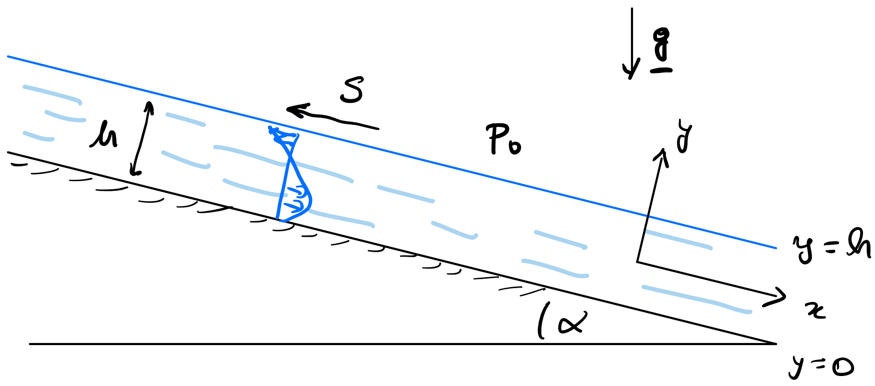
Test 2 Problem 1 - Solution

a)



$\underline{v} = v(y)\underline{e}_x$

b) Navier - Stokes:

unidirectional flow $\Rightarrow$ $(\underline{v}\cdot\nabla)\underline{v} = 0$

steady flow $\partial_t \underline{v} = 0$

In the x-direction $$0 = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 v}{\partial y^2} + \rho g \sin\alpha \qquad (1)$$

y-direction $$0 = -\frac{\partial p}{\partial y} - \rho g \cos\alpha \qquad (2)$$

Boundary conditions:

- continuous pressure $p = p_0$ on $y = h$ (3)
- no slip $v = 0$ on $y = 0$ (4)
- applied stress $\mu\frac{\partial v}{\partial y} = -S$ on $y = h$ (5)

From (2), using (3):

$$p = p_0 + \rho g \cos\alpha\,(h - y) \qquad \text{and} \qquad \frac{\partial p}{\partial x} = 0$$

From 1 $\qquad v = -\dfrac{\rho g \sin\alpha}{2\mu} y^2 + Ay + B$

From 4 $\qquad B = 0$

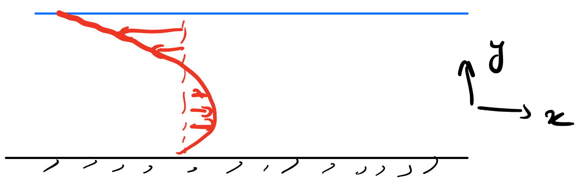
From 5 $\qquad -\rho g \sin\alpha \, h + \mu A = -S$

$$\Rightarrow \quad v = \frac{\rho g \sin\alpha}{2\mu} y(2h - y) - \frac{Sy}{\mu}$$

c) Volume flux

$$Q = \int_0^h v\,dy = \frac{\rho g \sin\alpha \, h^3}{3\mu} - \frac{Sh^2}{2\mu}$$

$$Q = 0 \quad \Rightarrow \quad S = \frac{2}{3} \rho g h \sin\alpha$$



d) Shear stress on lower plane is

$$\mu \left.\frac{dv}{dy}\right|_{y=0} = \frac{\rho g \sin\alpha}{2} (2h - 2\underset{0}{y}) - S$$

$$\text{Stress} = 0 \quad \Rightarrow \quad S = \rho g h \sin\alpha$$

$$v = -\frac{\rho g \sin\alpha}{2\mu} y^2$$