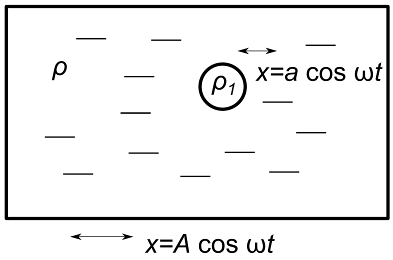
Hydrodynamics and Elasticity 2023/2024

Sheet 9

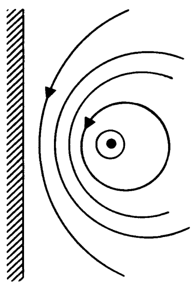
One of the problems will be handed in and marked.

Problem 1 A large container filled with an ideal, incompressible fluid of density ρ is performing sinusoidal oscillations of amplitude A under the action of an external force. Inside the container there is a small bubble of density ρ_1 (see the figure). What is the amplitude of the motion of the bubble? Neglect any gravitational effects.



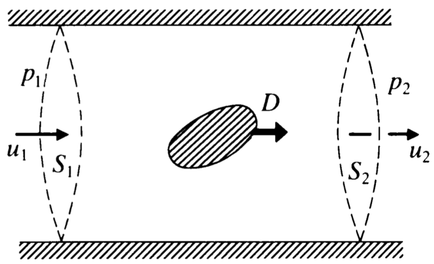
Problem 2 An inviscid and incompressible fluid occupies the region $x \geqslant 0$ bounded by a planar rigid wall at $x = 0$. At a point (d, y_0) there is a linear vortex of circulation Γ. Assuming that the vortex moves at the local flow velocity due to everything other than itself, use the method of images to show that the vortex moves downwards with a velocity

$$\frac{dy_0}{dt} = -\frac{\Gamma}{4\pi d}$$



Next, find the total force which the motion of the vortex exerts on the wall. Remember that – because the vortex is moving – the flow in the laboratory frame is not stationary. What would the force be if the vortex were fixed at $(d, 0)$?

Problem 3: D'Alembert's paradox Consider the steady flow of an ideal fluid around a 3D body which is placed in a long straight channel of uniform cross-section (see below). The body experiences the drag force D in the downstream direction.



(a) By integrating the Euler equation over an arbitrary fixed region V enclosed by a surface S, show that

$$-\int_S p\boldsymbol{n}\mathrm{d}S = \int_S \rho\boldsymbol{u}(\boldsymbol{u}\cdot\boldsymbol{n})\mathrm{d}S.$$

(b) Apply the obtained equation to the region in the figure. Find the net force in the downstream direction and equate it to the downstream component of the flux of momentum to find

$$D = \int_{S_1} (p_1 + \rho u_1^2) \mathrm{d}S - \int_{S_2} (p_2 + \rho u_2^2) \mathrm{d}S.$$

(c) Now check the assumptions and apply the Bernoulli streamline theorem to a streamline that runs along the channel walls to deduce that $D = 0$.

(*) Problem 4 The velocity field

$$u_r = \frac{Q}{2\pi r}, \quad u_\theta = 0$$

where Q is a constant, is called a line source flow if $Q > 0$ and a line sink if $Q < O$. Show that it is irrotational and that it satisfies $\nabla \cdot \mathbf{u} = 0$, save at $r = 0$, where it is not defined. Find the complex potential for such a flow.

Next, consider a mapping

$$Z = f(z)$$

where f is an analytic function of z. Provided that $f'(z_0) \neq 0$, points in the neighbourhood of $z = z_0$ are mapped by $Z = f(z)$, according to Taylor's theorem, in such a way that

$$Z - Z_0 = f'(z_0)(z - z_0) + O(z - z_0)^2$$

where $Z_0 = f(z_0)$. Use this to show that a line source of strength Q at $z = z_0$ is mapped into a line source of strength Q at $Z = Z_0$, provided that $f'(z_0) \neq 0$.

Next, consider fluid which occupies the region between two plane rigid boundaries at $y = \pm b$, and there is a line source of strength Q at $z = 0$. Find the complex potential $w(z)$ for the flow

1. by the method of images,
2. by using the mapping $Z = e^{\alpha z}$ with a suitably chosen $\alpha > 0$

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