Relativistic covariance and light-front electromagnetic vertex of a bound system

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Abstract

The consequences of the relativistic covariance violation in light-front calculations of the elastic electromagnetic form factors are analyzed. A consistent method to restore covariance of the light-front electromagnetic vertex of a bound system is proposed.

1. Introduction

At present the light-front (LF) dynamics is believed to be one of the most promising trends in theory of strong interactions. At the same time there exists one reasonable flaw of the approach, namely, the loss of relativistic covariance. This difficulty is encountered whenever a process involving bound systems is considered. However, sometimes it is possible to overcome it without complicated calculations or primitive "hand-made" methods. One of such problems, where the covariance playing an important role can be successively restored, is the one of the elastic electromagnetic form factors at large values of the momentum transfer, which is quite interesting from the practical point of view. We will give a detailed analysis of this problem and propose a general solution applicable to an arbitrary bound system.

2. Light-front electromagnetic vertex of a bound system

The form factors can be found by decomposition of the electromagnetic vertex (the current operator matrix element) in invariant amplitudes. For instanse, spin-1 system (say, deuteron) is described by three form factors $F_1(q^2)$, $F_2(q^2)$, $G_1(q^2)$:

$$< \lambda'|J_\rho|\lambda >= e^{*\lambda'}_\mu(p)\,\{(p+p')_\rho[F_1(q^2)g_{\mu\nu}+F_2(q^2)q_\mu q_\nu/2M^2]+$$
$$+G_1(q^2)[g_{\mu\rho}q_\nu - g_{\nu\rho}q_\mu]\}e^\lambda_\nu(p) \equiv e^{*\lambda'}_\mu(p')\,T^\rho_{\mu\nu}\,e^\lambda_\nu(p), \tag{1}$$

where $e^\lambda_\nu(p)$ is the polarization vector, p and p' are the initial and final momenta respectively, M is the deuteron mass, $q = p' - p$.

The current J_ρ in eq.(1) as any four vector operator must satisfy certain commutational relations with the Poincaré group generators. The latters contain interaction, hence, the current is a dynamical operator as well. As a result, the transformational properties of the current and the bound state wave function (w. f.) are compatible with each other; this leads to the covariance of the matrix element $<\lambda'|J_\rho|\lambda>$. In other words, in any reference frame there must be only three independent matrix elements (1).

In practice the dynamical current operator J_ρ is replaced by the free one, $J_\rho^{(0)}$, and the matrix elements $<\lambda'|J_\rho^{(0)}|\lambda>\equiv I^\rho_{\lambda',\lambda}$ are considered, for which the same decomposition (1) is used. If eq.(1) was true for the free current, then any triplet of independent matrix elements $I^\rho_{\lambda',\lambda}$ would give one and the same set of the form factors. As a matter of fact, this is not the case. It was explicitly demonstrated in ref.[1], where two solutions for the deuteron form factors were compared, based on the following sets of the free current matrix elements (so-called solutions "A" and "B"):

$$\text{(A)}\quad I^+_{1,1},\ I^+_{1,-1},\ I^+_{0,0};\qquad \text{(B)}\quad I^+_{1,1},\ I^+_{1,-1},\ I^+_{1,0}; \tag{2}$$

Here "+" stands for the plus-component $I^+=I^0+I^3$.

It was discovered[1] that the deuteron structure functions $A^{(A)}(q^2)$ and $B^{(A)}(q^2)$ calculated according to the solution "A" differ noticeably from $A^{(B)}(q^2)$ and $B^{(B)}(q^2)$ obtained by using the solution "B". So, the problem is how to get rid of the ambiguity in LF calculations of the electromagnetic form factors of a bound system.

As was explained in ref. 2, this ambiguity proceeds from the incorrectness of the covariant decomposition (1) for the matrix elements of the free current operator, calculated by using the LF w. f. for an interacting (bound) spin-1 system, because their transformation laws are incompatible with each other. In the theory on the null plane $t+z=0$ it means that $I^\rho_{\lambda',\lambda}$ depends on the choice of the z-axis direction. It is convenient to parametrize this dependence explicitly by introducing an *invariant* LF equation: $\omega x=0$, where ω is a four-vector: $\omega=(\omega_0,\vec{\omega})$, so that $\omega^2=0$, $\omega_0>0$. (The transition to the null plane case is achieved by setting $\omega=(1,0,0,-1)$). Now each matrix element $I^\rho_{\lambda',\lambda}$ is explicitly covariant but depends on the four-vector ω which participates in the decomposition of the former in invariant amplitudes, increasing their number (and, hence, the number of the form factors). Below we consider this situation in more detail, supposing the interaction to be C, P, T-invariant.

3. The form factor of a spinless system

A spinless system is described by only one electromagnetic form factor:

$$<\lambda'|J_\rho|\lambda>=(p+p')_\rho F(q^2)\delta_{\lambda,0}\delta_{\lambda',0}. \tag{3}$$

Because of the appearance of the four-vector ω the spin structure of the free LF electromagnetic vertex has more complicated form [2]:

$$I^\rho_{\lambda',\lambda}=[(p+p')_\rho F(q^2)+\omega_\rho\frac{(p+p')^2}{2(\omega p)}g_1(q^2)]\delta_{\lambda,0}\delta_{\lambda',0}. \tag{4}$$

We see that an extra term proportional to ω has appeared, so, we have *two* form factors instead of one. There is no any contradiction with general physical principles, since so far the contribution from the dynamical corrections to the free current (for instance, the pair creation by a photon) has not been taken into account. We will denote it as $< \lambda'|J_\rho - J_\rho^{(0)}|\lambda >\equiv \tilde{I}^\rho_{\lambda',\lambda}$. In ref. 2 in the framework of a diagrammatical approach, where this contribution was referred to as "the diagrams not expressed through the w. f.", it was shown that under the condition $\omega q = 0$ (equivalent to $q_+ = 0$ in the theory on the null plane), $\tilde{I}^\rho_{\lambda',\lambda}$ is proportional to ω_ρ, or, in other words, it can be represented as follows:

$$\tilde{I}^\rho_{\lambda',\lambda} = \omega_\rho \frac{(p+p')^2}{2(\omega p)} g_2(q^2)\delta_{\lambda,0}\delta_{\lambda',0}. \tag{5}$$

Since the matrix element $< \lambda'|J_\rho|\lambda >= I^\rho_{\lambda',\lambda} + \tilde{I}^\rho_{\lambda',\lambda}$ is covariant and does not depend on ω, the following equality must be valid:

$$g_1(q^2) + g_2(q^2) = 0. \tag{6}$$

So, the only role of the dynamical corrections to the free current is the cancellation of the ω-dependent (nonphysical) part of the free current matrix element, at the same time the ω-independent (physical) part remains unchanged. The general recipe of finding the form factors consists in the decomposition of the electromagnetic vertex in invariant amplitudes (including those dependent on the four-vector ω) and in the calculation of the coefficients at the ω-independent structures. Thus, the physical form factor given by $F(q^2)$ can be found by contracting the both sides of eq.(4) with ω_ρ:

$$F(q^2) = I^\rho_{0,0}\omega_\rho/2(\omega p). \tag{7}$$

On the null plane it is equivalent to taking I^+-component: $F(q^2) = I^+_{0,0}/2p_+$.

4. The form factors of a spin-1 system

Now we can apply the method described above for finding the form factors of a system with spin 1.

Because of the presence of the four-vector ω the decomposition of the free LF electromagnetic vertex in invariant amplitudes has the following form[2]:

$$I^\rho_{\lambda',\lambda} = e^{*\lambda'}_\mu(p')(T^\rho_{\mu\nu} + B^\rho_{\mu\nu})e^\lambda_\nu(p), \tag{8}$$

where $T^\rho_{\mu\nu}$ is given by eq.(1) and $B^\rho_{\mu\nu}$ contains eight new tensor structures depending on ω:

$$B^\rho_{\mu\nu} = \frac{M^2}{2(\omega p)}\omega_\rho(B_1 g_{\mu\nu} + B_2\frac{q_\mu q_\nu}{M^2} + B_3 M^2\frac{\omega_\mu\omega_\nu}{(\omega p)^2} + B_4\frac{q_\mu\omega_\nu - q_\nu\omega_\mu}{2(\omega p)})$$

$$+P_\rho(B_5M^2\frac{\omega_\mu\omega_\nu}{(\omega p)^2}+B_6\frac{q_\mu\omega_\nu-q_\nu\omega_\mu}{2(\omega p)})+B_7M^2\frac{g_{\mu\rho}\omega_\nu+g_{\nu\rho}\omega_\mu}{(\omega p)}+B_8q_\rho\frac{q_\mu\omega_\nu+q_\nu\omega_\mu}{2(\omega p)}. \quad (9)$$

Here B_{1-8} are invariant functions, $P=p+p'$. So, in this case there are eleven independent free current matrix elements (or eleven form factors), eight additional ones being the coefficients at the ω-dependent spin structures in eq.(9). Under the condition $\omega q = 0$ all the invariant functions in eqs.(8), (9) depend on q^2 only. It is convenient to extract the polarization vectors from the initial and final bound state w. f.'s: $\psi_\lambda = e^\lambda_\nu(p)\psi_\nu$, $\psi^*_{\lambda'} = e^{*\lambda'}_\mu(p')\psi_\mu$. Now the free LF electromagnetic vertex is corresponded by the tensor $<\mu|J^{(0)}_\rho|\nu> \equiv J^\rho_{\mu\nu}$, the contraction $e^{*\lambda'}_\mu(p')J^\rho_{\mu\nu}e^\lambda_\nu(p)$ coinciding with $I^\rho_{\lambda',\lambda}$. Taking into account the contribution from $\tilde{I}^\rho_{\lambda',\lambda}$ leads to the cancellation of all the ω-dependent structures in eq.(8). As a result, the physical form factors "sitting" in the tensor $T^\rho_{\mu\nu}$ can be obtained by the contractions[2]:

$$F_1 = J^\rho_{\mu\nu}\frac{\omega_\rho}{2(\omega p)}[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}-\frac{P_\mu\omega_\nu+P_\nu\omega_\mu}{2(\omega p)}+P^2\frac{\omega_\mu\omega_\nu}{4(\omega p)^2}], \quad (10)$$

$$\frac{F_2}{2M^2} = -J^\rho_{\mu\nu}\frac{\omega_\rho}{2(\omega p)q^2}[g_{\mu\nu}-2\frac{q_\mu q_\nu}{q^2}-\frac{P_\mu\omega_\nu+P_\nu\omega_\mu}{2(\omega p)}+M^2\frac{\omega_\mu\omega_\nu}{(\omega p)^2}-\frac{q_\mu\omega_\nu-q_\nu\omega_\mu}{2(\omega p)}], \quad (11)$$

$$G_1=\frac{1}{2}J^\rho_{\mu\nu}\{\frac{g_{\mu\rho}q_\nu-g_{\nu\rho}q_\mu}{q^2}+\frac{g_{\mu\rho}\omega_\nu+g_{\nu\rho}\omega_\mu}{2(\omega p)}+\frac{\omega_\rho}{2(\omega p)}[-P^2\frac{q_\mu\omega_\nu-q_\nu\omega_\mu}{2(\omega p)q^2}+\frac{q_\mu P_\nu-q_\nu P_\mu}{q^2}$$

$$+P^2\frac{\omega_\mu\omega_\nu}{2(\omega p)^2}-\frac{P_\mu\omega_\nu+P_\nu\omega_\mu}{2(\omega p)}]+P_\rho[\frac{q_\mu\omega_\nu-q_\nu\omega_\mu}{2(\omega p)q^2}-\frac{\omega_\mu\omega_\nu}{2(\omega p)^2}]-q_\rho\frac{q_\mu\omega_\nu+q_\nu\omega_\mu}{2(\omega p)q^2}\}, \quad (12)$$

where $J^\rho_{\mu\nu}$ is calculated by means of the diagram technique.

5. The role of nonphysical terms

The formulas (7) and (10)-(12) were tested[2] to be true in the framework of a model allowing independent solution for the form factors through the Bethe-Salpeter function. If we merely identify "by hands" the spin structure of the free LF electromagnetic vertex with that one of the Feynman covariant vertex (such a procedure was used in refs. 4-7), we will obtain not the physical form factors but their mixtures with nonphysical terms. Indeed, if instead of eq.(4) we represent $I^\rho_{\lambda',\lambda}$ for spinless case in the usual form as $I^\rho_{\lambda',\lambda} = (p+p')_\rho\tilde{F}(q^2)\delta_{\lambda,0}\delta_{\lambda,0}$, and, in order to find $\tilde{F}$, multiply both sides of the equality by $(p+p')_\rho$, then for $\tilde{F}$ we get:

$$\tilde{F} = F + g_1, \quad (13)$$

whereas the correct form factor is given by F. Just the same situation takes place for spin-1 case. For example, the solutions "A" and "B" (see eq.(2)) reproduce the form factors F_1 and F_2 only, G_1 containing nonphysical terms B_{5-7} [3]:

$$F_1^{(A)} = F_1^{(B)} = F_1, \quad F_2^{(A)} = F_2^{(B)} = F_2, \quad (13)$$

$$G_1^{(A)} = G_1 - B_6 + M(B_5 + B_7)/\sqrt{-q^2}, \quad G_1^{(B)} = G_1 + B_6. \qquad (14)$$

The nonphysical terms may distort noticeably the behaviour of the form factors at large momentum transfer[3]. Besides that, in spin-1 case they violate the current conservation: $q_\rho I^\rho_{\lambda',\lambda} \neq 0$ for some values of λ and λ'.

We see that to obtain a reliable result the nonphysical terms must be separated from the physical ones. It is hardly possible in the approach based on the noncovariant LF equation $t+z=0$. One may only hope to choose the matrix elements which are free from any contribution of the nonphysical terms. Such a "successful" choice was made in ref.[8], where $I^+_{1,1}$, $I^+_{1,-1}$ and $I^\perp_{1,1}$ - matrix elements of the deuteron current in the Breit frame were taken (here the symbol $\perp$ denotes any direction transversal to the momentum transfer $\vec{q}$). As was shown in ref. 3, in this case $e^{*\lambda'}_\mu(p')B^+_{\mu\nu}e^\lambda_\nu(p) = 0$ and $e^{*\lambda'}_\mu(p')B^\perp_{\mu\nu}e^\lambda_\nu(p) = 0$. But, of course, one can not assert *a priori* that it is really so. Having applied the covariant LF approach, we were able to separate covariantly the nonphysical terms and explained the ambiguities in the LF calculations of electromagnetic form factors. The formulas (7) and (10)-(12) are universal and applicable to a system with the arbitrary number of constituents. The solutions analogous to eqs.(7), (10)-(12) can also be obtained for a system with any value of spin (see, for example, ref. 2 for spin-1/2 case).

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