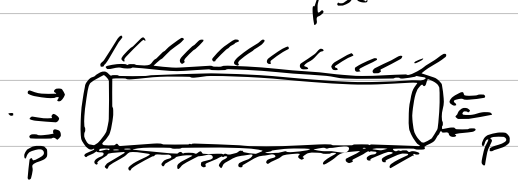
## Problem-3




$\Rightarrow T_{11} = -P$

**Notations**

Stress → $T_{ij}$ , displacement → $u_{ij}$

strain → $\varepsilon_{ij}$ , Poisson Ratio → $\nu$

Young Modulus → $E_Y$

**Soln**

① Displacement is allowed only in "Axial direction"

i.e $u_{11} \neq 0$

so strain tensor $\hat{\varepsilon} = \begin{vmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

Now using stress-strain relation.

$$T = \frac{E_Y}{(1+\nu)} \left\{ \hat{\varepsilon} + \frac{\nu}{(1-2\nu)} \, Tr\, \hat{\varepsilon} . \mathbb{1} \right\}$$

$$= \frac{E_Y}{(1+\nu)(1-2\nu)} \left\{ (1-2\nu)\, \hat{\varepsilon} + \nu\, Tr\, \hat{\varepsilon} . \mathbb{1} \right\}$$

$$T = \frac{E_Y}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu)\varepsilon_{11} & 0 & 0 \\ 0 & \nu\varepsilon_{11} & 0 \\ 0 & 0 & \nu\varepsilon_{11} \end{bmatrix}$$

state of stress — (1)

now given that $T_{11} = -P$

$\Rightarrow$ using eqn (1)

$$T_{11} = \frac{E_Y(1-\nu)\,\varepsilon_{11}}{(1+\nu)(1-2\nu)}$$

$$\Rightarrow (E_Y)_{eff} = \frac{T_{11}}{\varepsilon_{11}} = E_Y \frac{(1-\nu)}{(1+\nu)(1-2\nu)}$$

so $\boxed{(E_Y)_{eff} = E_Y \frac{(1-\nu)}{(1+\nu)(1-2\nu)}}$

Hence shown