Hydrodynamics and Elasticity 2023/2024

Sheet 2

One of the problems will be handed in and marked.

Problem 1 Consider a short 'needle', i.e. a section joining two points: $\boldsymbol{x}_0$ and $\boldsymbol{x}_0 + \boldsymbol{a}_0$. Show that the general form of deformation of such a needle may be written as

$$\boldsymbol{a} = \boldsymbol{a}_0 + \boldsymbol{\phi} \times \boldsymbol{a}_0 + \boldsymbol{E} \cdot \boldsymbol{a}_0,$$

where $\boldsymbol{\phi} = \frac{1}{2}(\nabla \times \boldsymbol{u})$, where $\boldsymbol{u}$ is the displacement, and $\boldsymbol{E}$ is the strain tensor. What is the interpretation of the second term?

Problem 2 Prove that the Levi-Civitta tensor $\boldsymbol{\epsilon}$ is an isotropic tensor of rank 3, i.e. its representation is basis-independent.

Problem 3 Consider a cylindrical rod of radius R, with its axis **parallel** to $\boldsymbol{e}_3$ in Cartesian coordinates. The rod is deforming according to

$$x_1 = x_1^0 - \alpha(t) x_2^0 x_3^0, \tag{1}$$

$$x_2 = x_2^0 + \alpha(t) x_1^0 x_3^0, \tag{2}$$

$$x_3 = x_3^0. \tag{3}$$

(a) Find, at time t, the position of particles which at tome $t = 0$ constituted: (i) the cross-section of the rod, at $x_3^0 = \text{const}$, (ii) a section of the cross-sectional radius, (iii) a section parallel to the cylinder axis, and located on its surface.
(b) Find the deformation field $\boldsymbol{u}$ and the strain tensor $\boldsymbol{E}$.

(*) Problem 4 In spherical coordinates (r, ϕ, θ), for a scalar f and vector field $\boldsymbol{A}$, find the form of differential operators:
(a) $\nabla \cdot A \equiv \text{div}\, A$,
(b) ∇f,
(c) $\nabla \times \boldsymbol{A} \equiv \text{rot}\, \boldsymbol{A}$,
(d) $\nabla^2 f$,
(e) $(\nabla \boldsymbol{A}) \equiv \text{Grad}\, \boldsymbol{A}$.

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