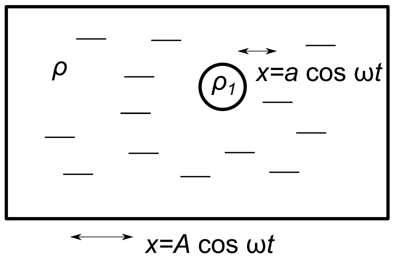
Hydrodynamics and Elasticity 2023/2024

Sheet 9

One of the problems will be handed in and marked.

Problem 1 A large container filled with an ideal, incompressible fluid of density ρ is performing sinusoidal oscillations of amplitude A under the action of an external force. Inside the container there is a small bubble of density ρ_1 (see the figure). What is the amplitude of the motion of the bubble? Neglect any gravitational effects.



Solution Let $X(t)$ denote the position of the container, and $x(t)$ the position of the ball. The total force acting on the ball from the fluid can be divided into two components. The first is associated with the non-uniformity of pressure in the container that arises due to the action of inertial forces. This is a force analogous to the buoyant force (Archimedes' principle) and is equal to $F_1 = V\rho\ddot{X}$, where V is the volume of the ball. The second is the drag force resulting from the unsteady motion of the ball relative to the fluid, given by $F_2 = -\frac{\rho V}{2}(\ddot{x} - \ddot{X})$, since the added mass for the ball is equal to half the mass of the fluid displaced by it. From the equation of motion of the ball

$$\rho_1 V\ddot{x} = F_1 + F_2 = \rho V\left(\ddot{X} - \tfrac{1}{2}(\ddot{x} - \ddot{X})\right)$$

From this, we get the desired amplitude

$$a = \frac{3\rho}{\rho + 2\rho_1}A$$

Alternative Solution:

The task can also be solved in the following way. First, we move to a frame of reference attached to the container. In this non-inertial frame, inertial forces act on both the fluid and the ball. In the fluid, these forces induce a pressure gradient, which is the source of an additional buoyant force acting on the ball. Thus, the equation of motion for the ball in the frame attached to the container is:

$$V(\rho_1 + \tfrac{1}{2}\rho)\ddot{x}' = -V(\rho_1 - \rho)\ddot{X}$$

where $x' = x - X$ (the position of the ball relative to the container). The situation is therefore analogous to that which occurs in a stationary vessel under the influence of gravity. Inserting the definition of x' we get:

$$\ddot{x} = \frac{3\rho}{\rho + 2\rho_1}\ddot{X}$$

and this is also the relationship between the amplitudes of oscillations.