The Effective Action for QCD at High Energies

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Abstract

I discuss the construction of the effective action for QCD suitable for the description of high-energy and small momentum transfer diffractive processes.

1. Introduction

In my talk I discuss the results obtained in collaboration with Roland Kirschner and Lev Lipatov which are the subject of Ref. 1 and Ref. 2.

As we learned from the talk by J. Bartels[3] in new experiments at HERA on electron-proton deep inelastic scattering one can probe the region of very small values of the Bjorken variable $x = -\frac{Q^2}{s} \sim 10^{-4}$, where s is the scattering energy squared and Q^μ being the momentum transferred by the photon. The results of the experiments show that the gluonic structure functions increase fairly strong for small x. Such a behaviour seems to be in agreement with the theoretical predictions based on the BFKL equation[4]. This equation is obtained in the leading logarithm approximation (LLA) which corresponds to the sum of perturbative contributions being series in the effective coupling constant $g^2 \ln \frac{1}{x} \ll 1$(g is the QCD coupling constant).
It is known that LLA violates unitarity so the growth of the number of gluons in the nucleon cannot continue forever. This means that one should apply the unitarization procedure which restores the unitarity by taking into account the screening effects and which goes beyond LLA.

The method of unitarization proposed by L. Lipatov[5] is based on the use of the effective lagrangian for QCD at high-energies and small momenta transfer i.e. in the multi-Regge kinematics (MRK). This lagrangian being simpler than original QCD lagrangian contains all important physical modes which are present in MRK. Its form reflects also the relationship of the four-dimensional QCD at high-energies with two-dimensional theories related to two-dimensional space of transverse momenta and the two-dimensional space of longitudinal momenta.

One can derive the effective lagrangian for QCD in several ways. Using the diagrammatic method (see Ref. 2) we derive from tree graphs in MRK the effective vertices for scattering and production of gluons and quarks. Then by an appropriate choice of the quark wave function and the gluon polarization vectors these vertices can be represented in a simple form.
One can also start directly from the original QCD lagrangian and try to eliminate modes of gluons and quarks which are not present in MRK. This elimination procedure can be performed either in the framework of the path integral formalism (see Ref. 2) or by means of the equations of motion (see Ref. 1). Below I shall describe the last method and for the simplicity of presentation we consider only the gluonic part of the QCD lagrangian.

2. Sketch of construction of the effective lagrangian

Because of the MRK it is natural to work in the light-cone gauge defined by one of the momenta p_A or p_B of the initial massless scattering particles. In the c.m.s. where

$$p_A^0 = p_B^0 = p_A^3 = -p_B^3 = \frac{\sqrt{s}}{2}, \qquad p_{A\perp} = p_{B\perp} = 0 \tag{1}$$

we choose for the definiteness the light-cone gauge

$$A_- \equiv A_0 - A_3 = p_B^\mu A_\mu = 0 \quad . \tag{2}$$

The gluonic part of the QCD lagrangian

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} \quad , \quad G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \tag{3}$$

depends quadratically on $A_+^a = A_0^a + A_3^a$ so one can eliminate this variable by means of the equations of motion.
In such a way we arrive to the lagrangian which depends only on transverse components A_ϱ^a, $\varrho = 1, 2$ of the four-potential

$$\begin{aligned} \mathcal{L} \;=\; & \frac{1}{2} A_\sigma^a \Box A^{a\sigma} - ig(\partial_- A^\sigma) T^a A_\sigma (\frac{1}{\partial_-} \partial_\varrho A^{a\varrho}) \\ & - \frac{g^2}{2} (\partial_- A_\sigma) T^a A^\sigma \frac{1}{\partial_-^2} (\partial_- A_\varrho) T^a A^\varrho - ig(\partial_\varrho A_\sigma^a) A^\varrho T^a A^\sigma \\ & + \frac{g^2}{4} A_\varrho T^a A_\sigma A^\varrho T^a A^\sigma \end{aligned} \tag{4}$$

where $x^\mp = x^0 \mp x^3$, $\partial_\mp = \frac{\partial}{\partial x^\mp}$, $\Box = 4\partial_- \partial_+ + \partial_\varrho \partial^\varrho$ and T^a denotes the generators of gauge group in the adjoint representation.

In Eq. (4) as it stays the fields contain all modes. On the other hand in the MRK the strongly virtual (heavy) modes of the fields $A_\varrho^{(s)}$

$$k^2 \simeq k_\parallel^2 \gg |k_\perp^2| \tag{5}$$

are not present since they are already integrated out and we left with the moderately virtual fields $A_\varrho^{(m)}$. This elimination of the heavy modes is performed within perturbation theory and in the following analysis we shall restrict ourselves to the first perturbative order.

Let us decompose A_ϱ as the sum of strongly and moderately virtual fields

$$A_\varrho = A_\varrho^{(s)} + A_\varrho^{(m)} \quad . \tag{6}$$

Substituting the decomposition (6) to the lagrangian (4) and neglecting the interference contribution between s- and m- fields we obtain as a kinetic part

$$\mathcal{L}^{kin} \cong 2A_\sigma^{(s)}\partial_+\partial_- A^{(s)\sigma} + \frac{1}{2}A_\sigma^{(m)}\Box A^{(m)\sigma} \quad . \tag{7}$$

As an interaction lagrangian for s-fields $\mathcal{L}^{(s)}$ we take those terms from the lagrangian (4) (after inserting (6)) which contain the enhancement factor in the MRK being the operator $\frac{1}{\partial_-}$ acting on the field with the smallest k_- momentum component. The resulting lagrangian has the form

$$\begin{aligned} \mathcal{L}^{(s)} &= 2A_\varrho^{(s)a}\partial_+\partial_- A^{(s)a\varrho} \\ &+ ig[A_\sigma^{(m)}T^a\partial_- A^{(s)\sigma} + A_\sigma^{(s)}T^a\partial_- A^{(m)\sigma}](\frac{1}{\partial_-}\partial_\varrho A^{(m)a\varrho}) \quad . \end{aligned} \tag{8}$$

The integration over $A_\varrho^{(s)}$ fields in Eq.(8) leads to the expression

$$\Delta\mathcal{L} = \frac{g^2}{4}A_\varrho^{(m)}T^a(\partial_- A^{(m)\varrho})(\frac{1}{\partial_+\partial_-}\partial_\sigma A^{(m)\sigma})T^a(\frac{1}{\partial_-}\partial_\eta A^{(m)\eta}) \quad . \tag{9}$$

The sum of lagrangian (4) involving only m-fields and formula (9) leads to the modified lagrangian $\mathcal{L}^{mod}$

$$\mathcal{L}^{mod} = \mathcal{L}|_{A\to A^{(m)}} + \Delta\mathcal{L} \quad . \tag{10}$$

We should never forget about the underlying MRK in which the k_- momentum components of the particles are ordered. In the case of Eq.(9), the k_- momentum components of the first two fields $A_\varrho^{(m)}$ are much bigger than the corresponding ones of the two last $A^{(m)}$'s .

After removing the heavy modes we separate the modes of $A_\varrho^{(m)}$ into a part involving Coulombic modes A'_ϱ obeying $|\ k_+k_-\ |\ll|\ k_\perp^2\ |$ and the part describing the produced particles A_ϱ with momenta satisfying $| k_+k_- | \approx | k_\perp^2 |$ (for which we keep the original notation). The kinetic term of the Coulombic modes involves only transverse derivatives so these modes describe the instantaneus Coulomb interaction.

With the help of the Coulombic modes A'_ϱ we can rewrite $\Delta\mathcal{L}$ from Eq.(9) as a product of the triple vertex from $\mathcal{L}$ (Eq.(4))

$$-ig(\partial_- A^{(m)}_\sigma)T^a A^{(m)\sigma}(\frac{1}{\partial_-}\partial_\varrho A'^{a\varrho}) \tag{11}$$

and the induced vertex $\Delta\mathcal{L}^{ind}$

$$\Delta\mathcal{L}^{ind} = \frac{ig}{4}(\partial_-\partial_\varrho A'^{a\varrho})\left(\frac{1}{\partial_+\partial_-}\partial_\sigma A^{(m)\sigma}\right)T^a\left(\frac{1}{\partial_-}\partial_\eta A^{(m)\eta}\right) \tag{12}$$

connected by the Coulombic propagators resulting from the kinetic part $\mathcal{L}^{kin}_{Coul}$

$$\mathcal{L}^{kin}_{Coul} = \frac{1}{2}A'^a_\varrho\partial_\sigma\partial^\sigma A'^{a\varrho} \quad . \tag{13}$$

Let us also note that the remaining terms resulting from the integration over A'_ϱ fields in Eqs. (11),(12) and (13) cancel the third term in $\mathcal{L}(A^{(m)})$ given by Eq. (4).
If we neglect the last nonsingular term in Eq.(4) we can write the effective lagrangian in the form

$$\begin{aligned}\mathcal{L}^{eff} &= \frac{1}{2}A^{(m)a}_\varrho\Box A^{(m)a\varrho} - ig(\partial_- A^{(m)}_\sigma)T^a A^{(m)\sigma}(\frac{1}{\partial_-}\partial_\varrho A'^{a\varrho}) \\ &\quad -ig(\partial_\varrho A^{(m)a}_\sigma)A^{(m)\varrho}T^a A^{(m)\sigma} + \frac{ig}{4}(\partial_-\partial_\sigma A'^{a\sigma})(\frac{1}{\partial_+\partial_-}\partial_\varrho A^{(m)\varrho})T^a(\frac{1}{\partial_-}\partial_\eta A^{(m)\eta}) \quad .\end{aligned} \tag{14}$$

It is convenient now to introduce the following notation for the Coulombic fields

$$A_+ = -\frac{1}{\partial_-}\partial_\sigma A'^\sigma \quad A_- = -2\frac{\partial_-\partial_\sigma}{\partial_\varrho\partial^\varrho}A'^\sigma \quad . \tag{15}$$

According to the above definitions the fields $A_\pm$ are dependent. Nevertheless we declare them in the following as being independent ones. This is needed in order to put effectively to zero the term arising from the integration over A'_ϱ discussed above and which cancels the corresponding term of the order g^2 from Eq.(4). In such a way we arrive to the following kinetic part of the gluonic effective lagrangian

$$\mathcal{L}_{kin} = \frac{1}{2}A^a_+\partial_\sigma\partial^\sigma A^a_- + \frac{1}{2}A^a_\sigma\Box A^{a\sigma} \quad . \tag{16}$$

Let us emphasize that the coefficient in the front of Coulombic part differs from the one obtained by the substitution of Eq. (15) to Eq. (13). It is fixed by requirement that the amplitudes obtained with the help of A'_ϱ fields coincide with the amplitudes calculated with the use of $A_\pm$ fields.

From the effective lagrangian (14) one can also read off the interaction terms. We substitute in Eq. (14) the decomposition $A^{(m)}_\rho = A'_\rho + A_\rho$ supplemented by introduction of the definitions (15). It is convenient to represent the result in the complex coordinates and using the analogous notation for the produced fields

$$\begin{array}{l} \varrho = x^1 + ix^2, \quad \varrho^* = x^1 - ix^2, \quad \partial = \frac{\partial}{\partial \varrho}, \quad \partial^* = \frac{\partial}{\partial \varrho^*} \\ A = A^1 + iA^2 \quad A^* = A^1 - iA^2 \quad . \end{array} \tag{17}$$

Moreover, we describe the produced particles in terms of the complex scalar fields ϕ^a (see Ref.5)

$$A = i\partial^*\phi \ , \ A^* = -i\partial\phi^* \quad . \tag{18}$$

After that the gluonic effective lagrangian is given as the sum

$$\mathcal{L}^{eff} = \mathcal{L}_{kin} + \mathcal{L}^{(R)}_{scat} + \mathcal{L}^{(L)}_{scat} + \mathcal{L}_{prod} + \mathcal{L}_{Coul} \quad . \tag{19}$$

In the sum (19) the term $\mathcal{L}_{kin}$ is obtained from Eq. (16)

$$\mathcal{L}_{kin} = -2A^a_+ \partial\partial^* A^a_- - \frac{1}{2}(\partial^*\phi^a)\Box(\partial\phi^{a*}) \quad . \tag{20}$$

The term $\mathcal{L}^{(R)}_{scat}$ describes the scattering off right particles i.e. on the A_+ field

$$\mathcal{L}^{(R)}_{scat} = -\frac{ig}{2}A^a_+\left[(\partial_-\partial^*\phi)T^a(\partial\phi^*) + (\partial_-\partial\phi^*)T^a(\partial^*\phi)\right] \quad . \tag{21}$$

The analogous expression corresponding to the scattering on the A_- field $\mathcal{L}^{(L)}_{scat}$ reads

$$\mathcal{L}^{(L)}_{scat} = -\frac{ig}{2}A^a_-[(\partial_+\partial^*\phi^*)T^a(\partial\phi) + (\partial_+\partial\phi)T^a(\partial^*\phi^*)] \quad . \tag{22}$$

The term $\mathcal{L}_{prod}$ describes the production of ϕ and ϕ^*

$$\mathcal{L}_{prod} = g[\phi^a(\partial A_-)T^a(\partial^* A_+) - \phi^{a*}(\partial^* A_-)T^a(\partial A_+)] \quad . \tag{23}$$

Finally, the part $\mathcal{L}_{Coul}$ contains the interaction vertices involving the Coulombic fields

$$\mathcal{L}_{Coul} = \frac{ig}{2}[(\partial\partial^* A^a_-)(\frac{1}{\partial_+}A_+)T^a A_+ + (\partial\partial^* A^a_+)(\frac{1}{\partial_-}A_-)T^a A_-] \quad . \tag{24}$$

Summarizing, we constructed the effective lagrangian for QCD from which one can reproduce in a very economic way the known results about the asymptotics of scattering amplitudes in the MRK and in the LLA. The lagrangian (19) posses many remarkable

properties. In particular, if we approximate the operator $\Box$ in (20) by $4\partial_+\partial_-$ then the scattering amplitudes resulting from the lagrangian (19) are given as the product of two scattering amplitudes related to the two-dimensional theories acting in the longitudinal space and in the transverse space.
We expect that the effective lagrangian (19) is a convenient starting point towards construction of a method which goes beyond the LLA.

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