

Isotropization vs. Equilibration & Azimuthal Fluctuations

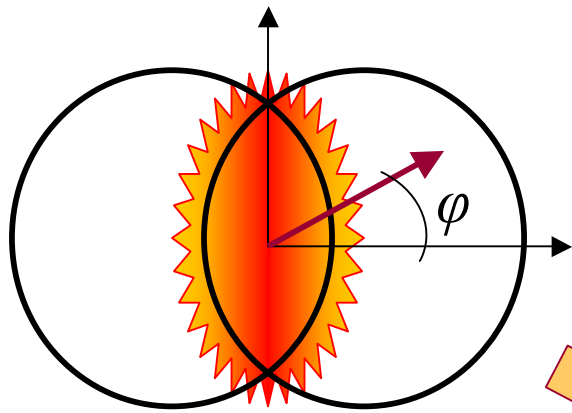
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- elliptic flow & fast equilibration
- instabilities driven isotropization
- equilibration vs. isotropization
- azimuthal fluctuations & preequilibrium

Evidence of equilibration at the early stage

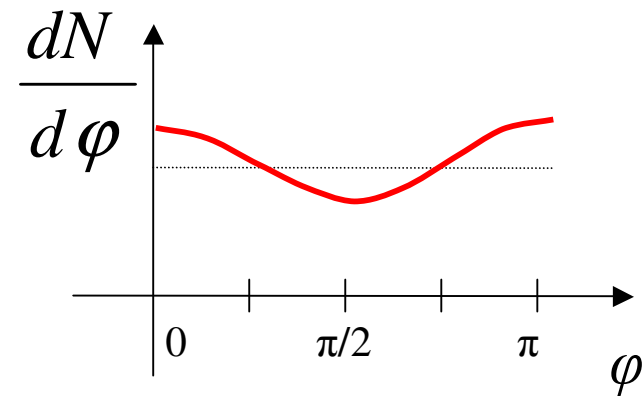
Success of hydrodynamic models in describing elliptic flow



Hydrodynamics

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = - \frac{\nabla p}{\rho}$$

**Hydrodynamic requires
local thermodynamical
equilibrium!**



Equilibration is fast

$$v_2 \sim \varepsilon = \left\langle \frac{x^2 - y^2}{x^2 + y^2} \right\rangle$$

Eccentricity decays due to the free streaming!

$$\varepsilon \searrow \Rightarrow v_2 \searrow$$



$$t_{\text{eq}} \leq 0.6 \text{ fm}/c$$

time of equilibration

Collisions are too slow

Time scale of hard parton-parton scattering

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g) T}$$

hard scattering ~ momentum transfer of order of T

either single hard scattering or multiple soft scatterings

$$t_{\text{eq}} \approx t_{\text{hard}} \geq 2.6 \text{ fm}/c$$

Scenarios of fast equilibration

- ▶ **Production mechanism of particles obeys equilibrium momentum distributions – instantaneous equilibration**

Schwinger mechanism:

$$\frac{d^2 n}{dp_T^2} \sim e^{-\frac{2\pi m_T^2}{eE}}$$

A. Białas, Phys.Lett. **B466**, 301 (1999)

W. Florkowski, Acta Phys. Pol. **B35**, 799 (2004)

D. Kharzeev & K. Tuchin, hep-ph/0501234

- ▶ **Equilibration is fast because quark-gluon plasma is strongly coupled**

sQGP

E.V. Shuryak, J. Phys. **G30**, S1221 (2004)

E.V. Shuryak & I. Zahed, Phys. Rev. **C70**, 021901 (2004)

- ▶ **Instabilities drive equilibration - as in the EM plasma**

Instabilities driven equilibration

The most important contributions

St. Mrówczyński,
Color Collective Effects At The Early Stage Of Ultrarelativistic Heavy Ion Collisions,
Phys. Rev. C **49**, 2191 (1994).

St. Mrówczyński,
Color filamentation in ultrarelativistic heavy-ion collisions,
Phys. Lett. B **393**, 26 (1997).

P. Romatschke and M. Strickland,
Collective modes of an anisotropic quark gluon plasma,
Phys. Rev. D **68**, 036004 (2003)

P. Arnold, J. Lenaghan and G.D. Moore,
QCD plasma instabilities and bottom-up thermalization,
JHEP **0308**, 002 (2003)

**Unstable
Mode
Analysis**

**Numerical
Simulations**

A. Rebhan, P. Romatschke and M. Strickland,
Hard-loop dynamics of non-Abelian plasma instabilities,
Phys. Rev. Lett. **94**, 102303 (2005)

A. Dumitru and Y. Nara,
QCD plasma instabilities and isotropization,
arXiv:hep-ph/0503121.

Instabilities driven equilibration

The most important contributions cont.

St. Mrówczyński and M. Thoma,
Hard Loop Approach to Anisotropic Systems,
Phys. Rev. D **62**, 036011 (2000)

P. Arnold and J. Lenaghan,
The abelianization of QCD plasma instabilities,
Phys. Rev. D **70**, 114007 (2004)

St. Mrówczyński, A. Rebhan and M. Strickland,
Hard-loop effective action for anisotropic plasmas,
Phys. Rev. D **70**, 025004 (2004)

**Effective
Action**

**Heavy-Ion
Phenomenology**

J. Randrup and St. Mrówczyński,
Chromodynamic Weibel instabilities in relativistic nuclear collisions,
Phys. Rev. C **68**, 034909 (2003)

P. Arnold, J. Lenaghan, G.D. Moore and L.G. Yaffe,
Apparent thermalization due to plasma instabilities in quark gluon plasma,
Phys. Rev. Lett. **94**, 072302 (2005)

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

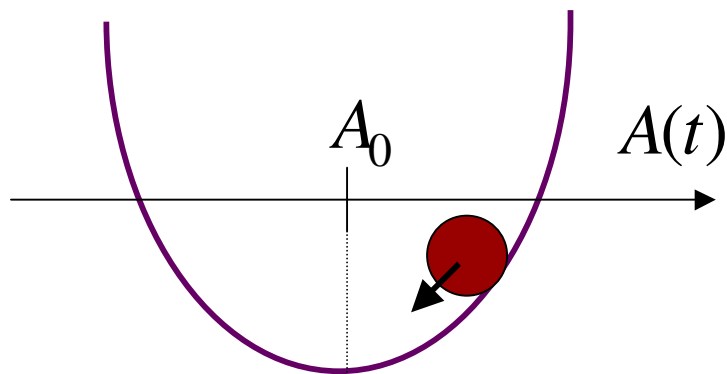
fluctuation

Instability

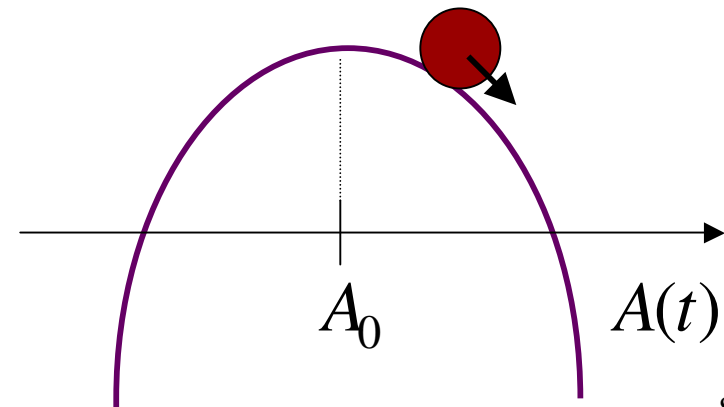
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Plasma instabilities

- ▶ instabilities in configuration space – **hydrodynamic instabilities**

- ▶ instabilities in momentum space – **kinetic instabilities**

Instabilities due to non-equilibrium momentum distribution

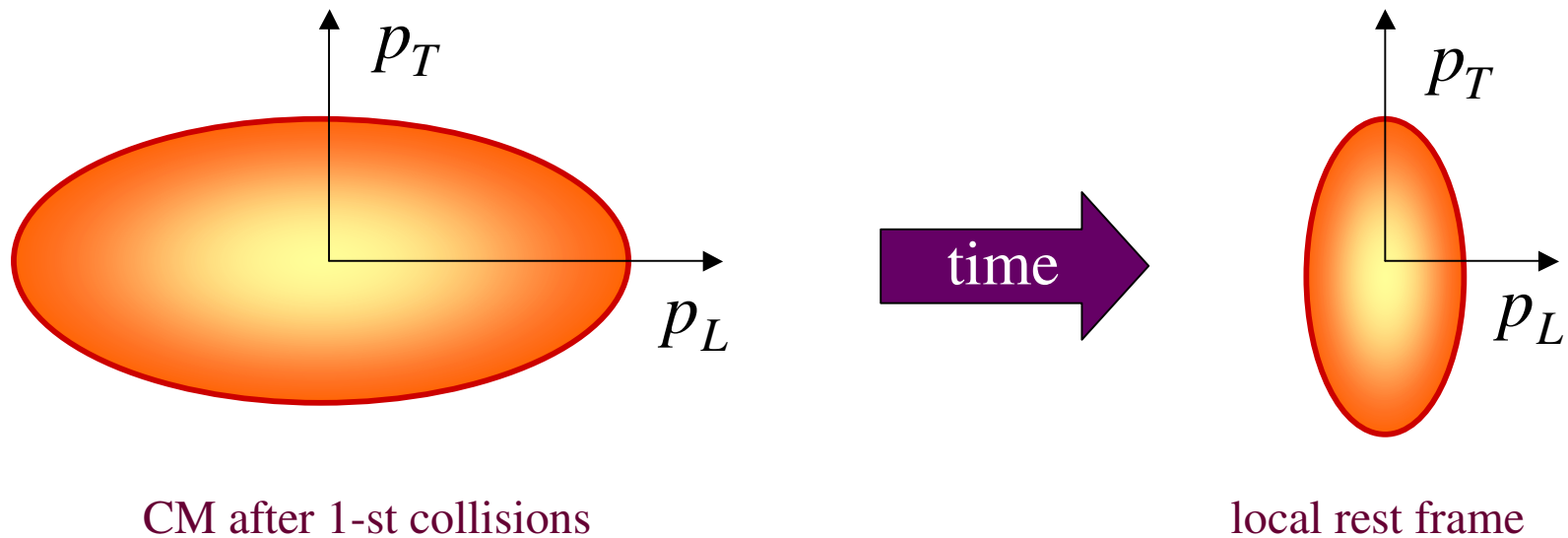
- ▶ **longitudinal modes** – $\mathbf{k} \parallel \mathbf{E}$, $\delta\rho \sim e^{-i(\omega t - \mathbf{k}\mathbf{r})}$

- ▶ **transverse modes** – $\mathbf{k} \perp \mathbf{E}$, $\delta\mathbf{j} \sim e^{-i(\omega t - \mathbf{k}\mathbf{r})}$

\mathbf{E} – electric field, \mathbf{k} – wave vector, ρ – charge density, \mathbf{j} - current

Momentum Space Anisotropy in Nuclear Collisions

Parton momentum distribution is initially strongly anisotropic

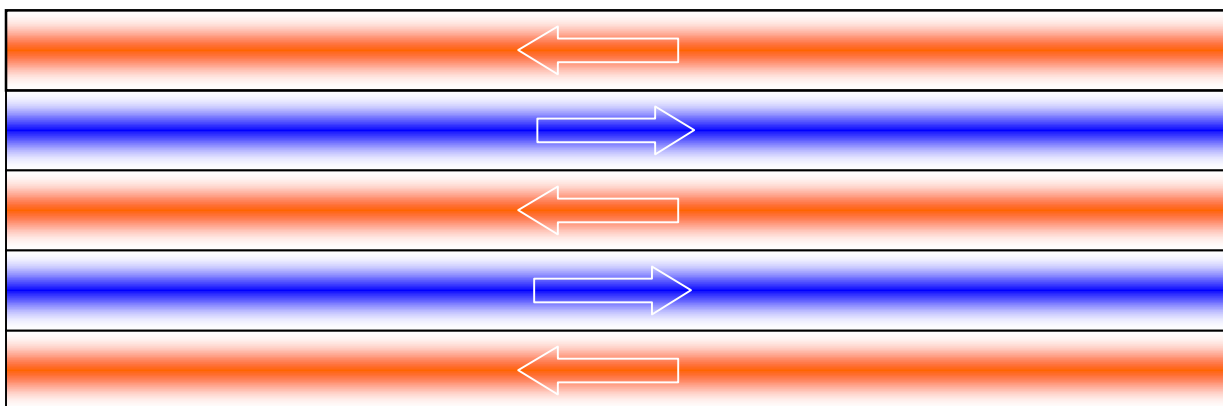


Seeds of instability

Current fluctuations

$$\langle j_a^\mu(x_1) j_b^\nu(x_2) \rangle = \frac{1}{2} \delta^{ab} \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x} - \mathbf{v}t)$$

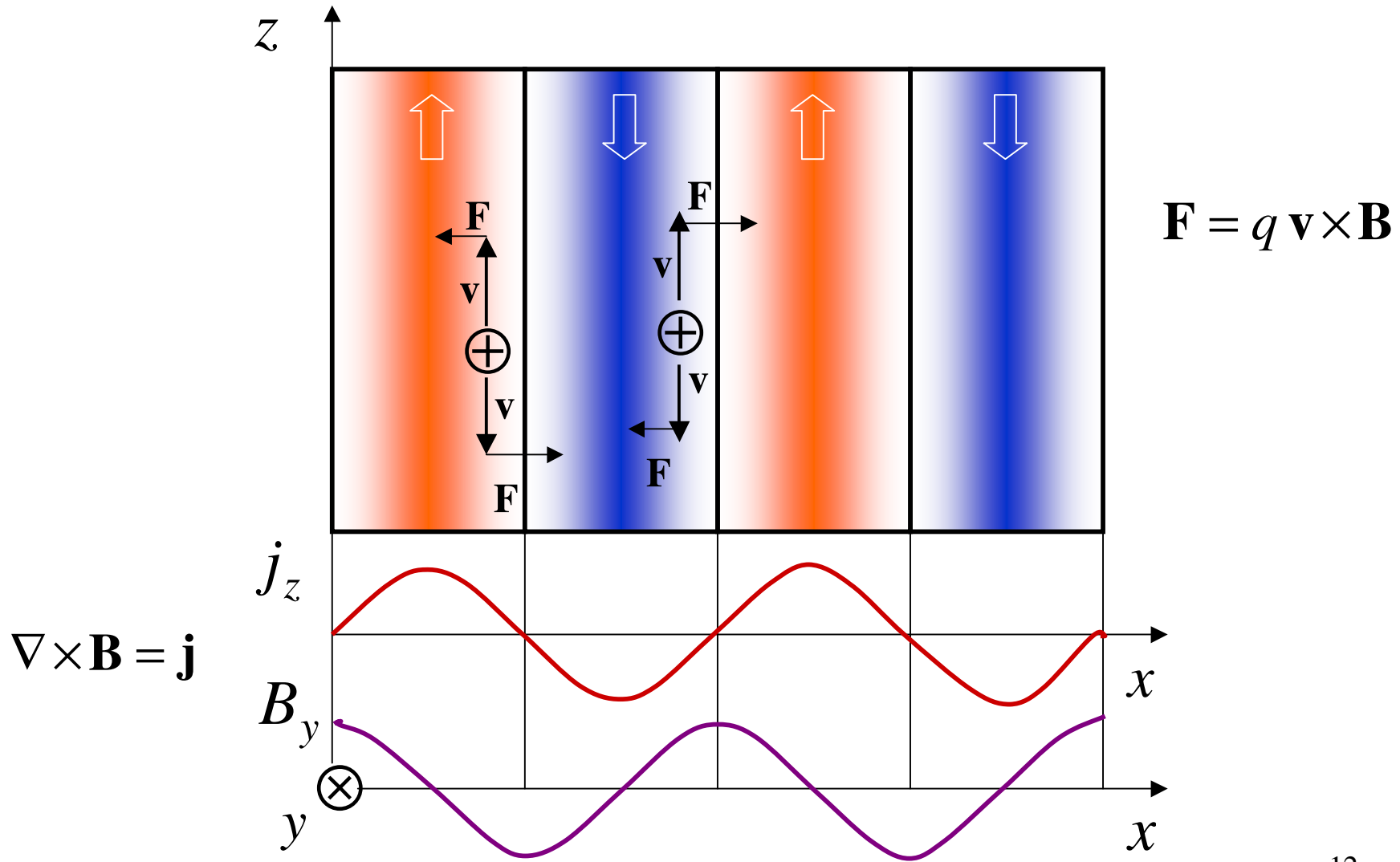
$$x_1 = (t_1, \mathbf{x}_1), \quad x_2 = (t_2, \mathbf{x}_2), \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$



Direction of the momentum surplus



Mechanism of filamentation



Dispersion equation

Equation of motion of chromodynamic field A^μ in momentum space

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] A_\nu(k) = 0$$

gluon self-energy

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k^\mu \equiv (\omega, \mathbf{k})$$

Instabilities – solutions with $\text{Im}\omega > 0$ $\Rightarrow A^\mu(x) \sim e^{\text{Im}\omega t}$

Dynamical information is hidden in $\Pi^{\mu\nu}(k)$. How to get it?

Transport theory – transport equations

$$\begin{array}{l}
 \text{fundamental} \left\{ \begin{array}{l}
 (p_\mu D^\mu - gp^\mu F_{\mu\nu}(x) \partial_p^\nu) Q(p, x) = C \\
 (p_\mu D^\mu + gp^\mu F_{\mu\nu}(x) \partial_p^\nu) \bar{Q}(p, x) = \bar{C}
 \end{array} \right. \\
 \text{adjoint} \left\{ \begin{array}{l}
 (p_\mu \mathcal{D}^\mu - gp^\mu \mathcal{F}_{\mu\nu}(x) \partial_p^\nu) G(p, x) = C_g
 \end{array} \right.
 \end{array}$$

free streaming

mean-field force

collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \dots], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu [Q, \bar{Q}, G]$$

mean-field generation

$$\text{collisionless limit: } C = \bar{C} = C_g = 0$$

Transport theory - linearization

fluctuation

$$Q(p, x) = Q_0(p) + \delta Q(p, x)$$

stationary colorless state $Q_0^{ij}(p) = \delta^{ij} n(p)$

$$|Q_0(p)| \gg |\delta Q(p, x)|, \quad |\partial_p^\mu Q_0(p)| \gg |\partial_p^\mu \delta Q(p, x)|$$

Linearized transport equations

$$p_\mu D^\mu \delta Q(p, x) - gp^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p) = 0$$

$$p_\mu D^\mu \delta \bar{Q}(p, x) + gp^\mu F_{\mu\nu}(x) \partial_p^\nu \bar{Q}_0(p) = 0$$

$$p_\mu \mathcal{D}^\mu \delta G(p, x) - gp^\mu \mathcal{F}_{\mu\nu}(x) \partial_p^\nu G_0(p) = 0$$

Transport theory – polarization tensor

$$\delta Q(p, x) = g \int d^4 x' \Delta_p(x - x') p^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p)$$



$$j^\mu[\delta Q, \delta \bar{Q}, \delta G]$$

$$p_\mu D^\mu \Delta_p(x) = \delta^{(4)}(x)$$



$$j^\mu(k) = \Pi^{\mu\nu}(k) A_\nu(k)$$

$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 2n_g(\mathbf{p})$$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2} \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu}{E} \left[g^{\nu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+} \right] \frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

Diagrammatic Hard Loop approach

$$\Pi^{\mu\nu}(k) = \left(\begin{array}{c} \text{Diagram 1: } \text{Wavy line } k \text{ enters from left, } k \text{ exits to right, } p \text{ on top arc, } p+k \text{ on bottom arc.} \\ \text{Diagram 2: } \text{Wavy line } k \text{ enters from left, } k \text{ exits to right, } p \text{ on top wavy loop, } p+k \text{ on bottom wavy loop.} \\ \text{Diagram 3: } \text{Wavy line } k \text{ enters from left, } k \text{ exits to right, } p \text{ on top wavy loop, } k \text{ on bottom wavy loop.} \end{array} \right)$$

Hard loop approximation: $k^\mu \ll p^\mu$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2} \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu}{E} \left[g^{\nu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+} \right] \frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

Dispersion equation

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} - \frac{1}{\omega^2} \Pi^{ij}(k) \quad \text{chromodielectric tensor}$$

$k^\mu \equiv (\omega, \mathbf{k})$

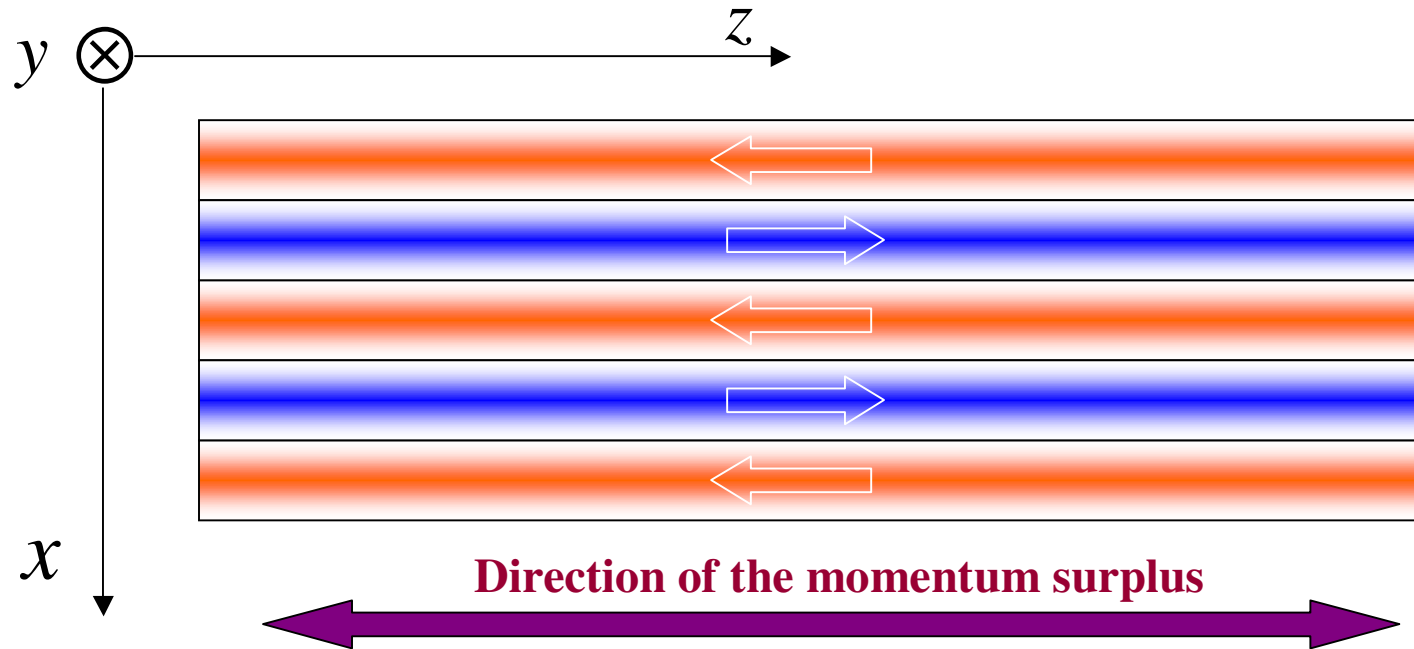
Dispersion equation

$$\det[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k)] = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{g^2}{2\omega} \int \frac{d^3 p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{k}\mathbf{v} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l} \left[\left(1 - \frac{\mathbf{k}\mathbf{v}}{\omega}\right) \delta^{lj} + \frac{k^l v^j}{\omega} \right]$$

$$\mathbf{v} \equiv \mathbf{p} / E$$

Dispersion equation – configuration of interest



$$\mathbf{j} = (0, 0, j), \quad \mathbf{E} = (0, 0, E), \quad \mathbf{k} = (k, 0, 0)$$

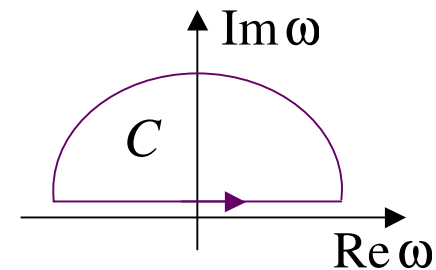
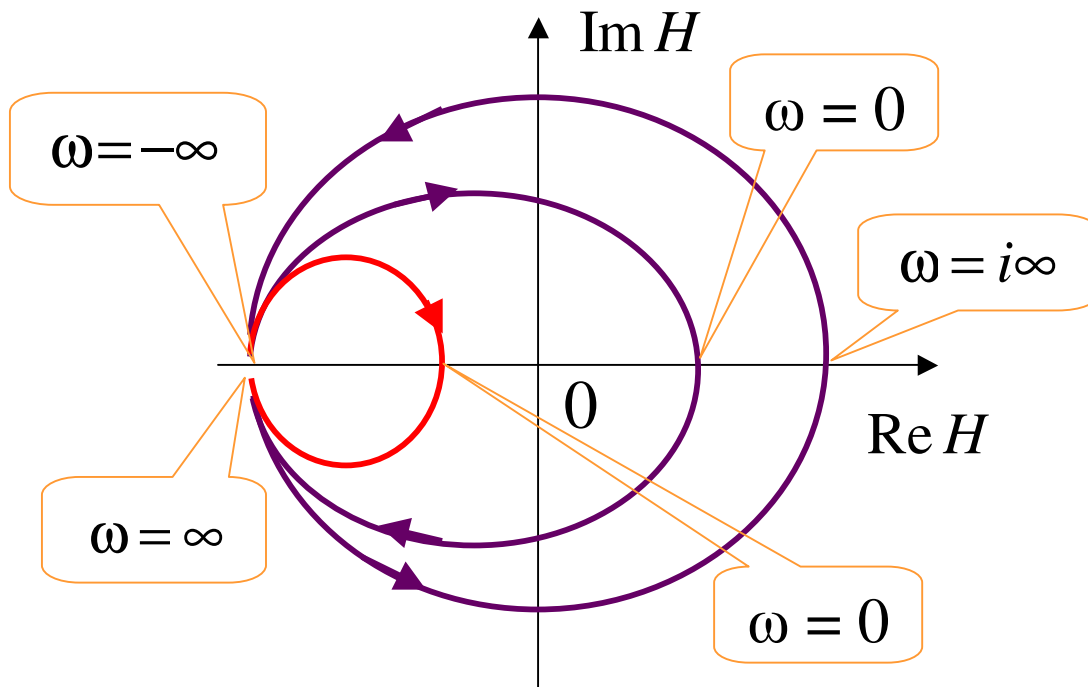
Dispersion equation

$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

Existence of unstable modes – Penrose criterion

$$H(\omega) \equiv k^2 - \omega^2 \varepsilon^{zz}(\omega, k)$$

$$\oint_C \frac{d\omega}{2\pi i} \frac{1}{H(\omega)} \frac{dH(\omega)}{d\omega} = \begin{cases} \oint_C \frac{d\omega}{2\pi i} \frac{d \ln H(\omega)}{d\omega} = \ln H(\omega) \Big|_{\phi=\pi^+}^{\phi=\pi^-} \\ \text{number of zeros of } H(\omega) \text{ in } C \end{cases}$$



There are unstable modes if

$$H(\omega = 0) < 0$$

Unstable solution

$$f(\mathbf{p}) = \frac{2^{1/2}}{\pi^{3/2}} \frac{\rho \sigma_{\perp}^4}{\sigma_{\parallel}} \frac{1}{(p_{\perp}^2 + \sigma_{\perp}^2)^3} e^{-\frac{p_{\parallel}^2}{2\sigma_{\parallel}^2}}$$

$$\rho = 6 \text{ fm}^{-3}$$

$$\alpha_s = g^2 / 4\pi = 0.3$$

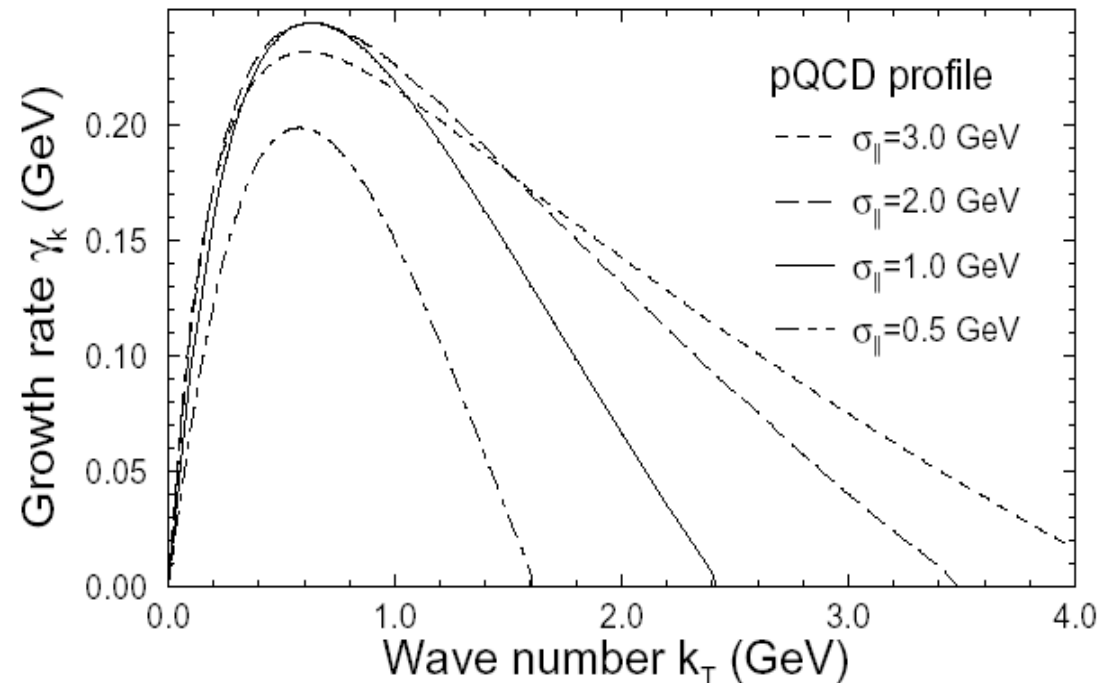
$$\sigma_{\perp} = 0.3 \text{ GeV}$$

$$k^2 - \omega^2 \epsilon^{zz}(\omega, k) = 0$$

solution

$$\omega(k) = \pm i \gamma_k$$

$$0 < \gamma_k \in \Re$$



Growth of instabilities – numerical simulation

Classical system of colored particles & fields

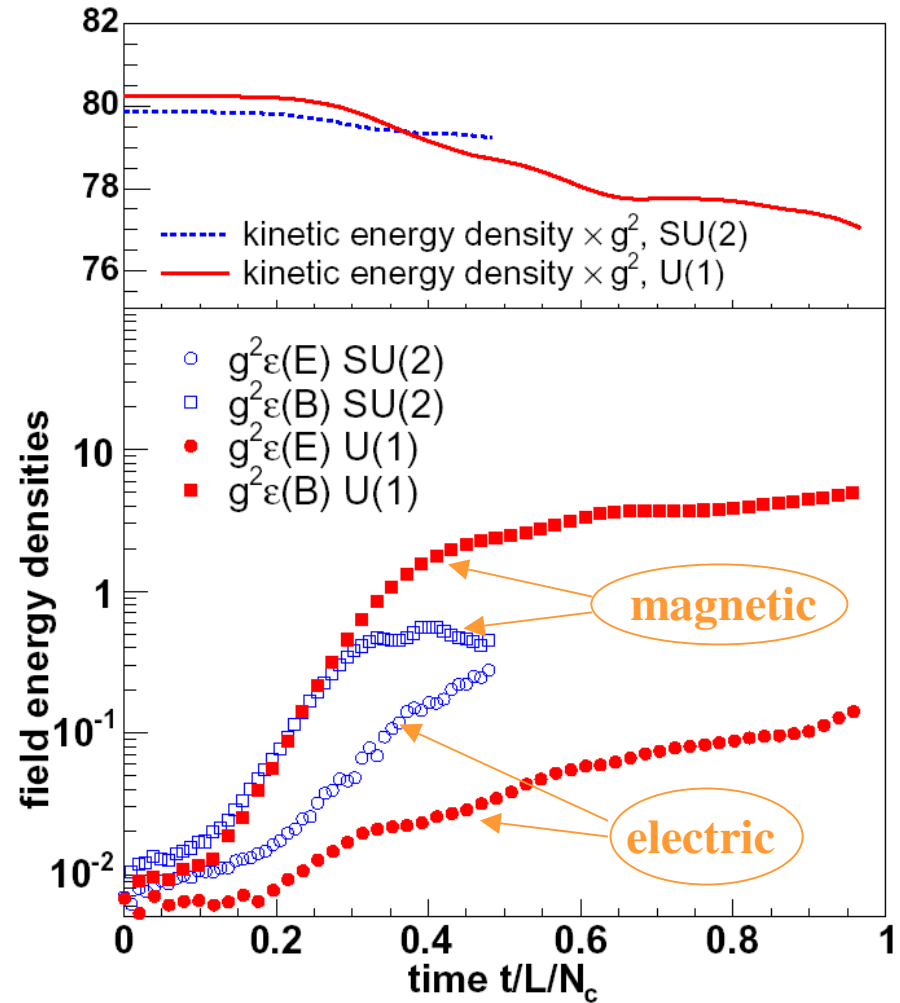
initial fields: Gaussian noise as in
Color Glass Condensate

initial particle distribution:

$$f_0(\mathbf{p}, \mathbf{x}) \sim \delta(p_x) e^{-\frac{\sqrt{p_y^2 + p_z^2}}{p_{\text{hard}}}}$$

$$p_{\text{hard}} = 10 \text{ GeV}$$

$$L = 40 \text{ fm} \quad \rho = 10 \text{ fm}^{-3}$$



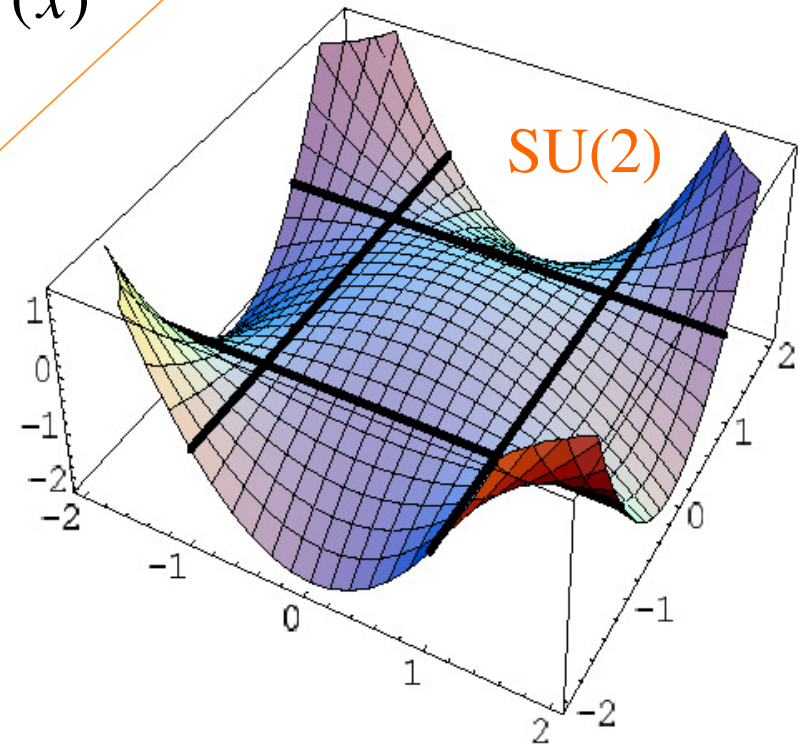
Abelianization

$$V_{\text{eff}}[\mathbf{A}^a] = -\mu^2 \mathbf{A}^a \cdot \mathbf{A}^a + \frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

the gauge $A_0^a = 0$, $A_i^a(t, x, y, z) = A_i^a(x)$

$$\begin{aligned} \mathcal{L}_{\text{YM}} &= -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} = -\frac{1}{2} \mathbf{B}^a \mathbf{B}^a \\ &= -\frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e) \end{aligned}$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a + \frac{g}{2} f_{abc} \mathbf{A}^b \times \mathbf{A}^c$$

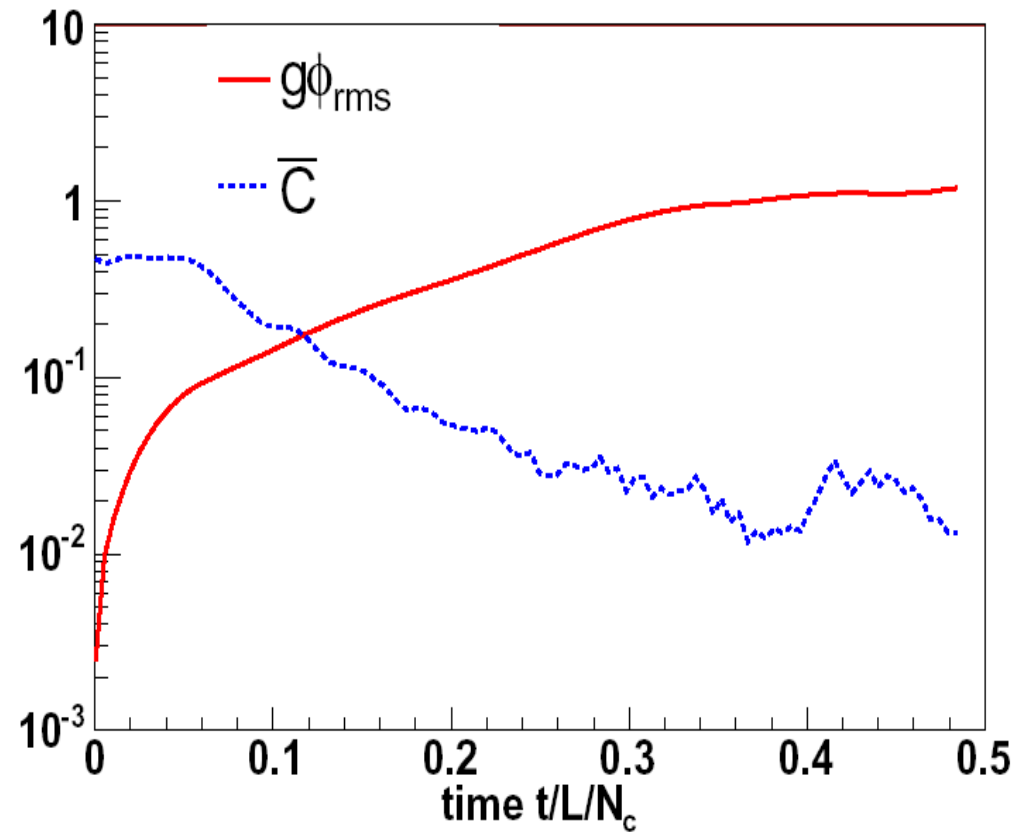


Abelianization – numerical simulation

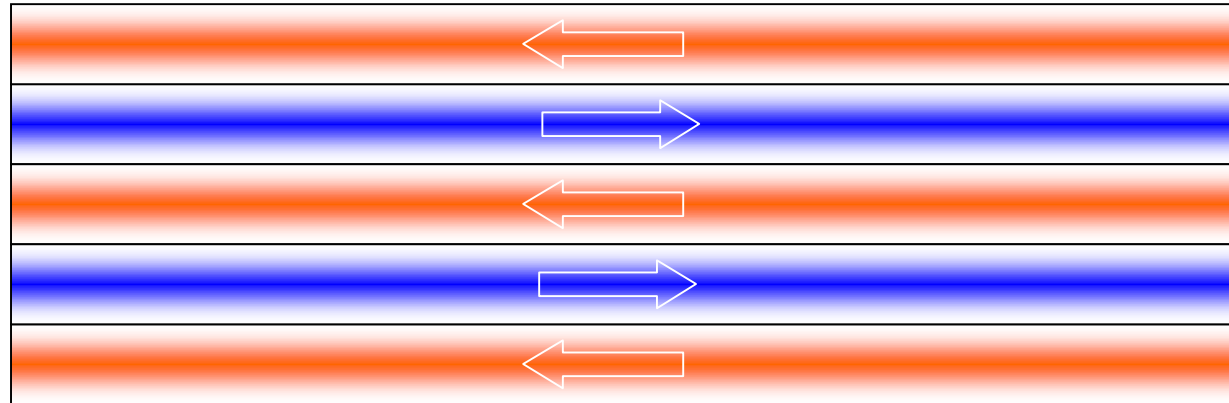
Classical system of colored particles & fields

$$\phi_{\text{rms}} = \sqrt{\int_0^L \frac{dx}{2L} \text{Tr}[\mathbf{A}^2]}$$

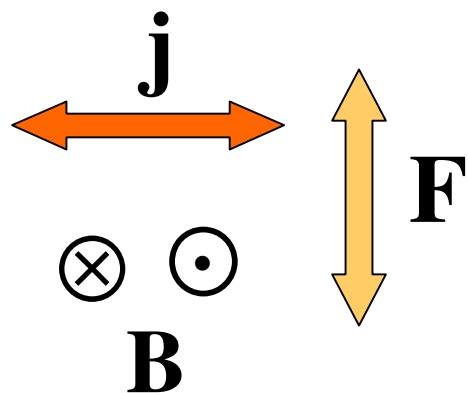
$$\bar{C} = \int_0^L \frac{dx}{L} \frac{\sqrt{\text{Tr}((i[A_y, A_z])^2)}}{\text{Tr}[\mathbf{A}^2]}$$



Isotropization - particles

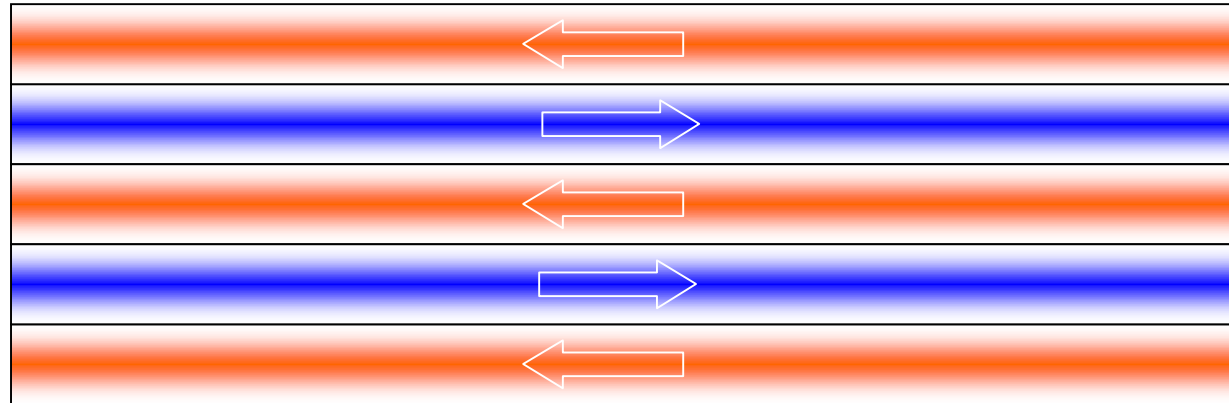


Direction of the momentum surplus

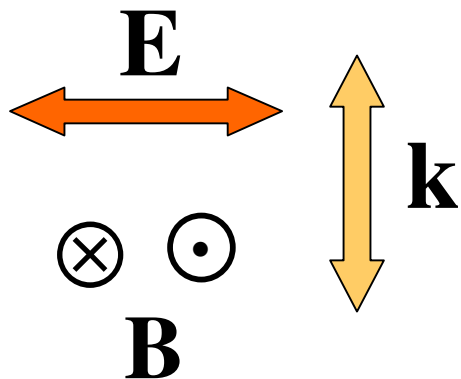


$$\Delta p = \int dt \mathbf{F}$$

Isotropization - fields



Direction of the momentum surplus



$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^a \times \mathbf{E}^a \sim \mathbf{k}$$

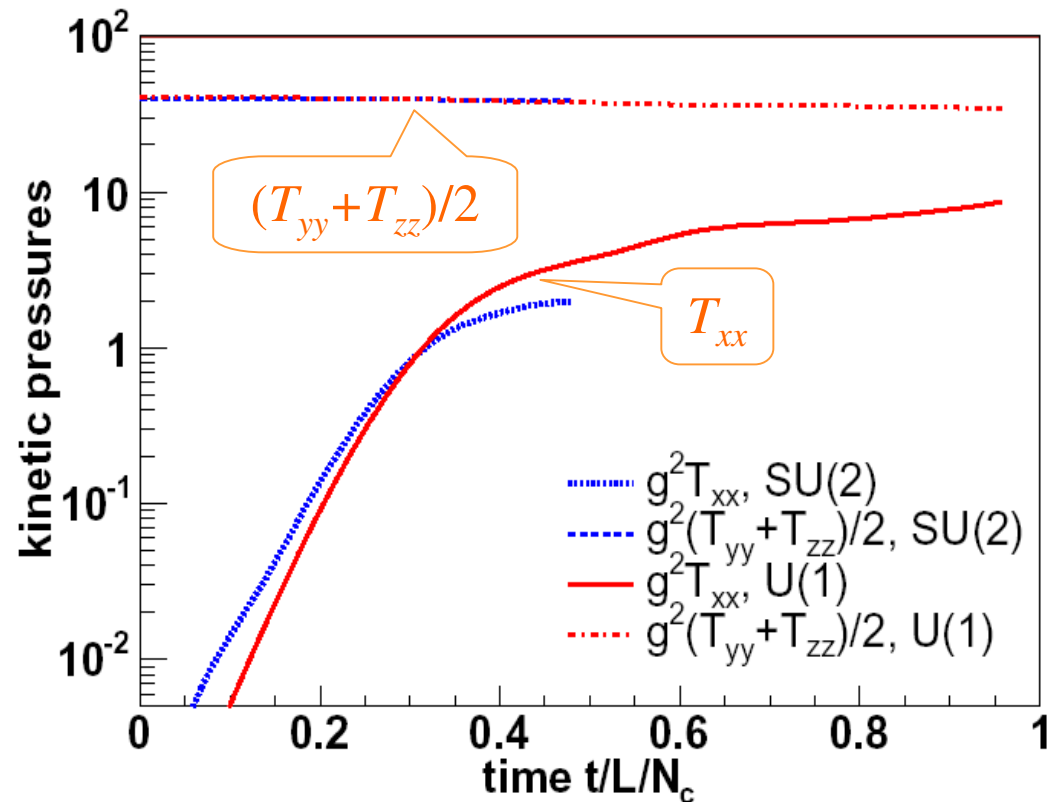
Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3 p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

Isotropy:

$$T_{xx} = (T_{yy} + T_{zz}) / 2$$



Isotropization vs. equilibration

Three comments:

Isotropization is a mean-field phenomenon which is not associated with the entropy production.

Collisions are needed for equilibration.

After the stage of instabilities, the system is in pre-equilibrium.

Collective flow in preequilibrium

Two comments:

Elliptic flow starts in preequilibrium stage.

Approximate hydrodynamics requires not equilibrium but merely isotropic momentum distribution.

Hydrodynamic equation

$$\partial_{\mu} T^{\mu\nu} = 0$$

Energy-momentum tensor of ideal fluid

$$T^{\mu\nu} = (\varepsilon + p)u^{\mu}u^{\nu} - p g^{\mu\nu}$$

Equation of state?

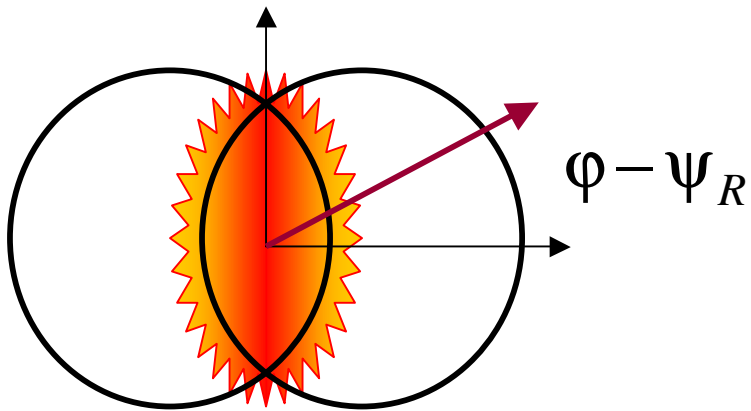
Equilibrium vs. preequilibrium

Q: How to distinguish equilibrium from preequilibrium collective flow?

A: Look for flow fluctuations.

Elliptic flow

$$P(\varphi) = \frac{1}{2\pi} \left[1 + 2 \sum_n v_n \cos(n(\varphi - \psi_R)) \right]$$



$$v_2 = \langle \cos(2(\varphi - \psi_R)) \rangle$$

Elliptic flow fluctuations

$$R \equiv \cos(\Psi_R - \Psi_E)$$

$$\langle v_2 \rangle = \frac{\langle \overline{\cos(2(\varphi - \Psi_E))} \rangle}{\langle R \rangle}$$

— averaging over particles
 ... from a single event

$$\text{Var}(v_2) \equiv \langle v_2^2 \rangle - \langle v_2 \rangle^2$$

$\langle \dots \rangle$ averaging over events

$$\text{Var}(v_2) = \frac{1}{\langle R \rangle^2} \left(\langle \overline{\cos(2(\varphi - \Psi_E))^2} \rangle - \langle \overline{\cos(2(\varphi - \Psi_E))} \rangle^2 \right)$$

Statistical noise

Due to the finite particle multiplicity

$$\text{Var}(v_2) = \frac{1}{2\langle R \rangle^2 \langle N \rangle} + \langle v_2 \rangle^2 \frac{\text{Var}(R)}{\langle R \rangle^2}$$

N number of particles used
to compute $\cos(\varphi - \psi_E)$

M number of particles used
to determine ψ_R

$$\psi_R - \psi_E \rightarrow 0$$

$$\langle (\psi_R - \psi_E)^2 \rangle \sim \frac{1}{\langle M \rangle}$$

$$\langle R \rangle^2 = \langle \cos(2(\psi_R - \psi_E)) \rangle^2 \approx \left(1 - 2\langle (\psi_R - \psi_E)^2 \rangle \right)^2 \approx \left(1 - \frac{2\alpha}{\langle M \rangle} \right)^2 \approx 1 - \frac{4\alpha}{\langle M \rangle}$$

$$\langle R^2 \rangle = \langle \cos^2(2(\psi_R - \psi_E)) \rangle \approx 1 - 4\langle (\psi_R - \psi_E)^2 \rangle \approx 1 - \frac{4\alpha}{\langle M \rangle}$$

$$\text{Var}(R) \sim \frac{1}{\langle M \rangle^2} \ll \frac{1}{\langle N \rangle}$$

Statistical noise

$$\delta v_2 = \frac{1}{\langle R \rangle \sqrt{2\langle N \rangle}}$$

$$\delta v_2 \equiv \sqrt{\text{Var}(v_2)}$$

$$\langle R \rangle \sim 1 \quad \& \quad \langle N \rangle \sim 10^3 \quad \Rightarrow \quad \delta v_2 \sim 10^{-2}$$

Fluctuations due to b - variation

$$\delta v_2 = \frac{d\langle v_2 \rangle}{db} \delta b$$

$$b \rightarrow N_p \quad N_p = 2Z \left(1 - \frac{b}{b_{\max}} \right)$$

$$\delta v_2 \approx 8 \times 10^{-4} \delta N_p$$

$$\delta N_p \sim 10 \Rightarrow \delta v_2 \sim 10^{-2}$$

Elliptic flow fluctuations

Statistical noise & b variation are under control

Dynamical elliptic flow fluctuations seem to be measurable

Integrated azimuthal fluctuations & Φ -measure

▶ single particle's variable $z = \varphi - \bar{\varphi}$ $\overline{\dots}$ inclusive averaging

▶ event's variable $Z = \sum_{i=1}^N (\varphi_i - \bar{\varphi})$ $\langle \dots \rangle$ averaging over events

$$\langle Z \rangle = 0$$

▶ Φ -measure

$$\Phi = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}$$

$\Phi = 0$ no correlations

Φ-measure of flow fluctuations

$$P_{\text{ev}}(\varphi) = \frac{1}{2\pi} \left[1 + 2 \sum_n v_n \cos(n(\varphi - \psi_n)) \right]$$

$$\Phi \approx \frac{3}{2\pi^2} \langle N \rangle \begin{cases} \left\langle \sum_n \left(\frac{v_n}{n} \right)^2 \right\rangle & \text{for } \langle \psi_n \psi_m \rangle = \langle \psi_n \rangle \langle \psi_m \rangle \\ & n \neq m \\ \left\langle \left(\sum_n \frac{v_n}{n} \right)^2 \right\rangle & \text{for } n\psi_n + \alpha_n = \psi_R \end{cases}$$

no flow fluctuations!

$$\langle N \rangle = 10^3, \quad v_1 = v_2 = 0.03, \quad v_n = 0, \quad n \geq 3$$

$$0.34 \leq \Phi \leq 0.62$$

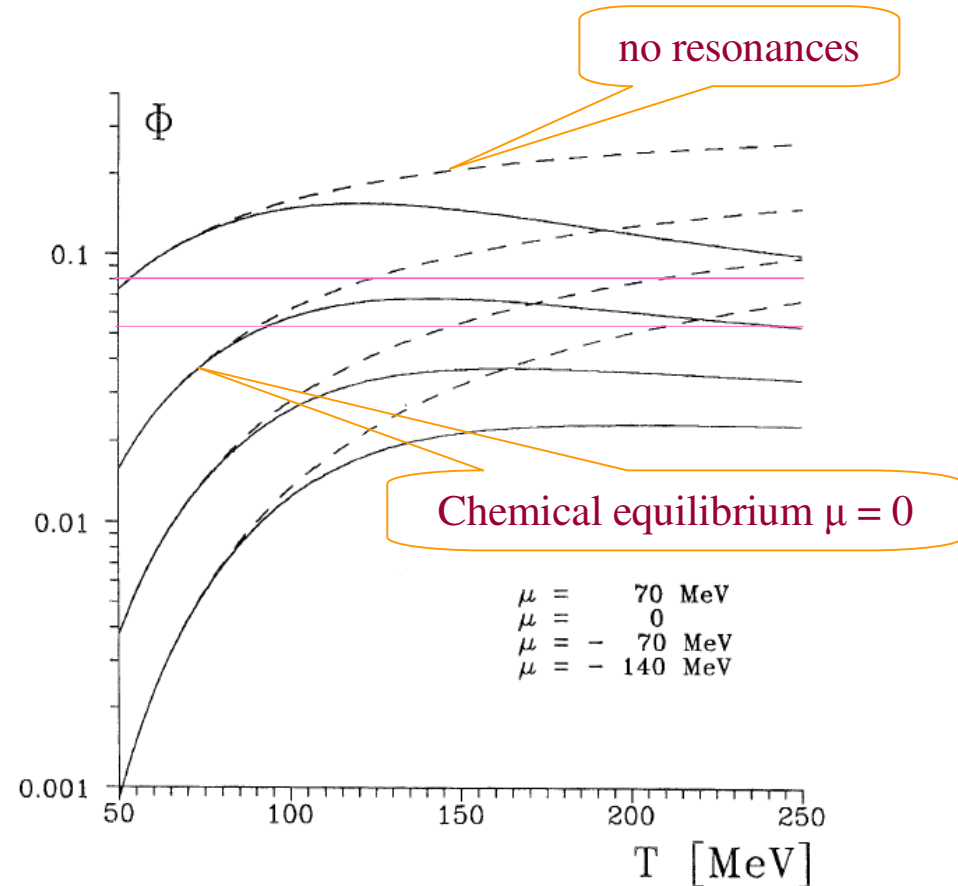
Non-flow azimuthal fluctuations

Bose-Einstein correlations

$$\Phi = \frac{\pi}{\sqrt{3}} \left(\sqrt{\frac{\tilde{\rho}}{\rho}} - 1 \right)$$

$$\rho = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta(E-\mu)} - 1}$$

$$\tilde{\rho} = \int \frac{d^3 p}{(2\pi)^3} \frac{e^{\beta(E-\mu)}}{(e^{\beta(E-\mu)} - 1)^2}$$



Conclusions

Azimuthal fluctuations can tell us whether the elliptic flow is generated in the fully equilibrated sQGP or in the pre-equilibrium pQGP driven by instabilities