# A Simple Confinement Mechanism for Light-Front Quantum Chromodynamics

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**Abstract**

Light-front field theory offers a scenario in which a constituent picture of hadrons may arise, but only if cutoffs that violate explicit covariance and gauge invariance are used. The perturbative renormalization group can be used to approximate the cutoff QCD hamiltonian, and even at lowest orders the resultant hamiltonian displays interesting phenomenological features. A general scheme for computing and using these hamiltonians is discussed and it is explicitly shown that a confining interaction appears when the hamiltonian is computed to second order.

## 1. Introduction and Basic Strategy

Quantum chromodynamics (QCD) is widely accepted as the fundamental theory of the strong interaction, but we are still unable to solve this theory in the low energy regime and obtain an accurate approximation for the structure of hadrons. This problem remains one of the most important unsolved problems in physics. The primary source of difficulty is that manifestly covariant and gauge invariant formulations of QCD yield a picture of hadrons as complicated many-body excitations on top of a complicated vacuum. In this picture we must solve coupled strongly-interacting many-body problems to obtain hadrons, in contrast to the simple few-body states found in the phenomenologically successful constituent quark model. Light-front field theory offers an alternative in which hadrons may be approximated as few-body bound states.

How can a constituent picture of hadrons arise in light-front field theory? The key is renormalization. [1] The issue is not whether hadrons contain arbitrarily many quarks and gluons added to a complicated vacuum. They do. The issue is whether the dynamical effects of almost all of the partons in a hadron and in the vacuum can be approximated by effective interactions in a light-front QCD hamiltonian that can be used to compute the dominant 'valence' structure of hadrons. To address this issue I would like the reader to consider how many-body states enter the structure of a hadron.

First, in a field theory with local interactions high energy many-body states do not decouple from low energy few-body states. If one uses perturbation theory to estimate the errors made when high energy states are simply removed by a cutoff, the answer is simple. The errors are infinite. For example, a single quark mixes perturbatively with high energy quark-gluon states, and the energy shift in second-order perturbation theory is infinite. This is an old story in field theory. We know that a regulator must be introduced, and if these high energy components are to be removed from the state so that it can be dominated by few-body components, this regulator must be a cutoff. However, after a cutoff is introduced, results are strongly dependent on the cutoff.

To remove cutoff dependence and properly account for the effects of high free-energy components, renormalization is required. The 'effective' hamiltonian becomes cutoff dependent, and must be designed to remove cutoff dependence from physical quantities such as masses and form factors. If the regulator does not violate the symmetries of the original theory, the resultant 'counterterms' will also respect these symmetries; and the only relevant and marginal operators that appear in the effective hamiltonian are canonical masses and couplings. Regulators that respect all the symmetries of QCD do not remove high energy states, and therefore do not yield a constituent picture. To obtain a constituent picture we are forced to use cutoffs that violate these symmetries. As a result, the effective QCD hamiltonian will contain operators that also violate these symmetries, forcing us to invent a renormalization procedure capable of identifying the extra relevant and marginal operators and fixing their strength. Wilson's renormalization group [2,3] suitably generalized for our problem offers the necessary tools.

Even if we can remove high free-energy components from physical states, and replace them with effective interactions, we are still faced with the fact that low free-energy many-body states do not typically decouple from low free-energy few-body states in QCD. The vacuum is supposed to be a complicated superposition dominated by low free-energy states. This second problem is what forces us to use light-front coordinates. In these coordinates we may be able to force the many-body states that appear as low free-energy states in equal time field theory to act like high energy states, so that the problem of replacing them with effective interactions resembles the renormalization problems we encounter when removing high free-energy states.

The principal observations that lead to this possibility are simple. First, the longitudinal momenta conjugate to the light-front longitudinal spatial coordinate are all positive,

$$p_i^+ \geq 0 \; . \tag{1}$$

Since every individual longitudinal momentum is positive, any state with many partons must contain some 'wee' partons; *i.e.*, partons with small longitudinal momentum fraction. The free energy of a particle in light-front coordinates is

$$p_i^- = \frac{\mathbf{p}_{\perp i}^2 + m^2}{p_i^+} \; . \tag{2}$$

This dispersion relation implies that a particle with small longitudinal momentum is a high energy particle, so that states containing wee partons are high energy states. Thus, we are left with the hope (possibly naive) that if we can successfully remove all high energy states in

QCD and replace them with effective interactions, we will be left with few-body states and a constituent picture of hadrons in light-front QCD.

It should be clear that the first step in this program is the calculation of effective interactions that result from the removal of high energy partons. This is the type of problem that led Wilson to develop his version of the renormalization group. For details I refer the reader to the excellent review articles by Wilson and Kogut [2,3], my recent article on light-front renormalization groups [4], and the recent articles by Głazek and Wilson [5] that develop the new similarity renormalization group. To identify the effective cutoff hamiltonian, we can directly study the cutoff dependence of the hamiltonian itself. The principal tool for this study is a renormalization group transformation. Given a hamiltonian with cutoff $\Lambda_0$, the transformation produces a new hamiltonian with cutoff $\Lambda_1$. These hamiltonians must produce equivalent results in some sense, and in the similarity renormalization group they are unitarily equivalent. By studying the properties of the transformation, we can try to identify a cutoff hamiltonian that is produced by an infinite number of transformations. If we find such a hamiltonian, by design it will produce the same results as a hamiltonian with an infinite cutoff; so it is a renormalized hamiltonian.

If the cutoff respects all the symmetries of the theory, the cutoff renormalized hamiltonian should be uniquely identified up to a few free masses and couplings, and irrelevant operators that can be ignored if the cutoff is sufficiently large. In massless QCD, a single running coupling will remain undetermined. On the other hand, if the cutoff violates these symmetries there will be many new candidate renormalized hamiltonians, because there are many relevant and marginal operators that violate the symmetries. Only one of these hamiltonians should restore these symmetries to physical quantities; so one strategy for finding the correct hamiltonian is to identify the relevant and marginal symmetry-breaking operators and tune their strengths to restore the symmetries. As far as I know, no exceptions to these rules have been found. The basic idea is that the complete set of symmetries determines the theory.

This procedure is confronted with serious problems in light-front QCD. First, in light-front field theory there are an infinite number of relevant and marginal operators because functions of longitudinal momenta appear in these operators. [1,4] This problem is due to the fact that longitudinal scaling is a boost, which is a Lorentz symmetry that cannot be broken. While boost invariance should be restored by only one choice of these functions, apparent ambiguities arise at finite orders of perturbation theory. The second problem is unique to non-abelian gauge theories. Many gauge-variant Green's functions are infrared divergent in QCD, and it is difficult to invent a scheme that can produce all the required counterterms without computing such Green's functions (*e.g.*, the quark-gluon vertex) at an intermediate stage. Wilson and I have devised *coupling coherence* to circumvent these problems. [6]

I refer to reader to the literature for details on coupling coherence, [4,6,7] and will provide only a sketch. The basic idea is that only the canonical masses and couplings should be independent functions of the cutoff, if the cutoff free hamiltonian respects the symmetries of the theory. All new relevant, marginal, and irrelevant couplings should depend on the cutoff only because they depend on these canonical couplings. The renormalization group equations determine how all the constants change with the cutoff; and when one inserts the ansatz that only a few constants depend on the cutoff, the remaining constants (including functions of longitudinal momentum fractions) are determined by the renormalization group equations. In practice we have only been able to apply coupling coherence to the perturbative renormalization group; but in all cases considered to date, the counterterms that result are exactly those required to restore the symmetries of the theory, even though no direct reference is made to these symmetries in the calculations. [4,6]

The conclusion we have reached is that, given a cutoff, the renormalization group and coupling coherence uniquely determine the hamiltonian to each order in the canonical coupling. In QCD this allows us to compute the effective hamiltonian as an expansion,

$$H_\Lambda = H^{(0)} + g_\Lambda H^{(1)} + g_\Lambda^2 H^{(2)} + \; \cdots . \tag{3}$$

I have suppressed the fact that there is also dependence on the running current quark masses; but the most important point is that the operators, $H^{(n)}$, depend on $\Lambda$ only because of their dimension (*e.g.*, a factor of $\Lambda^2$ for mass operators) and because of the cutoff functions. If the cutoff is chosen properly, as discussed below, we may be able to exploit asymptotic freedom to approximate the QCD hamiltonian by truncating this series at a finite order. It will almost certainly be necessary to further tune the strength of the relevant operators (i.e., the quark and gluon dispersion relations, and the chiral symmetry breaking quark-gluon vertex); but this is our starting point.

Having computed an approximate cutoff QCD hamiltonian, the next step is to study this hamiltonian non-perturbatively. An essential part of this step is the demonstration that the resultant low energy states are indeed dominated by few-body components so that a constituent picture arises. I am going to oversimplify this second step by ignoring the fact that we will need to push the cutoff as low as possible and confront the fact that the coupling becomes large as $\Lambda \rightarrow \Lambda_{QCD}$. Elsewhere in these proceedings, Wilson and Robertson discuss a strategy in which a sequence of weak-coupling calculations are extrapolated to this potentially large coupling, [8] a strategy first outlined by Wilson and collaborators. [1] For the purpose of this article it is sufficient to assume that the cutoff remains sufficiently large that the coupling does not become unmanageably large. If this is the case, we can simply use bound state perturbation theory to study our approximate QCD hamiltonian.

Once the cutoff is lowered to a suitable point where it is conceivable that the important many-parton components of hadron wave functions have been 'integrated out,' we must still deal with remaining interactions that involve parton emission and absorption. We assume that these interactions become unimportant at small cutoffs, so that it is possible to first approximate the hamiltonian by keeping only interactions in which parton number is conserved. This ansatz is quite reasonable from a variational point of view. If we consider a trial state in which a quark-antiquark pair are separated, the expectation value of the hamiltonian provides an upper bound on the energy. Any additional quarks and gluons in the wave function can only lower the energy. This means that if the hamiltonian is confining, there must be a two-body interaction which causes the energy to grow without bound as the pair is separated. In other words, the type of interactions we need to get a reasonable phenomenology without parton emission and absorption must actually appear in the hamiltonian, although there is no guarantee that they will appear in a perturbative approximation to the hamiltonian. We assume that few-body interactions largely determine the structure of hadrons, so that the additional interactions can be studied in bound state perturbation theory.

Given any hamiltonian, one can study bound states by first writing

$$H = H_0 + V \; . \tag{4}$$

To paraphrase Weinberg, you are free to choose any $H_0$ you please; but if you choose wrong, you'll be sorry. The main criteria in choosing $H_0$ are first that it be a reasonable approximation of $H$, so that bound state perturbation theory does not diverge; and second that it be solvable. Since the problem we will initially address is that of meson structure, this last restriction simply means that the quark-antiquark and quark-antiquark-gluon problems with two-body interactions should be tractable. This is not an overly severe restriction if one is willing to use a computer.

The strategy I will follow in this paper mirrors the strategy outlined above. I will compute the QCD hamiltonian to $\mathcal{O}(g^2)$ using a similarity renormalization group and coupling coherence, and I will then show that with a reasonable choice of $H_0$ this hamiltonian confines quarks and gluons. This result was first shown in Ref. [7].

There are two related questions one must ask to decide if the simple confinement mechanism survives. We first compute $H$ perturbatively by removing high energy states, and we must ask whether confining interactions that appear at low orders in this calculation survive to higher orders. We then use bound state perturbation theory to study the approximate hamiltonian, and this depends on an explicit choice of $H_0$. We must ask whether a choice of $H_0$ which includes confining interactions from $H$ leads to a reasonable bound state perturbative expansion. These questions are actually intertwined, but it is easier to study the second question using order-of-magnitude arguments than it is to study the first.

I must emphasize that to second order one can also force QED to be confining; but it is relatively straightforward to see that if $H_0$ is chosen to contain the confining interactions in QED, there are large perturbative corrections that cancel confinement. The fundamental observation is that even when confinement is included in $H_0$ for QED, photons are massless and not confined. This means photon exchange persists to arbitrarily large distances as charged particles separate and this photon exchange cancels the confining interaction. On the other hand, in QCD the same interactions that confine quarks appear in second order to confine gluons. This means that it is self-consistent to assume that the confinement mechanism survives because confinement turns off the long-range gluon exchanges that are necessary to cancel confinement. Hopefully this point will be clarified somewhat below.

## 2. Quark and Gluon Dispersion Relations from Coupling Coherence

The problem I want to address in this Section is the calculation of the one-body operators (i.e., the quark and gluon dispersion relations) to second-order in the QCD coupling constant. Since this constant does not run until third order, I only need to consider how the coupling runs to justify my choice of cutoffs. I can use any cutoff I want, but I would like to have some hope that the second-order results are not meaningless. This means I have to exploit asymptotic freedom to justify the first step in the analysis, the perturbative calculation of $H$.

If the cutoff is chosen properly, the QCD hamiltonian is approximately free when the cutoff is large. The free hamiltonian must be a fixed point (*i.e.,* a hamiltonian that does not change under the action of the transformation) for this to happen, which is actually rather easy to arrange since the transformation reduces to a scaling operation when applied to free hamiltonians. Near this fixed point, degrees of freedom with nearly the same free energy may still couple strongly to one another even when the coupling constant is small, which follows from nearly degenerate perturbation theory. Degrees of freedom that have drastically different free energy couple weakly. This means that if we want to exploit the fact that the coupling constant is small, the cutoff cannot remove the coupling between nearly degenerate degrees of freedom.

If the cutoff cuts through nearly degenerate degrees of freedom, we must solve a non-perturbative problem to replace the effects of their coupling with effective interactions. This is exactly what I want to avoid. Therefore, I am forced to use a cutoff on free energies. If the cutoff removes states (e.g., all particles with a free energy above some fixed value are removed), states just below the cutoff will couple strongly to states just above the cutoff, and we must again solve a non-perturbative problem to replace the effects of their coupling with effective interactions. For example, the quarks in a high energy pion interact strongly with one another. Therefore I am forced to use a cutoff that does not remove states, but instead removes only the direct coupling between states of drastically different free energy. In other words, the cutoffs must act at the vertices, preventing the free energy from changing by more than a fixed amount through a vertex. This is exactly the type of cutoff that the similarity transformation runs. In summary, to exploit asymptotic freedom I must use a cutoff that removes the coupling between states of drastically different free energy. The easiest cutoff functions to use in low order analytic calculations are step functions, which I will use here; although step functions introduce pathologies that are undesirable later.

Before proceeding to a brief discussion of the similarity transformation and coupling coherence, I want to point out a very interesting feature of the cutoff on free light-front energies. Light-front energy has the dimension of transverse momentum squared (the same as mass squared) divided by longitudinal momentum. Generically, our cutoff is $\Lambda^2/\mathcal{P}^+$, where $\Lambda$ has the dimension of mass. Transverse scale invariance is violated, leading to dimensional transmutation, and the QCD coupling constant is forced to run with $\Lambda$. However, our cutoff is $\Lambda^2/\mathcal{P}^+$ and contains an arbitrary longitudinal momentum scale, $\mathcal{P}^+$. If we succeed in renormalizing the theory, $\Lambda$ dependence will disappear from physical quantities, which means that $\mathcal{P}^+$ dependence will also disappear; but this will not happen exactly in a perturbative approximation, and because of this we will also find dependence on $\mathcal{P}^+$ in perturbative approximations. The appearance of this extra longitudinal momentum scale has profound implications for our program, some of which are illustrated and discussed below.

In order to use coupling coherence I need to study how the hamiltonian changes when the cutoff changes. I want to avoid a detailed derivation of a similarity transformation, [5] so I will just give the result I need through second order and show that it is easily understood. Let $H = h_0 + v$, where $h_0$ is a free hamiltonian and $v$ is cut off so that

$$\langle \phi_i | v | \phi_j \rangle = 0 \; , \tag{5}$$

if $|E_{0i} - E_{0j}| > \Lambda$; where $h_0 |\phi_i\rangle = E_{0i} |\phi_i\rangle$. If this cutoff is lowered to $\Lambda'$, the new hamiltonian matrix elements to $\mathcal{O}(v^2)$ are

$$\begin{aligned} H'_{ab} &= \langle \phi_a | h_0 + v | \phi_b \rangle \\ &- \sum_k v_{ak} v_{kb} \left[ \frac{\theta(|\Delta_{ak}| - \Lambda')\theta(|\Delta_{ak}| - |\Delta_{bk}|)}{E_{0k} - E_{0a}} + \frac{\theta(|\Delta_{bk}| - \Lambda')\theta(|\Delta_{bk}| - |\Delta_{ak}|)}{E_{0k} - E_{0b}} \right], \end{aligned} \tag{6}$$

where $\Delta_{ij} = E_{0i} - E_{0j}$ and $|E_{0a} - E_{0b}| < \Lambda'$. To follow the details of the discussion it is important to remember that there are implicit cutoffs in this expression because the matrix elements of $v$ have already been cut off so that $v_{ij} = 0$ if $|E_{0i} - E_{0j}| > \Lambda$. There are actually an infinite number of similarity transformations that will reduce the cutoff on how far off the diagonal matrix elements appear, but I will not discuss the additional constraints I have placed on the transformation to arrive at this result. They are not central to my discussion. I should note that I have fixed an error in my Brasil lectures, [7] where I used a transformation that does not completely avoid small energy denominators.

It is rather easy to understand this result qualitatively. We have removed the coupling between degrees of freedom whose free energy difference is between $\Lambda'$ and $\Lambda$, so the effects of these couplings are forced to appear in the new hamiltonian as direct interactions. To first order, the new hamiltonian is the same as the old hamiltonian, except that couplings between $\Lambda'$ and $\Lambda$ are now zero. To second order, the new hamiltonian contains a new interaction which sums over the second-order effects of couplings that have been removed. The second-order term in the new hamiltonian resembles the expression found in second-order perturbation theory, which is not surprising since the new hamiltonian must produce the same perturbative expansion for eigenvalues, cross sections, etc. as the original hamiltonian.

I have chosen the transformation so that it is always possible to find a coupling coherent hamiltonian to second order. To this order, we want the hamiltonian to reproduce itself, with the only change being $\Lambda \to \Lambda'$. The solution is found by noting that we need the partial sum above to be added to an interaction in $v$ that is expressed as a sum, so that the transformation merely changes the limits on the sum in a simple fashion. There are two possibilities. The first is

$$H_{ab} = \langle\phi_a|h_0+v|\phi_b\rangle - \sum_k v_{ak}v_{kb}\left[\frac{\theta(|\Delta_{ak}|-\Lambda)\theta(|\Delta_{ak}|-|\Delta_{bk}|)}{E_{0k}-E_{0a}}+\frac{\theta(|\Delta_{bk}|-\Lambda)\theta(|\Delta_{bk}|-|\Delta_{ak}|)}{E_{0k}-E_{0b}}\right], \quad (7)$$

and the second is

$$H_{ab} = \langle\phi_a|h_0+v|\phi_b\rangle + \sum_k v_{ak}v_{kb}\left[\frac{\theta(\Lambda-|\Delta_{ak}|)\theta(|\Delta_{ak}|-|\Delta_{bk}|)}{E_{0k}-E_{0a}}+\frac{\theta(\Lambda-|\Delta_{bk}|)\theta(|\Delta_{bk}|-|\Delta_{ak}|)}{E_{0k}-E_{0b}}\right]. \quad (8)$$

Note that the $v$ in these expressions is the same as that above only to first order. The coupling coherent interaction in $H$ is written as a power series in $v$ which reproduces itself under the transformation, except the cutoff changes. In higher orders the canonical variables would also run.

Given the generic coupling coherent hamiltonian to second order, it is a conceptually simple exercise to compute the coherent QCD hamiltonian to second order. For a second-order calculation it is sufficient to assume that $v$ contains all canonical QCD interactions. Space does not permit me to list the canonical QCD hamiltonian, so I must again refer the reader to the literature for details. [1,7,9,10] It is not necessary to be careful in the derivation of the canonical hamiltonian, because coupling coherence will take care of details. It is sufficient to naively derive the canonical hamiltonian in light-cone gauge, $A^+=0$, and insert cutoffs on free energy transfer in each of the vertices. The next step is to compute the $\mathcal{O}(g^2)$ corrections using Eq. (7) or Eq. (8). To decide which of these equations to use one must in principle go to higher orders, but in practice it is usually obvious which choice is correct. In the remainder of this section I will discuss the $\mathcal{O}(g^2)$ corrections to the one-body operators in the QCD hamiltonian.

First consider the second-order correction to the quark self-energy. This results from the quark mixing with quark-gluon states whose energy is above the cutoff. If we assume that the light-front energy transfer through the quark-gluon vertex must be less than $\Lambda^2/\mathcal{P}^+$, the coupling coherent self-energy for quarks with zero current mass is

$$\Sigma_\Lambda(p) = \frac{g_\Lambda^2 C_F \Lambda^2}{4\pi^2\mathcal{P}^+}\left\{\ln\left(\frac{p^+}{\epsilon\mathcal{P}^+}\right)-\frac{3}{4}\right\}+\mathcal{O}(\epsilon)\ . \quad (9)$$

Let me first describe the variables that appear in this result and then turn to a discussion of two important features. The quark has longitudinal momentum $p^+$, while the longitudinal momentum scale in the cutoff is $\mathcal{P}^+$. This coupling coherent solution comes from the second generic solution above, Eq. (8), in which one sums over states below the cutoff. This sum becomes an integral in the continuum theory, and I have completed the quark-gluon loop integral to obtain this result.

The first and most interesting feature of this result is that I have been forced to introduce a second cutoff,

$$p_i^+ > \epsilon \mathcal{P}^+ \ , \tag{10}$$

which restricts how small the longitudinal momenta of any particle can become. Without this second cutoff on the loop momenta, the self-energy is infinite, even with a cutoff on free energies. This second cutoff should be thought of as a longitudinal resolution. As we let $\epsilon \rightarrow 0$ we resolve more and more wee partons, and in the process we should confront effects normally ascribed to the vacuum. In this case the wee gluons are responsible for giving the quark a mass that is literally infinite. Theorists who insist on deriving intuition from manifestly gauge invariant calculations may find this interpretation repugnant, but within the framework of a light-front hamiltonian calculation it is quite natural. It is gauge invariance that is not natural, a heretical conclusion that will put light-front theorists on the defensive until we solve non-perturbative problems that have not been solved with other methods.

This second, infrared cutoff poses a problem. If we introduce a second cutoff, shouldn't we introduce a second renormalization group transformation to run this cutoff and find the new counterterms required by it? The oversimple answer I will need here is 'no.' The divergences that require us to introduce $\epsilon$ appear only in second-order diagrams and subdiagrams, so they look like super-renormalizable divergences that can be removed to all orders by a few counterterms. In principle the infrared divergences could require us to introduce complicated functions of transverse momentum, a possibility emphasized by Wilson. However, in perturbation theory we find that these divergences always cancel without the need for counterterms that violate transverse locality. [11] I will assume here that we can maintain such cancellations at all stages of our calculation.

When we compute $H_\Lambda$ perturbatively, the cancellations are those of perturbation theory. For example, the divergence in the quark mass is canceled by perturbative mixing of the quark with quark-gluon states, until the cutoff approaches $\Lambda_{QCD}$. There is no phenomenological reason to believe that such a cancellation can persist as the cutoff approaches $\Lambda_{QCD}$, because there are no free massless gluons. This means that the perturbative cancellations at high energies must be replaced by new cancellations at low energies, cancellations that do not require mixing between few-body and many-body states. [11] These cancellations are related to confinement, as we will see. When we study the cutoff hamiltonian in bound state perturbation theory the need to maintain precise cancellation of all infrared divergences places severe constraints on our choice of $H_0$.

So, for the purposes of this paper, I will assume that the infrared divergences are simple enough that we can introduce the cutoff $\epsilon$, and take it to zero at the end of the calculation. The reason that this answer is oversimple is because eventually one discovers that parton-parton interactions diverge as longitudinal momenta go to zero. This can be understood by thinking about the fact that the cutoff is $\Lambda^2/\mathcal{P}^+$, so that reducing the longitudinal momentum of a pair of partons that interact is equivalent to lowering the cutoff $\Lambda$; and $g_\Lambda$ increases as $\Lambda$ decreases. This issue is extremely important, and I am avoiding it because it is complicated and because I do not have a full solution to this problem. I will only add one cryptic remark. If a renormalization group is used to run a cutoff on longitudinal momenta, the full interacting QCD hamiltonian must be a fixed point of the longitudinal transformation because longitudinal scaling is a Lorentz symmetry that cannot be violated. [4]

If we want to let $\epsilon \to 0$, we must face the fact that the quark self-energy diverges even when $\Lambda$ is finite and identify a new cancellation mechanism. There are two possibilities. First, the divergences could be canceled in the energy of a physical quark. Second, the energy of a single quark could remain infinite with the divergences only being canceled in color singlet states. The first possibility is clearly the one required in QED. However, there is no experimental evidence for a finite mass quark; so we can explore the possibility that only color singlets have finite mass. This can only happen if there are infrared divergent interactions that exactly cancel the infrared divergent part of the self-energy, and I will show that this does indeed happen in a second-order analysis of QCD. Roughly speaking, the self-energy of a monopole diverges but that of a neutral dipole does not.

The second interesting feature of the above ‘mass’ is that it produces a dispersion relation which differs from that of current masses. Normally a mass produces an energy of the form $m^2/p^+$, where $p^+$ is the longitudinal momentum of the parton; but here we find $\Lambda^2/\mathcal{P}^+$. This means that the energy does not diverge like $1/p^+$, but at this order is independent of the parton momentum.

A nearly identical calculation reveals the second-order self-energy of gluons, and again we find that the dominant term goes like

$$\frac{g_\Lambda^2 \Lambda^2}{\mathcal{P}^+} \ln\left(\frac{p^+}{\epsilon \mathcal{P}^+}\right) . \tag{11}$$

I do not list the exact expression because it is not important. In QED we find that the photon mass in the cutoff hamiltonian is infrared finite, and we expect that it is exactly canceled by mixing with electron-positron pairs. However, in QCD the gluon mass is infinite and cannot be canceled by mixing with gluon pairs if there are no free massless gluons. Here the story is almost identical to that for quarks. If this divergence is not canceled by such mixing, there must be a divergent interaction involving gluons that allows it to be canceled in color singlet states. Once again, this is exactly what happens in a second-order analysis of QCD.

### 3. Confinement from Coupling Coherence

In addition to one-body operators we find quark-quark, quark-gluon, and gluon-gluon interactions in the second-order coupling coherent hamiltonian. As we lower the cutoff, we remove gluon exchange interactions, and these are replaced by direct interactions. The analysis of all of these interactions is nearly identical, so I will only consider the quark-antiquark interaction.

As stated above, in light-front coordinates partons with small longitudinal momentum are high energy partons, and this has important consequences for the light-front hamiltonian. We find important tree level counterterms that are not encountered in equal time coordinates. In equal time coordinates a high free-energy gluon has a large momentum, and its exchange produces quarks with large momenta and therefore high energy. In light-front coordinates the exchange of a wee gluon changes the momentum of quarks by a small amount, allowing them to have low energy. As a result we find second-order two-body interactions between low energy quarks generated by the removal of coupling to high energy gluons, and these interactions are crucial for producing a constituent picture in light-front coordinates. Since these interactions are dominated by the exchange of wee gluons, I will concentrate on the part of the quark-antiquark interaction that diverges as the longitudinal momentum exchange goes to zero.

In the case of gluon exchange between a quark and antiquark, we need the coupling coherent solution in Eq. (7). The exchange of high energy gluons is removed by the cutoff and replaced in second order by a direct interaction with cutoffs projecting on all intermediate state energies above the cutoff. We must add the canonical instantaneous gluon exchange interaction to this induced interaction. The induced interaction is

$$
\begin{aligned}
V_\Lambda \;=\; & -4 g_\Lambda^2 C_F \sqrt{p_1^+ p_2^+ k_1^+ k_2^+}\; \frac{q_\perp^2}{(q^+)^3} \\
& \times \Bigg[ \frac{\theta(|p_1^- - p_2^- - q^-| - \Lambda^2/\mathcal{P}^+)\;\; \theta(|p_1^- - p_2^- - q^-| - |k_2^- - k_1^- - q^-|)}{p_1^- - p_2^- - q^-} \\
& + \frac{\theta(|k_2^- - k_1^- - q^-| - \Lambda^2/\mathcal{P}^+)\;\; \theta(|k_2^- - k_1^- - q^-| - |p_1^- - p_2^- - q^-|)}{k_2^- - k_1^- - q^-} \Bigg] \\
& \times \theta\Big(\Lambda^2/\mathcal{P}^+ - \mid p_1^- + k_1^- - p_2^- - k_2^- \mid\Big) \, . \qquad (12)
\end{aligned}
$$

Here the initial and final quark (antiquark) momenta are $p_1$ and $p_2$ ($k_1$ and $k_2$), and the exchanged gluon momentum is $q$. The energies are all determined by the momenta, $p_1^- = p_{\perp 1}^2/p_1^+$, etc. This part of the interaction is independent of the spins, which remain unchanged.

This interaction can be further simplified for our analysis by noting that we are interested only in its most singular part, for which $q^+$ is extremely small. In this case $|q^-|$ is much larger than $p_i^-$ and $k_i^-$, leading to the approximation

$$
\begin{aligned}
V_\Lambda \;\approx\; & 4 g_\Lambda^2 C_F \sqrt{p_1^+ p_2^+ k_1^+ k_2^+}\; \left(\frac{1}{q^+}\right)^2 \theta(|q^-| - \Lambda^2/\mathcal{P}^+) \\
& \times \theta\Big(\Lambda^2/\mathcal{P}^+ - \mid p_1^- + k_1^- - p_2^- - k_2^- \mid\Big) \, . \qquad (13)
\end{aligned}
$$

The entire analysis can be made without making this approximation, and the results are the same.

We also need the instantaneous gluon exchange interaction,

$$V_{instant} = -4g_\Lambda^2 C_F \sqrt{p_1^+ p_2^+ k_1^+ k_2^+} \left(\frac{1}{q^+}\right)^2 \times \theta\Big(\Lambda^2/\mathcal{P}^+ - \mid p_1^- + k_1^- - p_2^- - k_2^- \mid\Big) . \quad (14)$$

The final cutoff on each of these interactions is the same, requiring the quark-antiquark energy to change by less than the cutoff. Since this final cutoff appears everywhere and is unimportant for the discussion, I will drop it. In addition to the cutoffs I have displayed, the same cutoff on longitudinal momenta used in the last Section must be added; so that all longitudinal momenta are required to exceed $\epsilon\mathcal{P}^+$.

Adding the above interactions and inserting the cutoff on longitudinal momenta we find

$$V_{singular} = -4g_\Lambda^2 C_F \sqrt{p_1^+ p_2^+ k_1^+ k_2^+} \left(\frac{1}{q^+}\right)^2 \theta(\Lambda^2/\mathcal{P}^+ - |q^-|)\, \theta(|q^+| - \epsilon\mathcal{P}^+) . \quad (15)$$

The most singular part of the one-gluon exchange operator cancels the instantaneous interaction above the cutoff, leaving us with the instantaneous exchange potential below the cutoff. If $\Lambda \approx \Lambda_{QCD}$, we expect further gluon exchange to be suppressed, and we are left with this singular interaction between the quark and antiquark.

The next step in the analysis is to take the expectation value of this interaction between arbitrary quark-antiquark states. The first cutoff forces $|q^+|/\mathcal{P}^+ > q_\perp^2/\Lambda^2$, and the second cutoff forces $|q^+| > \epsilon\mathcal{P}^+$. We see that $|q^+|$ can reach its lower limit only when $q_\perp^2 < \epsilon\Lambda^2$, and as $\epsilon \to 0$ the singularity is suppressed because of this phase space restriction; but it is not removed. The expectation value is

$$\langle\Psi_2(P)|V_{singular}|\Psi_1(P)\rangle = -4g_\Lambda^2 C_F \int \frac{dp_1^+ d^2p_{\perp 1}}{16\pi^3} \frac{dp_2^+ d^2p_{\perp 2}}{16\pi^3} \phi_2^*(p_2)\, \phi_1(p_1) \times \left(\frac{1}{q^+}\right)^2 \theta(\Lambda^2/\mathcal{P}^+ - |q^-|)\, \theta(|q^+| - \epsilon\mathcal{P}^+) , \quad (16)$$

where as usual $q = p_1 - p_2$ and $q^- = q_\perp^2/q^+$. The wave functions for the relative motion of the quark-antiquark pair are $\phi_1$ and $\phi_2$, and I have suppressed their dependence on the total momentum $P = p_1 + k_1$. I have not displayed the delta function normalization associated with center-of-mass motion. To evaluate the singular part of this integral, change variables to

$$Q = \frac{p_1 + p_2}{2} \;\; , \;\; q = p_1 - p_2 , \quad (17)$$

and expand the wave functions about $q = 0$. Only the leading term diverges, and it is

$$\langle \Psi_2(P)|V_{singular}|\Psi_1(P)\rangle = -4g_\Lambda^2 C_F \int \frac{dQ^+ d^2Q_\perp}{16\pi^3} \phi_2^*(Q)\, \phi_1(Q)$$
$$\times \int \frac{dq^+ d^2q_\perp}{16\pi^3} \left(\frac{1}{q^+}\right)^2 \theta(\Lambda^2/\mathcal{P}^+ - |q^-|)$$
$$\times\ \theta(|q^+| - \epsilon\mathcal{P}^+)\, \theta(\eta\mathcal{P}^+ - |q^+|) \ +\ finite\ . \quad (18)$$

$\eta$ is an arbitrary constant that simply prevents $|q^+|$ from becoming too large, and it does not matter since the divergence comes only from small $|q^+|$. Completing the final integral we obtain

$$\langle \Psi_2(P)|V_{singular}|\Psi_1(P)\rangle = -\frac{g_\Lambda^2 C_F \Lambda^2}{2\pi^2\mathcal{P}^+} \log\Big(\frac{1}{\epsilon}\Big) \int \frac{dQ^+ d^2Q_\perp}{16\pi^3} \phi_2^*(Q)\phi_1(Q) + finite. \quad (19)$$

Unless $\phi_1$ and $\phi_2$ are the same, this vanishes by orthogonality. If they are the same, this is exactly the same expression we obtain for the expectation value of the quark plus antiquark divergent mass operators; except with the opposite sign. Therefore, there is a divergence in the quark-antiquark interaction that is independent of their relative motion and which exactly cancels the divergent masses! These cancellations only occur for color singlets, and they occur for any color singlet state with an arbitrary number of quarks and gluons. Moreover, these cancellations appear directly in the hamiltonian matrix elements, so we can take the $\epsilon \to 0$ limit before diagonalizing the matrix.

This is half of the simple confinement mechanism. At this point it is possible to obtain finite mass hadrons even though the parton masses diverge. However, since the cancellations are independent of the relative parton motion, we must study the residual interactions to see if they are confining. Since I am interested in the long-range interaction, I will study the fourier transform of the potential and compute $V(r) - V(0)$ so that the divergent constant in which we are no longer interested is canceled. The details of computing a fourier transform are not illuminating, so I will simply list the results.

$$V_{singular}(r) - V_{singular}(0) \to \frac{g_\Lambda^2 C_F \Lambda^2}{4\pi^2\mathcal{P}^+}\ \log(|x^-|)\ , \quad (20)$$

when $x_\perp = 0$ and $|x^-| \to \infty$; and

$$V_{singular}(r) - V_{singular}(0) \to \frac{g_\Lambda^2 C_F \Lambda^2}{2\pi^2\mathcal{P}^+}\ \log(|x_\perp|)\ , \quad (21)$$

when $|x_\perp| \to \infty$ and $x^- = 0$. This potential is not rotationally symmetric, but it diverges logarithmically in all directions.

If the potential is not rotationally symmetric, how can rotational symmetry be restored? To answer this question, remember that the generators of rotations in light-front field theory contain interactions that change parton number. We expect the physical states in which a quark and antiquark are separated by a large distance to contain gluons. There is no reason to assume that the gluon content of these states is the same when the state is rotated, so rotational symmetry will be restored in highly excited states only if we allow additional partons. This complicates our attempt to derive a constituent picture, but we only need the constituent picture to work well for low-lying states. The intermediate range part of the potential is rotationally symmetric, and we may expect the ground state hadrons to be dominated by the valence configuration.

Isn't the confining potential supposed to be linear and not logarithmic? As far as I know there is no conclusive evidence that the long-range potential is linear, and heavy quark phenomenology shows that a logarithmic potential can work quite well. [12] The fact is that we know extremely little about highly excited states with large color dipole moments, and it is not clear that measurable quantities are ever sensitive to such states. In any case, I do not want to argue that these calculations show that the long-range potential in light-front QCD is logarithmic. Higher order corrections could produce powers of logarithms that add up to produce a linear potential.

The important point with which I will conclude is that $H$ contains a confining interaction that we are free to include in $H_0$, giving us some hope of finding a reasonable bound state perturbation theory for hadrons that resembles the bound state perturbation theory that has been successfully applied to the study of atoms.


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