Hydrodynamics and Elasticity 2023/2024

Sheet 5

One of the problems will be handed in and marked.

Problem 1 *Principal stresses* Consider the problem of maximal shear stresses for the stress tensor $\boldsymbol{T}$. Let us introduce the basis which diagonalises the stress tensor, so that $\boldsymbol{T} \cdot \boldsymbol{e}_i = T_i \boldsymbol{e}_i$. For a unit vector $\boldsymbol{n} = (n_1, n_2, n_3)$ find in this case the stress vector $\boldsymbol{t} = \boldsymbol{T} \cdot \boldsymbol{n}$. Next, decompose this vector into its normal and shear component. Show that the magnitude of the shear force is given by

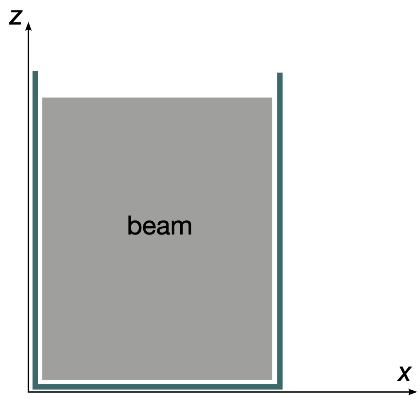
$$t_s^2 = |\boldsymbol{T} \cdot \boldsymbol{n}|^2 - (\boldsymbol{n} \cdot \boldsymbol{T} \cdot \boldsymbol{n})^2.$$

Find the maximal shear stress by seeking the maximum t_s with an additional condition of $|\boldsymbol{n}| = 1$. Find the direction of the vector $\boldsymbol{n}$ corresponding the the maximal shear and show that the maximal shear stress is

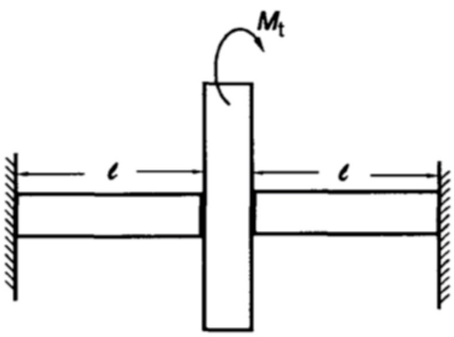
$$T_s^{\max} = \frac{1}{2}(T_{\max} - T_{\min}),$$

where $T_{\max}$ and $T_{\min}$ are the maximal and minimal eigenvalue of the stress tensor.

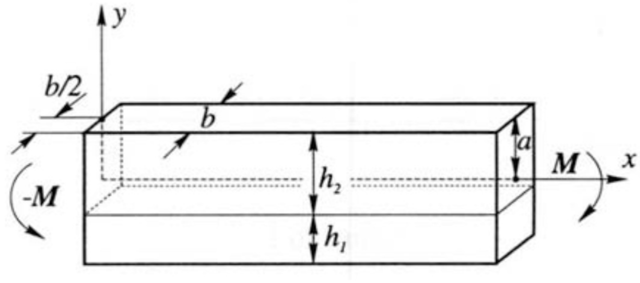
Problem 2 Consider an infinitely long (along the y axis) elastic beam with a rectangular cross-section of height h in a gravitational field $\boldsymbol{g} = (0, 0, -1)$. The beam is placed in a container, whose walls are perfectly rigid and perfectly slippery. The plane $z = 0$ is the bottom of the container. The walls allow the beam to move along their surfaces but do not allow motion in the direction perpendicular to their plane.
Find the stresses and deformation of the beam under its own weight. The density of the beam material is ρ_0. Sketch the shape of the deformed beam and the stress distribution. Is there a characteristic length scale associated with such a deformation?



Problem 3 In the figure below, a twisting torque M_t is applied to the rigid disc in the middle. Find the twisting moments transmitted to the circular shafts on either side of the disc.



(*) Problem 4 A beam with rectangular cross section consists of two layers of different materials connected so that it functions as whole (bimetallic beam, see figure). Its width is b, the thickness of the layer of the material with Young's modulus E_1 equals h_1, the second layer has thickness h_2 and Young's modulus E_2. The beam is in a state of pure bending with moment $\boldsymbol{M}$ in a vertical plane. Determine the distribution of the stresses T_{11} and location of the neutral axis a.



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