation of the operators as functions of the Casimir operator of the holomorphic linear conformal transformations acting on x_1, x_2.

$$C_{\Delta_1\Delta_2} = x_{12}^2\partial_1\partial_2 + 2x_{12}(\Delta_1\partial_2 - \Delta_2\partial_1) + (\Delta_1 + \Delta_2)(1 - \Delta_1 - \Delta_2). \tag{4}$$

Here $\Delta = \frac{1}{2}(\delta + s)$. The operators are given in terms of the functions

$$\chi_\lambda(z) = \sum_{\ell=0}^{\infty}\left(\frac{2(\ell+\lambda)+1}{(\ell+\lambda)(\ell+\lambda+1) - z} - \frac{2}{\ell+1}\right) \tag{5}$$

in the following form ($A = G, F, \tilde{F}$)

$$H_A = \frac{1}{2}\left(\chi_{\lambda_A}(C_{\Delta_1,\Delta_2}) + (\chi_{-\lambda_A}(C_{\Delta_1,\Delta_2})\right), \quad P_{12}G_{F/G} = [C_{0,1/2} - \frac{1}{4}]^{-1/2}. \tag{6}$$

λ_A depends on the type of the interaction as follows, $\lambda_G = \lambda_F = 0, \lambda_{\tilde{F}}^{(\omega)} = \frac{1-\omega}{2}$.

The problem of r-reggeon exchange at large N can be represented in the form of a one-dimensional lattice spin model with r sites. Instead of usual $SU(2)$ spins one has representations of the holomorphoic linear conformal transformation ($SL(2,R)$) with scaling dimensions δ and conformal spin s. Among all nearest neighbour interactions there are unique hamiltonians (depending on δ and s at the neighbouring sites) for which the system is completely integrable. In the case of only gluons it is known [6,7] that this is just the operator H_G (3) derived from QCD. We expect that this generalizes to the other cases. This is obvious for $H_{\tilde{F}}$ and requires still some mathematical investigation for the cases of reggeons of different types (i.e. if $\lambda_A \neq 0$ as for $H_F^{(\omega)}$).

These theoretical results are essential steps in going beyond the leading logarithmic approximation and to calculate the corrections for unitarization. This is needed for calculating from QCD how the preasymptotics of the leading logarithmic approximation observed presently at

$x \sim 10^{-4}$ gradually turns over at still smaller values of x into the true asymptotics matching the Froissart bound.

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References

1. I. Abt et al., H1 Collab., Nucl. Phys. **B407** , 515 (1993);
M. Derrick et al., Zeus Collab., Phys. Lett. **B316**, 412 (1993).
2. V.S. Fadin, E.A. Kuraev and L.N. Lipatov, Phys. Lett. **60B** , 50 (1975);
ZhETF **71**, 840 (1976); *ibid* **72**, 377 (1977);
Y.Y. Balitski and L.N. Lipatov, Sov. J. Nucl. Phys. **28**, 882 (1978).
3. R. Kirschner, *Low x physics and perturbative Regge asymptotics*, (hep-ph 9408258), to appear as DESY preprint and in the proceedings of the Tennessee Intern. Symp. on Radiative Corrections 1994.
4. R. Kirschner, *Reggeon interactions in perturbative QCD*, (hep-th-9407085), to appear as DESY preprint and in Zeitschrift f. Phys. **C**.
5. R. Kirschner, "Regge asymptotics of scattering with flavour exchange in QCD", preprint DESY 94-090.
6. L.N. Lipatov, *High-energy asymptotics of multi-colour QCD ...*, Padova preprint DFPD 93/TH/70 (1993).
7. L.D. Faddeev and G.P. Korchemski, *High-energy QCD as a completely integrable model*, Stony Brook preprint ITP-SB-94-14.