

# Relativistic Nuclear Physics

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## Abstract

We quote arguments from discussions about relativistic nuclear physics for a claim that the light-front Hamiltonian approach and the similarity renormalization scheme may provide a missing element in a theory of baryons and mesons. The key point is that the meson-baryon vertex form factors can be correlated with the number and type of interaction terms in the Hamiltonian, in agreement with quantum mechanics and special relativity. No attempt is made to quote extensive literature on relativistic nuclear physics.

## 1. Introduction

Volodia Karmanov and Jean Mathiot have presented new results for the deuteron electromagnetic structure using some light-front wave functions and current operators.<sup>1</sup> These results may lead to a better understanding of how the deuteron absorbs virtual photons. At momentum transfers of the order of 0.5 GeV one expects the relativistic components of the wave functions to become important. The expectation is that measurements at CEBAF will see these components. Many other relativistic effects in nucleon-meson dynamics will be measured at CEBAF. This is the realm of relativistic nuclear physics in the few-body case and a potentially interesting area of application for the light-front Hamiltonian approach combined with the similarity renormalization group techniques. Relativistic hadron-nucleus or heavy-ion collisions were only briefly mentioned during the discussion.

Relativistic nuclear physics (RNP) is in a situation opposite to physics of quarks and gluons (QCD). In RNP, there are models of NN and  $\pi$ N interactions that can reproduce data with remarkable accuracy but there is a shortage of a unified dynamical scheme that could apply to the whole range of processes involving nucleons and mesons, for example, when energies cross the pion threshold and pions begin to move with large velocities. In QCD, the high energy limit is governed by asymptotic freedom that leads to a perturbative theory of hard processes, but the theory does not provide much insight into the low energy hadronic interactions that are most important in RNP.

## 2. Vertex form factors

In RNP, nucleons emit and absorb several mesons,  $\pi$ ,  $\rho$ ,  $\omega$  and others, and one often considers a  $\sigma$ -meson that is an effective degree of freedom which mediates interactions that involve more than one pion at a time. The interaction strength is modulated by vertex form factors. These factors are thought of as representing complicated structure of the interacting hadrons but their major role is that they regulate divergences that otherwise would make the models

meaningless. One finds strong dependence of the model results on the cutoff parameters. Special values of the cutoff have to be chosen in order to bring the results close to physical data. The corresponding coupling constants are large. The cutoff parameters are on the order of one to several nucleon masses depending on the kind of fit that one performs. Thus, two features appear. One is that the mesons, especially pions, may have momenta much larger than their masses. This is a typical situation and to our best knowledge there is no relativistic Hamiltonian theory that could handle this situation in a systematic way. The second feature is that the interaction terms may represent physical interactions of mesons and nucleons if and only if the special cutoff parameters are chosen and this means that the cutoffs are related to the number and kind of interaction terms that are present in the Hamiltonian. Both features indicate a need for a theoretical scheme for constructing relativistic Hamiltonians for strongly interacting particles. We need some theory to gain control (1) on the vertex regulators where they differ from unity, (2) on the intermediate states with mesons that are created and destroyed in the interactions and produce corrections to the vertices, masses and currents, and (3) on Lorentz contraction, delay and off-shell effects.

The belief that the meson-nucleon vertex form factors in RNP represent the composite nature of the hadrons that are built from quarks and gluons puts the blame for lack of a systematic theory on QCD. However, the same problems are typical to quark and gluon structure of hadrons as well. Constituent quarks are presumably soft composite objects, their interactions are not known precisely and are approximated by nonrelativistic models while the quark momenta can be large in comparison to their masses. Exact QCD is not able to tell us at the moment what Hamiltonian should be considered for relativistic mesons interacting with nucleons.<sup>2</sup>

### 3. Similarity factors in Hamiltonians

The similarity renormalization scheme for Hamiltonians is defined in Ref. 3. The main feature that makes the scheme potentially useful for applications to RNP is that the scheme is based on the calculation of an effective Hamiltonian which contains a new factor. Namely, interaction terms in the effective Hamiltonian are multiplied by regulating factors that correspond to the vertex form factors from RNP. The similarity renormalization group tells us how to calculate an effective Hamiltonian for one value of the cutoff parameter in the vertices from the knowledge of another effective Hamiltonian at another value of the cutoff. The Hamiltonians will differ not only by the different values of the cutoffs but they will also contain different interaction terms and the strength of different terms will depend on the cutoff. Once the number of Fock sectors is limited one can perform the similarity transformation using computers. It will be very hard to do precise calculations but, one should be able to find out what is the structure of terms required when the cutoff parameter is changed. Then, the cutoff dependence will be largely removed by playing with the new terms and one will start focusing the accuracy of the fits on the physical parameters instead of fitting the cutoff to 5 digits or so. One additional technique that helps in finding the correlation is weak coupling expansion. For example, one can start from Yukawa theory with a small coupling, or a  $\sigma$ -model, or more complicated theories with vector

particles (massive vector particles require a special treatment of the longitudinal polarization) and use perturbation theory. One can investigate the structure of effective Hamiltonians with small cutoffs of the order of the fermion masses, that correspond to the initial Hamiltonian with a very large cutoff and some counterterms, by expanding in the small coupling constant. Then, one can extrapolate to larger values of the coupling. The extrapolation may fail numerically if the coupling dependence of the Hamiltonians is not smooth but it will indicate what kind of terms have to be considered in order to be able to vary the cutoff. The cutoff is not to be fitted precisely and physical results should be stable against changing the cutoff because the introduction of the cutoff reflects our lack of knowledge and fitting the cutoff is merely pretending that suddenly the lack of knowledge disappeared. Another important aspect of the cutoff independence issue is special relativity. The cutoff makes the interactions nonlocal and spoils covariance. The counterterms to the cutoff dependence contain parts with unknown coefficients. One can fit the coefficients to restore covariance as far as possible. In other words, the wave functions corresponding to fixed eigenvalues and effective operators will contain parameters to be constrained by symmetry requirements. For example, without knowing the current operator for bound (or off-shell) nucleons and without choosing special values for the parameters in the wave functions, one can hardly discuss physics beyond the leading nonrelativistic domain.

#### **4. Arguments**

We summarize arguments in support of a claim that the dynamics of mesons and baryons can be systematically investigated using new light-front Hamiltonian techniques that were discussed during this workshop.

The first argument is that the meson-baryon vertex form factors can be treated as regulating factors in the similarity renormalization scheme<sup>3</sup> and the scheme can be used to correlate these factors with the interaction terms in the Hamiltonian. This is a very much desired feature because otherwise one is left with an overwhelming number of ambiguities in the theory. The usual fits to data allow arbitrary changes of parameters while they are, in fact, strongly correlated, and the key parameters need to be isolated and understood.

The second argument is that the light-front Hamiltonians are intrinsically relativistic. Their structure can manifestly obey seven Poincare symmetries instead of only six that are manifest in Hamiltonians of the instant form of dynamics. The remaining three dynamical symmetries are the translation in the light-front time, generated by the light-front Hamiltonian, and two transformations that change the light-front hyperplane in space-time. Therefore, obtaining fully covariant results is not easy, but it is not impossible when counterterms are introduced.<sup>4</sup>

The third argument is that the Hamiltonian approach offers a possibility to solve the strong coupling problem as an eigenvalue problem for the Hamiltonian. This is one of the dominant reasons for thinking that little alternative to the Hamiltonian approach is possible for strongly interacting systems. Many methods of diagonalization of matrices are available. A number of approximate variational methods can be used. And most importantly from our point of view, one can understand the quantum mechanical structure of the states in question. This understanding

provides insights that can be used in making suitable simplifications that are needed in practice.

The fourth argument is based on the fact that the light-front Hamiltonian approach may be applied to QCD.<sup>2</sup> Therefore, one can argue that if QCD provides a deeper understanding of the meson-baryon dynamics then, the Hamiltonian one will construct for mesons and baryons will be an effective Hamiltonian for QCD and it is not excluded that the derivation of the effective Hamiltonian for RNP from light-front QCD will be possible.

The fifth argument is that very simple models, analogous to Lee model, despite obvious deficiencies, are able to reproduce some properties of nucleons. We consider it to be surprising and encouraging, taking into account the primitive structure of the existing models.<sup>6</sup> We shall discuss an example in the next Section.

## 5. Model

Sasha Bylev, Jurek Przeszowski and myself have studied a simple model Hamiltonian that describes fermions interacting with bosons.<sup>6</sup> The model is an extension of the model from Refs. 4 and 5. The original Hamiltonian from Ref. 4 contains a cutoff and some counterterms required to obtain covariant results for the boson-fermion scattering. Note, that a special triviality problem in the model prevents sending the cutoff to infinity. We have extended that model by including isospin degrees of freedom and replacing sharp cutoffs by smooth regularization factors in the interaction terms. Isospin factors modify some renormalization conditions and the triviality bounds. Introduction of the smooth regularization leads to finite nucleon radii. With the sharp cutoff the radii are infinite, although the infinities are not visible in the form factors when momentum transfers are large in comparison to the inverse of the cutoff parameter. Charged bosons contribute to the physical fermion electromagnetic structure. We have compared results of the extended model with data for nucleons and pions.

The model fails to explain the pion-nucleon scattering data. This was no surprise, since the model is very simple. Nevertheless, and this was surprising to us, it is able to reproduce qualitatively some features of the electromagnetic form factors of nucleons. We have found that the most probable extension of the model that is required by data is the inclusion of two-meson states in the dynamics. The two-meson states are needed to include chiral-symmetry and obtain crossing and better angular dependence of the scattering amplitude. Then, the challenge will be to demonstrate that many meson states are not essential in explaining data.

The following table shows some of our results for nucleon form factors.<sup>6</sup> The radii and magnetic moments are given in the units of fm and Bohr magnetons, respectively. The first two columns were fitted to obtain the best possible results in the next columns. The rows correspond to the best fit for proton, for neutron and for both nucleons, respectively.

$\Lambda/m$	$\alpha_{phys}$	$\kappa^p$	$r_M^p$	$r_E^p$	$\kappa^n$	$r_M^n$	$(r_E^n)^2$
0.87	20.2	1.30	1.06	0.87	- 2.40	1.01	$-(0.85)^2$
3.38	4.2	0.37	0.52	0.41	- 1.15	0.54	$-(0.40)^2$
3.89	3.8	0.32	0.49	0.39	- 1.07	0.52	$-(0.37)^2$
experimental		1.79	0.84	0.84	- 1.91	0.84	$-(0.34)^2$

Note, that we have purposely not included the “experimental” value for the coupling constant since our model needs the two-meson states to be included before one will be able to define such a constant in the Hamiltonian by comparison with the pion-nucleon scattering data.

Our important observation in the model is that **the cutoff parameters and the coupling constants are close to the triviality bounds**. The bare couplings in the model are orders of magnitude larger than the physical coupling. Therefore, it is unlikely that a reasonable Hamiltonian for pions interacting with nucleons can be found without a well defined procedure that can control strong dynamical cancellations that lead to the physical picture.

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