

# Light-Front QCD and the Constituent Quark Model

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## Abstract

A general strategy is described for deriving a constituent approximation to QCD, inspired by the constituent quark model and based on light-front quantization. Some technical aspects of the approach are discussed, including a mechanism for obtaining a confining potential and ways in which spontaneous chiral symmetry breaking can be manifested.

## 1. Introduction

In order to help organize our thinking about QCD and our understanding of hadronic physics it may be useful to group some relevant issues into three broad categories. These categories are certainly not meant to be a complete set, but are meant to help guide us in our approach to the problems we must face when attempting to solve QCD.

In the first category we have what we might call exact representations of QCD, for example, the complete set of Schwinger-Dyson equations for QCD, or the continuum limit of lattice QCD. We shall also include in this category “exact” light-front theory, by which we mean light-front quantized QCD including all necessary effects of vacuum degrees of freedom (also known as “zero modes,” though this phrase has several distinct meanings in light-front quantization). This theory has a nontrivial vacuum state due to the presence of zero longitudinal momentum particles. Correctly incorporating these into the theory from the beginning is a difficult problem, and is a subject of ongoing research efforts.

In the second category we have simple pictures of hadronic physics, each of which may roughly correspond to one or more of the exact representations. In this group we have, for example, truncation to the first Schwinger-Dyson equation, or the strong coupling limit of lattice QCD. Corresponding loosely to the exact light-front theory we have several simple pictures, among them the infinite momentum frame and the closely related parton model. The picture we shall particularly emphasize in connection with light-front physics, however, is that of the constituent quark model (CQM). In the CQM only a minimum number of constituents required by the symmetries are used to build each hadron, and the vacuum is trivial. The possibility of a connection to light-front QCD follows from

the simplicity of the vacuum on the light front, as will be discussed below. The strong-coupling limit of lattice QCD is also linked to the constituent quark model, but in a lattice version. The CQM discussed here exists in a continuum, with confining potentials rather than the stringlike confinement of lattice QCD.

In the third category we have the problem of building bridges between the simple pictures and the exact representations of QCD. In lattice QCD, for example, a bridge between the strong coupling and continuum limits is provided by the Monte Carlo technique. The problem we wish to focus on here is that of building a bridge between the CQM and exact light-front QCD.

We should emphasize from the outset an important point concerning the various models and theories. It is that, while the various exact representations must be precisely equivalent in the physics they predict, there is no such requirement for the various simple pictures. All of these have been obtained by making drastic and sometimes completely uncontrolled approximations to the full theory, and afterwards there clearly need be no relation between them. This is not to say, of course, that understanding in detail the relationship of a model to the full theory—that is, the bridge—might not lead to ways of incorporating important physics from others of the models.

Let us begin with the observation that in a certain sense the discovery of the formal rules of QCD represented a step backwards. In the CQM, starting before QCD was developed, hadrons are simple bound states of a few quark constituents. The up and down quarks are taken to have a mass roughly equal to half the average mass of the nonstrange mesons, around 300 MeV. Were it present in the model, the gluon would also be assigned a rather large mass, because we have not yet seen any low-mass gluonium. The vacuum is trivial. In this picture confinement is implemented by an *ad hoc* potential. There is no gluon emission or absorption since there are no gluons. There is no need to understand the role of renormalization in this model, because with *ad hoc* two-body potentials there is no need for renormalization. However, a major failing of the CQM is that it tells us nothing about mechanisms for chiral symmetry breaking.

QCD was a step backwards in the sense that it forced upon us a complex and mysterious vacuum. In QCD, because the effective coupling grows at long distances, there is always copious production of low-momentum gluons, which immediately invalidates any picture based on a few constituents. Of course, this step was necessary to understand the nature of confinement and of chiral symmetry breaking, both of which imply a nontrivial vacuum structure. But for 20 years we have avoided the question: Why did the CQM work so well that no one saw any need for a complicated vacuum before QCD came along?

A bridge between equal-time quantized QCD and the equal-time CQM would clearly be extremely complicated, because in the equal-time formalism there is no easy *nonperturbative* way to make the vacuum simple. Thus a sensible description of constituent quarks and gluons would be in terms of quasiparticle states, i.e., complicated collective excitations above a complicated ground state. Understanding the relation between the

bare states and the collective states would involve understanding the full solution to the theory. On the light front, however, simply implementing a cutoff on small longitudinal momenta suffices to make the vacuum completely trivial. Thus we immediately obtain a constituent-type picture, in which all partons in a hadronic state are connected directly to the hadron, instead of being simply disconnected excitations in a complicated medium. Whether or not the resulting theory allows reasonable approximations to hadronic states to be constructed using only a *few* constituents is an open question. However, we propose to regard the relative success of the CQM as a reason for optimism.

The price we pay to achieve this constituent framework is that the renormalization problem becomes considerably more complicated on the light front. Counterterms for divergences arising from large transverse momenta involve entire functions of longitudinal momenta, and vice versa. The situation is further complicated by the fact that the regulators we are forced to use, which must be nonperturbative and applicable in a Hamiltonian framework, are neither Lorentz nor gauge invariant. It is in this way that the familiar “Law of Conservation of Difficulty” manifests itself in our approach.

In the remainder of this paper we shall sketch the approach taken by the group at OSU, which is presented in more detail in Ref. [1]. (Discussion of some newer developments can be found in the recent lecture notes of R. Perry [2].) The bridge that we hope to construct includes a renormalization procedure that takes us from a field theoretic bare Hamiltonian to one that is strongly cut off and is much closer to a CQM-like model. We shall set up the CQM side of the bridge by introducing constituent masses for both quarks and gluons, and then examining the case that the running coupling is small at the scale of these masses. We thereby obtain a theory that can be studied in much the same way bound states are analyzed in QED. This means using one perturbation theory to study the renormalization problem and another to compute bound state properties. We must in the end extrapolate back to the physical value of the coupling. The bridging to QCD is this extrapolation process.

We shall begin by describing some general aspects of the approach and the overall strategy. We then discuss two key ingredients of the program: first, the appearance of a confinement mechanism starting from the canonical light-front Hamiltonian for QCD; and second, the nature and realization of chiral symmetry breaking on the light front. The problems of confinement and chiral symmetry breaking have been the two major barriers to building the bridge we seek, because they prevented us from making a straightforward connection to a weak-coupling picture. We hope to explain how these barriers may be overcome in light-front QCD. There are many other important details and caveats that we shall not discuss here; we present only some bare essentials in oversimplified form. The reader is referred to Ref. [1] for more details.

## 2. Generalities

In order to establish some notation and to have a starting point for the discussion, let us begin by sketching the canonical light-front Hamiltonian for QCD. We choose the light-cone gauge  $A_a^+ = 0$  and finesse (i.e., ignore for now) the problem of the zero modes.

There is first of all a free part

$$H_{free} = \int dk^+ d^2 k_\perp \left\{ \sum_\lambda \frac{k_\perp^2 + \mu^2}{k^+} a_{k\lambda}^\dagger a_{k\lambda} + \sum_\sigma \frac{k_\perp^2 + m^2}{k^+} (b_{k\sigma}^\dagger b_{k\sigma} + d_{k\sigma}^\dagger d_{k\sigma}) \right\}, \quad (1)$$

which counts the light-front energy of each constituent. Here  $\lambda$  and  $\sigma$  index gluon and quark polarizations, respectively, and color indices have been suppressed. Note that we have included a mass term for the gluons as well as the quarks (although we include only transverse polarization states for the gluons). We have in mind here that all masses that occur in  $H_{free}$  should roughly correspond to constituent rather than current masses. There are two points that should be emphasized in this regard.

First, cutoff-dependent masses for both the quarks and gluons will be needed anyway as counterterms. This occurs because all the cutoffs we have at our disposal for nonperturbative Hamiltonian calculations violate both equal-time chiral symmetry and gauge invariance. These symmetries, if present, would have protected the quarks and gluons from acquiring this kind of mass correction. Instead, in the calculations we are discussing both the fermion and gluon self-masses are quadratically divergent in a transverse momentum cutoff  $\Lambda$ .

The second point is more physical. When setting up perturbation theory (more on this below) one should always keep the zeroth order problem as close to the observed physics as possible. Furthermore, the division of a Hamiltonian into free and interacting parts is always completely arbitrary, though the convergence of the perturbative expansion may hinge crucially on how this division is made. Nonzero constituent masses for both quarks and gluons clearly comes closer to the phenomenological reality (for hadrons) than do massless gluons and nearly massless light quarks.

Next we have the various interactions, among which are the standard quark-gluon and gluon self-couplings. There are also ‘‘instantaneous’’ interactions, four-point operators that arise when certain constrained field components are eliminated by their equations of motion. Elimination of  $A_a^-$  results in the instantaneous gluon interactions:

$$H_g = -2g^2 \int dx^- d^2 x_\perp \left\{ (\psi_+^\dagger T^a \psi_+) \left( \frac{1}{\partial^+} \right)^2 (\psi_+^\dagger T^a \psi_+) \right. \\ \left. + (f^{abc} A_b^i \partial^+ A_c^i) \left( \frac{1}{\partial^+} \right)^2 (\psi_+^\dagger T^a \psi_+) + \frac{1}{4} f^{abc} f^{ade} (A_b^i \partial^+ A_c^i) \left( \frac{1}{\partial^+} \right)^2 (A_d^j \partial^+ A_e^j) \right\}, \quad (2)$$

while elimination of the constrained components of  $\psi$  gives rise to the instantaneous

fermion term:

$$H_f = g^2 \int dx^- d^2x_\perp \psi_+^\dagger \alpha_\perp \cdot A_\perp \left( \frac{1}{i\partial^+} \right) \alpha_\perp \cdot A_\perp \psi_+ . \quad (3)$$

Here  $\psi_+ \equiv \frac{1}{2}\gamma^0\gamma^+\psi$  is the dynamical part of the Fermi field, and the  $T^a$  are the generators and the  $f^{abc}$  the structure constants of color  $SU(3)$ .

Now, the presence of a nonzero gluon mass has important consequences. First, it automatically stops the running of the coupling below a scale comparable to the mass itself. This allows us to (arbitrarily) start from a small coupling at the gluon mass scale so that perturbation theory is everywhere valid, and only extrapolate back to the physical value of the coupling at the end. The quark and gluon masses also provide a kinematic barrier to parton production; the minimum free energy that a massive parton can carry is  $\frac{m^2}{p^+}$ , so that as more partons are added to a state and the typical  $p^+$  of each parton becomes small, the added partons are forced to have high energies. Finally, the gluon mass eliminates any infrared problems of the conventional equal-time type.

Next let us discuss cutoffs. When contemplating cutoffs there are certain general issues to be considered, for example whether to mix the transverse and longitudinal momentum cutoffs or keep them separate. This question is sharpened with the realization that longitudinal scale invariance must be an exact Lorentz symmetry of the theory, while transverse scale invariance is broken by masses and the cutoff itself. For the present we shall imagine a simple cutoff on constituent energies, that is, requiring

$$\frac{p_\perp^2 + m^2}{p^+} < \frac{\Lambda^2}{P^+} \quad (4)$$

for each constituent in a given Fock state. Here  $P^+$  is a parameter that sets a longitudinal scale.<sup>1</sup> Note that some Fock states have high energies due to the presence of a few high-energy partons, while others have high energies due to the presence of very many low-energy partons. Because of the presence of masses, however, Eq. (4) results in a small- $p^+$  cutoff, so that the total number of partons in a state of given total  $p^+$  is bounded. Thus the full quantum field theory is reduced to an ordinary quantum mechanical many-body problem.

Imposing (4) does not completely regulate the theory, however; there are additional small- $p^+$  divergences coming from the instantaneous terms in the Hamiltonian. We shall regulate these by treating them as if the instantaneous exchanged gluons and quarks were actually constituents, and were required to satisfy condition (4).

Other cutoff schemes are of course also possible, and may in fact be preferable for more refined calculations than the ones we shall discuss here. One can impose a cutoff

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<sup>1</sup>We write the cutoff in this way in order to make explicit that the RHS of (4), which in normal rest frames is measured in units of mass, actually scales like a transverse mass squared divided by a longitudinal mass. For a discussion of the scaling behavior of various quantities and power counting on the light front see Ref. [1].

on the invariant mass of Fock states, which is potentially useful because it respects those Lorentz symmetries that are kinematic on the light front, in particular longitudinal boost invariance. A disadvantage of this scheme is that it leads to spectator-dependence in counterterms. Another possibility is based on a transverse lattice, perhaps coupled with a momentum discretization in the  $(x^+, x^-)$  plane.

We can now outline the first stage of the program. Having stopped the running of the coupling below the constituent mass scale, we arbitrarily take it to be small at this scale, so that perturbation theory is valid at all energy scales. We can now use power counting to identify all relevant and marginal operators (relevant or marginal in the renormalization group sense). Because of the cutoffs we must use, these operators are not restricted by Lorentz or gauge invariance. Because we have forced the vacuum to be trivial, the effects of spontaneous chiral symmetry breaking must be manifested in explicit chiral symmetry breaking effective interactions. (We shall return to this below.) This means the operators are not restricted by chiral invariance either. There are thus a large number of allowed operators. Furthermore, since transverse divergences occur for any longitudinal momentum, the operators that remove transverse cutoff dependence contain functions of dimensionless ratios of all available longitudinal momenta. That is, many counterterms are not parameterized by single coupling constants, but rather by entire functions of longitudinal momenta. A precisely analogous result obtains for the counterterms for light-front infrared divergences; these will involve entire functions of transverse momenta.

The counterterm functions can in principle be determined by requiring that Lorentz and gauge invariance be restored in the full theory. Alternatively, we might invoke the idea of coupling coherence [3]. The basic idea is that the coefficients of the operators that appear in the effective Hamiltonian are not arbitrary; they are in principle computable in terms of the one coupling parameter  $g$  that characterizes the relativistic theory, along with the quark mass parameters. Thus as the cutoff is varied the running of these coefficients should occur only through their dependence on a single running coupling constant  $g_\Lambda$ . Requiring this to be the case can be used to fix many of the couplings. In fact, in all examples worked out so far coupling coherence automatically yields the correct counterterms necessary to restore, e.g., Lorentz invariance, and in the case of the  $\sigma$  model can also be used to fix the strengths of symmetry breaking operators that arise as a result of spontaneous symmetry breaking.

The cutoff Hamiltonian, with renormalization counterterms, will thus be given as a power series in  $g_\Lambda$ :

$$H(\Lambda) = H^{(0)} + g_\Lambda H^{(1)} + g_\Lambda^2 H^{(2)} + \dots , \quad (5)$$

where all dependence on the cutoff  $\Lambda$  occurs through the running coupling  $g_\Lambda$ , and cutoff-dependent masses. The next step is to make this Hamiltonian look more like that of a CQM by performing a similarity transformation on it.

### 3. A CQM-Like Low-Energy Hamiltonian

The next stage in building a bridge from the CQM to QCD is to establish a connection between the *ad hoc*  $q\bar{q}$  potentials of the CQM and the complex many-body Hamiltonian of QCD (see Eqs. (2), (3), and (5)). We shall illustrate in a very simplified way how this connection can be established. The full formalism needed is described in Ref. [4].

In lowest order the canonical QCD Hamiltonian contains gluon emission and absorption terms, including emission and absorption of high-energy gluons. Since a gluon's energy is  $\frac{k_\perp^2 + \mu^2}{k^+}$  for momentum  $k$ , a high-energy gluon can result either if  $k_\perp$  is large or  $k^+$  is small. But in the CQM, gluon emission is ignored and only low-energy states matter. How can one overcome this double disparity? The answer is that we can change the initial cutoff Hamiltonian  $H(\Lambda)$  by applying a unitary transformation to it. We imagine constructing a transformation  $U$  that generates a new effective Hamiltonian  $H_{\text{eff}}$ :

$$H_{\text{eff}} = U^\dagger H(\Lambda) U . \quad (6)$$

We then *choose*  $U$  to cause  $H_{\text{eff}}$  to look as much like a CQM as we can.

The essential idea is to start out as though we were going to diagonalize the Hamiltonian  $H(\Lambda)$ , except that we stop short of computing actual bound states. A complete diagonalization would generate an effective Hamiltonian  $H_{\text{eff}}$  in diagonal form; all its off-diagonal matrix elements would be zero. Furthermore, in the presence of bound states the fully diagonalized Hamiltonian would act in a Hilbert space with discrete bound states as well as continuum quark-gluon states. In a confined theory there would only be bound states. What we seek is a compromise: an effective Hamiltonian in which some of the off-diagonal elements can be nonzero, but in return the Hilbert space for  $H_{\text{eff}}$  remains the quark-gluon continuum that is the basis for  $H(\Lambda)$ . No bound states should arise. All bound states are to occur through the diagonalization of  $H_{\text{eff}}$ , rather than being part of the basis in which  $H_{\text{eff}}$  acts.

To obtain a CQM-like effective Hamiltonian, we would ideally eliminate all off-diagonal elements that involve emission and absorption of gluons or of  $q\bar{q}$  pairs. It is the emission and absorption processes that are absent from the CQM, so we should remove them by the unitary transformation. However, we would allow off-diagonal terms to remain within any given Fock sector, such as  $q\bar{q} \rightarrow q\bar{q}$  off-diagonal terms or  $qqq \rightarrow qqq$  terms. This means we allow off-diagonal potentials to remain, and trust that bound states appear only when the potentials are diagonalized.

Actually, as discussed in Ref. [4], we cannot remove all the off-diagonal emission and absorption terms. This is because the transformation  $U$  is sufficiently complex that we only know how to compute it in perturbation theory. Thus we can reliably remove in this way only matrix elements that connect states with a large energy difference; perturbation theory breaks down if we try to remove, for example, the coupling of low-energy quark to a low-energy quark-gluon pair. We therefore introduce a second cutoff parameter  $\frac{\lambda^2}{P^+}$ ,

and design the similarity transformation to remove off-diagonal matrix elements between sectors where the energy *difference* between the initial and final states is greater than this cutoff. For example, in second order the effective Hamiltonian has a one-gluon exchange contribution in which the intermediate gluon state has an energy above the running cutoff. Since the gluon energy is  $\frac{k_{\perp}^2 + \mu^2}{k^+}$ , where  $k$  is the exchanged gluon momentum, the cutoff requirement is

$$\frac{k_{\perp}^2 + \mu^2}{k^+} > \frac{\lambda^2}{P^+} . \quad (7)$$

This procedure is known as the “similarity renormalization group” method. For a more detailed discussion and for connections to renormalization group concepts see Ref. [4].

#### 4. Solving $H_{\text{eff}}$ : Two Perturbation Theories

The result of the similarity transformation is to generate an effective light-front Hamiltonian  $H_{\text{eff}}$ , which must be solved nonperturbatively. Guided by the assumption that a constituent picture emerges, in which the physics is dominated by potentials in the various Fock space sectors, we can proceed as follows.

We first split  $H_{\text{eff}}$  anew into an unperturbed part  $H_0$  and a perturbation  $V$ . The principle guiding this new division is that  $H_0$  should contain the most physically relevant operators, e.g., constituent-scale masses and the potentials that are most important for determining the bound state structure. All operators that change particle number should be put into  $V$ , as we anticipate that transitions between sectors should be a small effect. This is consistent with our expectation that a constituent picture results, but this must be verified by explicit calculations. Next we solve  $H_0$  nonperturbatively in the various Fock space sectors, using techniques from many-body physics. Finally, we use bound-state perturbation theory to compute corrections due to  $V$ .

We thus introduce a second perturbation theory as part of building the bridge. The first perturbation theory is that used in the computation of the unitary transformation  $U$  for the incomplete diagonalization. The second perturbation theory is used in the diagonalization of  $H_{\text{eff}}$  to yield bound-state properties. R. Perry in particular has emphasized the importance of distinguishing these two different perturbative treatments [2]. The first is a normal field-theoretic perturbation theory based on an unperturbed free field theory. In the second perturbation theory a different unperturbed Hamiltonian is chosen, one that includes the dominant potentials that establish the bound state structure of the theory. Our working assumption is that the dominant potentials come from the lowest-order potential terms generated in the perturbation expansion for  $H_{\text{eff}}$  itself. Higher-order terms in  $H_{\text{eff}}$  would be treated as perturbations relative to these dominant potentials.

It is only in the second perturbative analysis that constituent masses are employed for the free quark and gluon masses. In the first perturbation theory, where we remove transitions to high-mass intermediate states, it is assumed that the expected field theoretic



masses can be used, i.e., near-zero up and down quark masses and a gluon mass of zero. Because of renormalization effects, however, there are divergent mass counterterms in second order in  $H(\Lambda)$ .  $H_{\text{eff}}$  also has second-order mass terms, but they must be finite—all divergent renormalizations are accomplished through the transformation  $U$ . When we split  $H_{\text{eff}}$  into  $H_0$  and  $V$ , we include in  $H_0$  both constituent quark and gluon masses and the dominant potential terms necessary to give a reasonable qualitative description of hadronic bound states. Whatever is left in  $H_{\text{eff}}$  after subtracting  $H_0$  is defined to be  $V$ .

In both perturbation computations the same expansion parameter is used, namely the coupling constant  $g$ . In the second perturbation theory the running value of  $g$  measured at the hadronic mass scale is used. In relativistic field theory  $g$  at the hadronic scale has a fixed value  $g_s$  of order one; but in the computations an expansion for arbitrarily small  $g$  is used. It is important to realize that covariance and gauge invariance are violated when  $g$  differs from  $g_s$ ; the QCD coupling at any given scale is not a free parameter. These symmetries can only be fully restored when the coupling at the hadronic scale takes its physical value  $g_s$ .

To assure maximum effectiveness of the perturbative computations it is convenient to alter the coupling dependencies of major terms in the interaction  $V$ . For example,  $V$  contains the difference between the constituent quark and gluon masses used in  $H_0$  and the masses in  $H_{\text{eff}}$  itself, a difference which is of order one. But since this difference only has to be exact for  $g = g_s$  we can convert this term to a second order term by multiplying it by  $(g/g_s)^2$ , turning it into a genuine perturbation. Likewise some of the second-order potential terms in  $H_0$  (second order meaning  $\mathcal{O}(g^2)$ ) can be multiplied by  $(g/g_s)^2$  to make them of fourth order in  $g$  when  $g$  is small, if this is necessary to simplify the bound state analysis for small  $g$ . These modifications can also cause violations of covariance for  $g \neq g_s$ .

## 5. Confinement

The conventional wisdom is that any weak-coupling Hamiltonian derived from QCD will have only Coulomb-like potentials, and certainly will not contain confining potentials. Only a strong-coupling theory can exhibit confinement.

The conventional wisdom is wrong. Robert Perry made this discovery, as he will report in his contribution to these proceedings [2,5]. When  $H_{\text{eff}}$  is constructed by the unitary transformation of Eq. (6), with  $U$  determined by the “similarity renormalization group” method,  $H_{\text{eff}}$  has an explicit confining potential already in second order! We shall explain this result below. However, first we should give the bad news. If quantum electrodynamics (QED) is solved by the same process as we propose for QCD, then the effective Hamiltonian for QED has a confining potential too. In the electrodynamic case, the confining potential is purely an artifact of the construction of  $H_{\text{eff}}$ , an artifact which disappears when the bound states of  $H_{\text{eff}}$  are computed. Thus the key issues, discussed below, are to understand how the confining potential is cancelled in the case of electrodynamics, and

then to establish what circumstances would prevent a similar cancellation in QCD.

The confining potential that appears in  $H_{\text{eff}}$  in both QED and QCD is easily understood if we recall a cancellation that occurs between the infrared divergences of the instantaneous gluon (or photon) exchange potential and the perturbative one gluon exchange diagram. (This same cancellation occurs for photons in QED.) Stripped to its essence, the terms that cancel behave like

$$\text{Instantaneous exchange :} \quad -\frac{1}{(q^+)^2} \quad (8)$$

$$\text{One gluon exchange :} \quad \frac{1}{(q^+)^2} \frac{q_\perp^2}{q_\perp^2} + \text{terms of order } \frac{1}{q^+} \quad (9)$$

where  $q^+$  is the exchanged longitudinal momentum and  $q_\perp$  is the exchanged transverse momentum. The ratio  $(q_\perp^2/q_\perp^2)$  results only for small  $q^+$ ; otherwise the denominator is more complicated. When the two terms are added the  $(\frac{1}{q^+})^2$  terms cancel, leaving no terms more singular than  $\frac{1}{q^+}$  at small  $q^+$ .

In the similarity renormalization group procedure, the explicit one gluon exchange diagram is generated only for  $q^+$  and  $q_\perp$  values for which the gluon energy is greater than the running energy cutoff  $\frac{\lambda^2}{P^+}$ . The gluon energy is  $\frac{q_\perp^2}{|q^+|}$ , where the absolute value of  $q^+$  appears to keep the energy positive and no mass occurs because the gluon is massless in the first stage computation of  $H_{\text{eff}}$ . As a result of this energy formula, the instantaneous gluon exchange term  $(\frac{1}{q^+})^2$  remains uncanceled if  $\frac{q_\perp^2}{|q^+|}$  is less than  $\frac{\lambda^2}{P^+}$ . This corresponds to a potential in position space that behaves like

$$v(x^-, x_\perp) = \int_{-\epsilon}^{\epsilon} dq^+ \int_0^{|\mathbf{q}_\perp| < \sqrt{\lambda^2 |q^+| / P^+}} d^2 q_\perp e^{iq^+ x^- - iq_\perp \cdot x_\perp} \frac{1}{(q^+)^2}. \quad (10)$$

(We have arbitrarily bounded the  $q^+$  integration between  $\pm\epsilon$  because only the small- $q^+$  behavior of the potential has been computed. The remainder of the potential will not be important.) This potential has a divergent constant term: when  $x^- = x_\perp = 0$ ,

$$v(0, 0) = \int_{-\epsilon}^{\epsilon} dq^+ \frac{\pi \lambda^2 |q^+|}{P^+} \frac{1}{(q^+)^2}, \quad (11)$$

which is logarithmically divergent. It turns out that this divergence cancels against self-energy divergences for color-singlet states, and is therefore irrelevant. We then compute the difference  $v(x^-, x_\perp) - v(0, 0)$ . The  $q_\perp$  integration when carried out gives

$$v(x^-, x_\perp) - v(0, 0) = \int_{-\epsilon}^{\epsilon} dq^+ \frac{\pi \lambda^2 |q^+|}{P^+} \left\{ e^{iq^+ x^-} f\left(\sqrt{\frac{\lambda^2 |q^+|}{P^+}} |x_\perp|\right) - 1 \right\} \frac{1}{(q^+)^2}, \quad (12)$$

where

$$f(|y_\perp|) \equiv \frac{1}{\pi} \int_0^{|q_\perp| < 1} d^2 q_\perp e^{iq_\perp \cdot y_\perp} . \quad (13)$$

If either  $x^-$  or  $|x_\perp|$  is very large (the appropriate limit to study for confinement) then one can verify that the exponential term

$$e^{iq^+ x^-} f\left(\sqrt{\frac{\lambda^2 |q^+|}{P^+}} |x_\perp|\right) \quad (14)$$

is approximately unity if  $q^+$  is so small that both  $|q^+ x^-|$  and  $\sqrt{\frac{\lambda^2 |q^+|}{P^+}} |x_\perp|$  are small, so that in this limit the integrand is negligible. However, once  $q^+$  is large enough so that either  $|q^+ x^-|$  or  $\sqrt{\frac{\lambda^2 |q^+|}{P^+}} |x_\perp|$  is large, then the exponential term is negligible and we are left with a logarithmic integral:

$$- \int_{-\epsilon}^{\epsilon} dq^+ \frac{\pi \lambda^2}{P^+} \frac{1}{|q^+|} \left( |q^+| > \frac{1}{|x^-|} , |q^+| > \frac{P^+}{\lambda^2 |x_\perp|^2} \right) . \quad (15)$$

The result is that the potential grows logarithmically whenever either  $|x^-|$  or  $|x_\perp|$  or both are large! If  $\frac{\lambda^2 |x_\perp|^2}{P^+} > |x^-|$  then the potential behaves as  $\ln(\frac{\lambda^2 |x_\perp|^2}{P^+})$ , otherwise it behaves as  $\ln|x^-|$ . Thus we obtain a logarithmic confining potential along any direction in the three-dimensional  $(x^-, x_\perp)$  space. For further details and discussion see Refs. [2,5].

Now we come to the crucial question. How is this confining potential cancelled in the case of QED, and why won't the same cancellation mechanism apply to QCD also?

In the case of QED the cancellation comes from the obvious source: one-photon intermediate states with photon energies below the cutoff. The effective Hamiltonian  $H_{\text{eff}}$  for QED contains one photon emission and absorption matrix elements for photons with energies below the cutoff. When the remaining one-photon states are eliminated by perturbation theory, the confining potential is cancelled and the normal Coulomb potential is all that remains.

What about QCD? The corresponding states would be  $q\bar{q}g$  states (where  $q$  stands for a quark and  $g$  a gluon) with gluon energies below the cutoff  $\frac{\lambda^2}{P^+}$ . Now we must be careful, however. If we imagine eliminating such states by perturbation theory we will obtain a contribution of the usual second-order form

$$\frac{\langle q\bar{q}|V|q\bar{q}g\rangle \langle q\bar{q}g|V|q\bar{q}\rangle}{E_{q\bar{q}} - E_{q\bar{q}g}} , \quad (16)$$

where  $E$  is the energy of an unperturbed state. This assumes that we start with a  $q\bar{q}$  state and make a transition to a  $q\bar{q}g$  state. The most simple-minded computation we can make with this perturbative form is to use only the free part of  $H_0$  to define both the  $q\bar{q}$  and  $q\bar{q}g$  states and their unperturbed energies  $E_{q\bar{q}}$  and  $E_{q\bar{q}g}$ . In this case the energy denominator

reduces to the gluon energy. But now our expectation for QCD, if it represents the physics of hadrons correctly, is that only a rather large gluon mass can come close to representing the physics. Hence the energy denominator would include a factor  $q_{\perp}^2 + \mu^2$ , where  $\mu$  is the gluon mass. The presence of this mass changes the one-gluon exchange term from

$$\frac{1}{(q^+)^2} \frac{q_{\perp}^2}{q_{\perp}^2} \quad (17)$$

to

$$\frac{1}{(q^+)^2} \frac{q_{\perp}^2}{q_{\perp}^2 + \mu^2} . \quad (18)$$

This means that if  $|q_{\perp}|$  is much less than  $\mu$  the one-gluon exchange term determined from  $H_{\text{eff}}$  is far too small to cancel the instantaneous term. However, when one examines the range of  $q_{\perp}$  that contributes significantly to the logarithmic growth of the potential as  $x^-$  or  $x_{\perp}$  become large, one finds that this range involves only very small values of  $|q_{\perp}|$ , small enough so that one-gluon exchange is negligible.

This argument is very simplistic, based on an artificially constructed free Hamiltonian with all potentials ignored. Examining the analysis with care, the crucial question is to determine the difference  $E_{q\bar{q}} - E_{q\bar{q}g}$  when the  $q\bar{q}g$  state includes an infrared gluon: a gluon with  $q^+$  and  $q_{\perp}$  small, and with  $\frac{q_{\perp}^2}{q^+}$  of order the cutoff  $\frac{\lambda^2}{P^+}$ . This means looking at the full effective Hamiltonian  $H_{\text{eff}}$  in the  $q\bar{q}$  sector (including potential terms) to determine  $q\bar{q}$  eigenvalues and then examining the  $q\bar{q}g$  sector, including potentials also, to determine how much the presence of an infrared gluon increases the energy eigenvalues obtained. Is this increase enough to prevent cancellation of instantaneous gluon exchange when  $q^+$  and  $q_{\perp}$  are both small? Our working assumption is that at the relativistic value of the coupling  $g_s$ , the  $q\bar{q}g$  sector will indeed have sufficiently higher energy eigenvalues than the  $q\bar{q}$  sector to block cancellation.

## 6. Spontaneous Chiral Symmetry Breaking

Another area of crucial importance to the CQM-QCD bridge is the issue of spontaneous chiral symmetry breaking. There are two troubling problems regarding spontaneous chiral symmetry breaking. The first is that the vacuum is often presumed to be trivial in light-front field theory. But how can a trivial vacuum be also a spontaneously broken vacuum of any symmetry? The second question is how any spontaneous breaking effects could appear in a weak-coupling approach, given that in the weak coupling limit there is no basis for spontaneous breaking of  $SU(3) \times SU(3)$  to occur?

There is a third question which is troubling to many newcomers to light-front physics, namely how can one talk about chiral symmetry at all in an effective Hamiltonian which has nonzero constituent masses for the up and down quarks even when chiral symmetry

is supposed to be exactly conserved? This last question puzzles only newcomers to the field because veterans of light-front research know that on the light front chiral symmetry is the same as helicity conservation, and in the free-field limit this is an exact symmetry for any value of the quark mass. The only breaking of light-front chiral symmetry occurs in one of the gluon emission and absorption terms, namely the term that is linear in the quark mass and explicitly combines helicity flip with gluon emission or absorption. Thus the question of chiral symmetry breaking in  $H_{\text{eff}}$  concerns the interactions in  $H_{\text{eff}}$ , not its free field structure.

One often turns to the  $\sigma$  model for inspiration on issues of spontaneous chiral symmetry breaking. In this model the mechanism behind the spontaneous breaking is that the  $\sigma$  field acquires a vacuum expectation value (VEV). In momentum space it is the zero momentum mode of  $\sigma$  that acquires the VEV. However, all claims that the vacuum of a light front field theory is trivial depend on elimination or disregard of the zero momentum modes. If these modes are allowed, an arbitrarily complex vacuum built of zero longitudinal momentum constituents becomes possible. In the case of the  $\sigma$  model one either (a) includes the zero momentum modes, making it possible to have a nontrivial vacuum in which  $\sigma$  has a VEV, or (b) declares the zero modes to be absent but permits explicit chiral symmetry breaking terms to appear in the Hamiltonian, of the kind a shift of  $\sigma$  by a constant would generate. See Appendix A of Ref. [1] for more details.

By analogy with the  $\sigma$  model we propose that zero momentum modes have an equally crucial role in spontaneous chiral symmetry breaking for QCD. After some extensive search we have established a tentative working assumption, that spontaneous chiral symmetry breaking is represented by a shift in the zero longitudinal momentum mode of the *lower* (constrained) components of  $\psi$  and  $\bar{\psi}$ . These shifts are not constants, however. Instead they are shifts by composite operators of the form  $\psi A$  or  $\bar{\psi} A$ , with appropriate spin matrices. These shifts lead to new explicit chiral breaking interactions in the canonical QCD Hamiltonian, in particular new chiral-breaking components in the instantaneous quark exchange term in the canonical Hamiltonian.

It is only a working assumption that spontaneous chiral symmetry breaking is represented by explicit chiral breaking terms in the instantaneous quark exchange term. The proof has to be that there exists a locally conserved chiral current despite the explicit breaking terms. Unfortunately, we can only expect exact current conservation when the coupling constant has its relativistic value  $g_s$ . For all smaller values of  $g$  there will be explicit nonconservation of the currents. Thus until we can extrapolate reliably from small  $g$  to  $g_s$  we will have no way of testing our working assumptions about how spontaneous chiral symmetry breaking is realized.

## 7. Concluding Remarks

The construction of the QCD effective Hamiltonian described above has already been

carried out through second order in perturbation theory [1]. The resulting theory is closely analogous to the phenomenological CQM. The next important step to be taken is to extend this work to fourth order in  $g$ , where we expect to begin to see the phenomenology replaced by true QCD effects. This step is also important to see whether the program is practical and whether the results actually improve. A useful first test of these Hamiltonians might be in the study heavy quark systems, where one presumably does not have to have complete control over confinement and chiral symmetry breaking to obtain reasonable results.

There are of course other important technical aspects of the light-front approach that we have not touched on here. The renormalization issues in particular have only been sketched very superficially. There has been significant work on formulating other light-front renormalization groups in addition to the similarity renormalization scheme [6]. Power counting on the light front is significantly different than at equal times, and is discussed in Ref. [1].

There are many open problems, both of a practical and conceptual nature. One area that has not received sufficient attention concerns the nature of instantons and the resolution of the  $U(1)_A$  problem in the light front framework. There are also the problems associated with the vacuum degrees of freedom, or zero modes. In the formulation we have described these are excluded from the outset, so there can be no direct confrontation of dynamical effects produced by these degrees of freedom. Work on incorporating them into the theory should help us understand the kinds of operators that can appear in the effective Hamiltonian for the cutoff theory. Finally, work on constructing nonperturbative realizations of the renormalization group on the light front has barely begun.

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