Twierdzenia o pochodnej sumy, różnicy, iloczynu, ilorazu funkcji różniczkowalnych.

Pochodna sumy:

$$(f(x)+g(x))' = \lim_{h\to 0}\frac{f(x+h)+g(x+h)-(f(x)+g(x))}{h} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} = f'(x)+g'(x)$$

Pochodna różnicy:

$$(f(x)-g(x))' = \lim_{h\to 0}\frac{f(x+h)-g(x+h)-(f(x)-g(x))}{h} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} - \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} = f'(x)-g'(x)$$

Pochodna iloczynu:

$$(f(x)\cdot g(x))' = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h} =$$
$$= \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)+f(x)g(x+h)-f(x)g(x+h)}{h} =$$
$$= \lim_{h\to 0}\frac{f(x)(g(x+h)-g(x))+g(x+h)(f(x+h)-f(x)}{h} = f(x)\cdot\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} + g(x)\cdot\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f(x)\cdot g'(x)+f'(x)\cdot g(x)$$

Pochodna ilorazu:

$$(\frac{f(x)}{g(x)})' = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)}-\frac{f(x)}{g(x)}}{h} = \lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{h\cdot g(x)g(x+h)} =$$
$$= \lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)+f(x)g(x)-f(x)g(x)}{h\cdot g(x)g(x)} =$$
$$= \lim_{h\to 0}\frac{g(x)(f(x+h)-f(x))-f(x)(g(x+h)-g(x))}{h\cdot (g(x))^2} = \frac{g(x)}{(g(x))^2}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} - \frac{f(x)}{(g(x))^2}\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x)^2)},\ \ gdy\, g(x)\neq 0$$

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