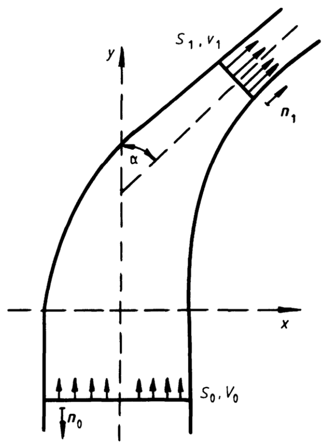
Hydrodynamics and Elasticity 2023/2024

Training Sheet

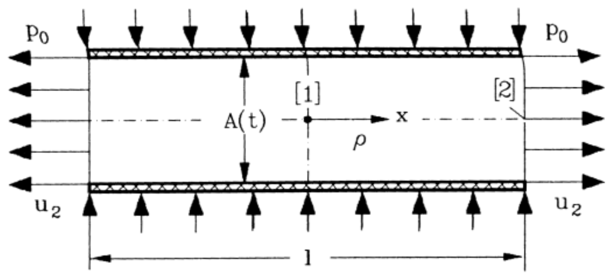
Problem 1 The stationary flow of an ideal incompressible fluid with density ρ is rotated by an angle α by a tube of a variable cross-section and ejected into a vacuum (see the figure). Find the force acting on the bend in the tube. Consider the flow as uniform at the cross-sections S_0 and S_1: the inlet velocity is equal to v_0.



Problem 2 Determine the friction force acting on an oscillating rigid plane, covered with a layer of fluid of thickness h and viscosity η, the upper surface of which is a free surface. The plane oscillates harmonically with amplitude A and frequency ω.

Problem 3 Show that an array of N identical point vortices of circulation Γ, placed equally about a circle of radius a, will rotate at a constant angular frequency Ω. Find the value of Ω.

Problem 4 The walls of a cylindrical pipe filled with ideal, incompressible fluid are squeezed uniformly, so that its cross-sectional area, $A(t)$, decreases in time. As the pipe is squeezed, the water begins to flow out of its ends. We assume that the x component of the flow field is of the form $v_x(x, y, z, t) = v_x(x, t)$ (with x axis directed along the pipe). For $A(t) = A_0 \cos \omega t, 0 \leqslant t < \pi/2\omega$ find $v(x, t)$ and the pressure difference between points [1] i [2] (see the figure). The total length of the pipe is l and the atmospheric pressure is equal to p_0.



Problem 5 Two viscous, incompressible fluids flow between two parallel planes located at $z = -h/2$ and $z = h/2$ under the influence of a pressure gradient directed along the x-axis: $\frac{dp}{dx} = \text{const}$. One of the fluids (with viscosity μ_1 and density ρ_1) occupies the region $-h/2 < z \leqslant 0$, while the other (with viscosity μ_2 and density ρ_2) occupies the region $0 < z < h/2$. Find the velocity field of both fluids. Discuss and interpret the boundary cases: $\mu_1 \gg \mu_2$, $\mu_1 \ll \mu_2$, and $\mu_1 = \mu_2$.

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