Exclusive Electroproduction of Φ Mesons in the ZEUS Experiment

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Abstract

Electroproduction of Φ mesons in $ep \to ep\Phi$ ($\Phi \to K^+K^-$) reaction, in the kinematical range 0.25 GeV^2 $< Q^2 <$ 0.85 GeV^2, 25 GeV $< W <$ 90 GeV, $|t| <$ 0.6 GeV^2 has been studied using the ZEUS detector. Integrated cross section of $\sigma_{\gamma^* p \to \phi p}$=343.0 $\pm$ 25.0$(stat.)^{+34}_{-32}(syst.)$ nb for the entire kinematical range has been obtained. The dependence of the cross section on W and Q^2 has also been studied and ratio $R = \sigma_L/\sigma_T$ determined.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Understanding the structure of the matter has always been one of the main goals of physics. A significant progress in this domain was made by discovery of the electron by Thomson in 1897. In 1911 Rutherford carried out an experiment of scattering α particles on a thin gold foil which led to discovery of the atomic structure.

Further research has led to discovery of the constituents of the nucleus - protons and neutrons and later many other particles. In the 1964 Murray Gell-Mann and George Zweig independently put forward a hypothesis that strongly interacting particles, hadrons, consist of more fundamental objects which they called quarks. Introduction of quarks made possible understanding of the variety of known particles.

Structure of the nucleons (proton, neutron) has been studied in the lepton (electrons, muons ,neutrinos) scattering experiments. Experiments performed in the '70-s in the American centre SLAC (Stanford Linear Acceleration Center) confirmed the hypothesis of the quark structure of hadrons.

In order to obtain higher values of the centre-of-mass energy, construction of the electron-proton collider was proposed. It has been built in the scientific centre DESY (Deutsches Elektronen-SYnchrotron) in Hamburg. The HERA accelerator collides the electron beam of energy up to 30 GeV with the proton beam of energy 820 GeV, reaching centre-of-mass energy $\sqrt{s} \approx 300$ GeV. The main investigations concern :

- diffraction
- photoproduction
- the proton structure functions
- testing Quantum Chromodynamics (QCD)
- *ep* scattering with exchange of charged currents
- search for signals of new hypothetic phenomena beyond the Standard Model (substructure of the quarks and leptons, leptoquarks)

Particular attention is paid to exclusive vector meson production : $ep \to epV^0$ ($V^0 = \rho^0, \omega, \Phi, J/\psi$). It was observed in fixed target experiments [1, 2] that these processes reveal features characteristic of diffraction: weak energy dependence of cross sections, an exponential fall-off of the differential cross section $d\sigma/dt$ (t is the square of the four-momentum transfer at the proton vertex) and conservation of helicity in the s-channel (SCHC). The cross sections for exclusive photoproduction ($Q^2 \approx 0$, where Q^2 is the square of the four-momentum of the virtual photon), of light vector mesons were measured in numerous fixed target experiments [1, 2, 3, 4] and also at HERA [5, 6, 7].

These results are well described in the framework of the Regge theory (soft phenomena) [8] and the Vector Meson Dominance (VMD) model [9]. However both fail to describe

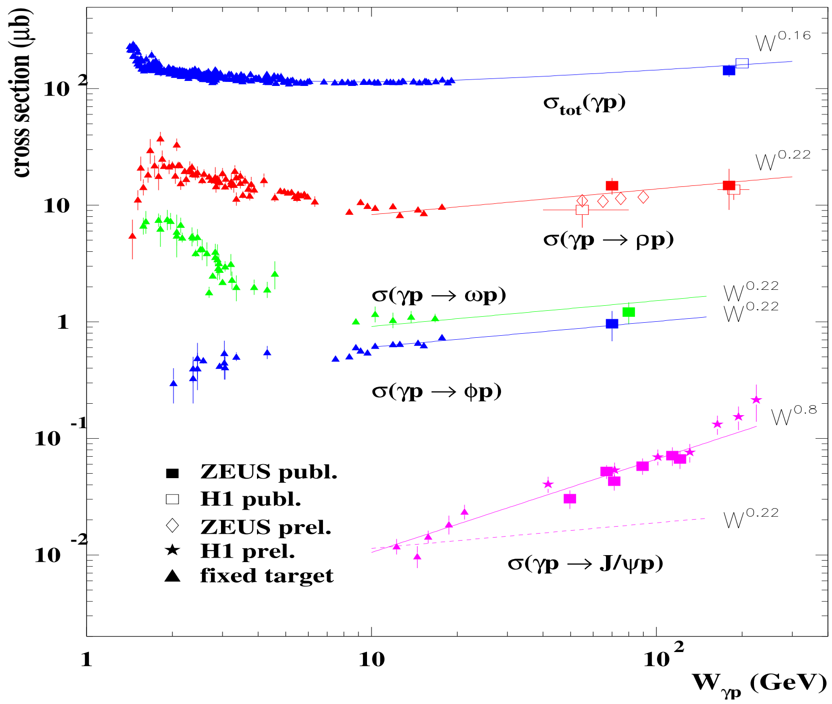

Figure 1.1:

the rapidly raising cross sections for exclusive photoproduction of heavy vector mesons (e.g. J/ψ) [10, 11, 12]. Which is on the other hand well described in the framework of the perturbative QCD (pQCD) [17, 18] (hard phenomena). In the calculations of exclusive J/ψ production, the large scale of the process, which justifies the perturbative approach, is provided by the mass of J/ψ ($M_{J/\psi} \simeq 3.1$ GeV). It is expected that the pQCD may become applicable also to exclusive production of light vector mesons, at sufficiently high values of Q^2 ($Q^2 \gtrsim 20$ GeV^2). The recent measurements of exclusive production cross sections for ρ^0 and Φ in deep inelastic scattering (DIS) for $Q^2 \gtrsim 5$ GeV^2 [13, 14, 15] may be interpreted as an indication that the energy dependence of cross sections may be stronger then in photoproduction (the data cover the Q^2 range up to $Q^2 \sim 27$ GeV^2). The interplay of soft and hard phenomena and the onset of hard phenomena in exclusive vector meson production still remains to be clarified. This task requires measurements of cross sections in a wide range of Q^2. The purpose of this analysis was to measure the cross sections for exclusive Φ production, $ep \to ep\Phi$, in the low Q^2 region (0.25 GeV$^2 < Q^2 <$ 0.85 GeV^2), which remained unexplored, for the centre-of-mass energies of the photon-proton system 25 GeV $<$ W $<$ 90 GeV. This kinematical region is important because it lies between the photoproduction and DIS. Recent results from exclusive ρ^0 production analysis [16] indicate that the energy dependence of the cross sections in the above region of Q^2 is similar to that measured in photoproduction and is well described by the Regge phenomenology. Because the mass of the Φ meson ($M_\Phi \simeq 1.02$ GeV) is larger then that of ρ^0 ($M_\rho \simeq 0.77$ GeV) it is interesting to compare the behaviour of the cross sections as a function of energy in these two cases.

Another motivation for the present analysis was to investigate a possibility of using particle identification (by means of energy loss, dE/dx) in cross section determination.

This analysis was performed using the data collected with the ZEUS detector in the 1996 running period. Small values of Q^2 became accessible for the physical analysis owning to the installation of the Beam Pipe Calorimeter (BPC) which enables detection of positrons scattered at very small angles (17 mrad $< \theta <$ 35 mrad).

Chapter 2

Electroproduction of Φ mesons

2.1 Kinematics

Fig. 2.1 shows the diagram describing scattering of electrons on protons in the first order of perturbation theory with the exchange of the electroweak gauge boson $(\gamma^*, Z^0, W^\pm)$ [1]. The variables are defined as follows :

k	:four-momentum of the incoming positron
k'	:four-momentum of the scattered positron
P	:four-momentum of the incoming proton
P'	:four-momentum of the scattered proton
q=k-k'	:four-momentum of the exchanged gauge boson

Using the above notation the following kinematical variables are introduced to describe the *ep* interactions :

- Centre-of-mass energy, $\sqrt{s}$:

$$s \equiv (k+P)^2 = m_e^2 + m_p^2 + 2(E_e E_p - \vec{p}_e \vec{p}_p) \simeq 4E_e E_p \quad , \tag{2.1}$$

  where E_e and E_p are the positron and proton beam energies respectively. In 1996 these were : E_e=27.5 GeV, E_p=820 GeV, resulting in the centre-of-mass energy of $\sqrt{s} \simeq 300$ GeV.

- The negative four-momentum squared of the virtual photon, Q^2 :

$$Q^2 \equiv -q^2 = -(k-k')^2 = -2m_e^2 + 2kk' = -2m_e^2 + 2(E_e E_e' - \vec{p}_e \vec{p}_e\,') \quad , \tag{2.2}$$

  where E_e', $\vec{p}_e\,'$ are energy and momentum of the scattered electron.

[1] γ^* denotes the virtual photon, beyond the mass threshold ($Q^2 \neq 0$)

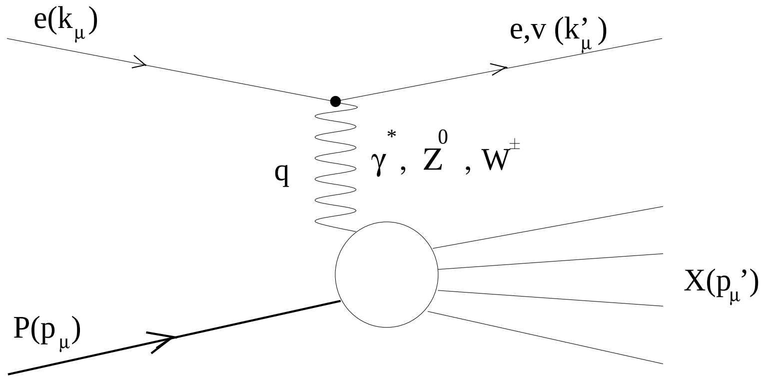

Figure 2.1: *The first order diagram describing process of $ep \to e\ (\nu)X$ scattering. For the description of variables - see text.*

- the Bjorken scaling variable, x :

$$x \equiv \frac{Q^2}{2P \cdot q} \quad , \tag{2.3}$$

which has the interpretation of the fraction of the total momentum of the proton carried by the struck quark (as viewed in the infinite momentum frame).

- the Bjorken variable, y :

$$y \equiv \frac{P \cdot q}{P \cdot k} = \frac{\nu}{\nu_{max}} \quad , \tag{2.4}$$

where ν is the electron energy transfered to the proton in the proton rest frame and is given by :

$$\nu \equiv (E_e - E'_e)_{p\,rest\,frame} = \frac{P \cdot q}{m_p}$$

and $\nu_{max} \approx s/2m_p$ is the maximal energy that can be transfered to the proton (electron energy in the proton rest frame).

- Energy of the γ^* p centre-of-mass system, W :

$$W^2 \equiv (P+q)^2 = m_p^2 - Q^2 + 2P \cdot q = m_p^2 - Q^2 + ys \quad . \tag{2.5}$$

- four-momentum transfer squared at the proton vertex, t :

$$t \equiv (P - P')^2 \ . \tag{2.6}$$

Fig. 2.2 shows the diagram of the vector meson production in ep interactions with the Pomeron exchange.

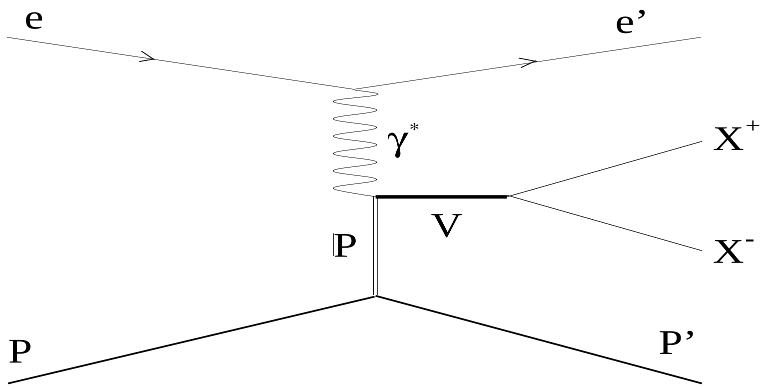

Figure 2.2: *Diagram of the vector meson electroproduction with the Pomeron exchange.*

2.2 Cross Sections

2.2.1 *ep* cross section

The differential cross section of ep neutral current interaction via the γ^* exchange can be expressed in terms of two structure functions F_1 and F_2 by the following formula [39]:

$$\frac{d\sigma}{dydQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(F_1\left(1-\frac{2m_e^2}{Q^2}\right)xy + F_2\left(\frac{1-y}{y}+\frac{m_p^2 x}{s}\right)\right) \quad , \tag{2.7}$$

where α is the electromagnetic coupling constant. In the quark-parton model (QPM) the structure functions F_1 and F_2 are expressed as :

$$2F_1(x) = \sum_i e_i^2(q_i(x) + \bar{q}_i(x)) \tag{2.8}$$

$$F_2(x) = \sum_i e_i^2 x(q_i(x) + \bar{q}_i(x)), \tag{2.9}$$

where $q_i(x)(\bar{q}_i(x))$ denote the probability of finding a quark(antiquark) of flavour i and charge e_i carrying fraction x of the proton four-momentum and the summation is done over all quark flavours. In this model structure functions depend only on the scaling variable x. The following relation, called Callan-Gross relation holds [44]:

$$2xF_1(x) = F_2(x) \quad . \tag{2.10}$$

2.2.2 $\gamma^* p$ cross section

The discussion of $\gamma^* p$ interaction is restricted to the proton rest frame, thus four-momentum of the photon is $q = (\nu, \vec{q})$. The total cross section for scattering real photons (γ) of polarisation ϵ_λ off unpolarised protons was calculated from the Feynman rules to be :

$$\sigma^{tot}(\gamma p \to X) = \frac{4\pi^2\alpha}{K}\epsilon_\lambda^{\mu^*}\epsilon_\lambda^\nu W_{\mu\nu} \quad , \tag{2.11}$$

where K is the flux factor. In case of the real photons $K = \nu$ and only transversal polarisations are possible $\lambda = 1, 2$. In order to obtain expression for $\sigma^{tot}(\gamma^* p \to X)$ one requires to know K and ϵ^{μ}_{λ} for the case of the virtual photons. The polarisation in this case is assumed to be polarisation of the massive spin 1 particle :

$$\epsilon^{\mu}_{\pm 1} = \pm\frac{1}{\sqrt{2}}(0, 1, \pm i, 0) \qquad \epsilon^{\mu}_{0} = \pm\frac{1}{\sqrt{Q^2}}(\sqrt{\nu^2 + Q^2}, 0, 0, \nu) \ . \tag{2.12}$$

The most popular choice of the flux factor is based on the convention by Hand [43]. It requires the flux factor K to be equal to the energy of the real photon needed to create an equivalent centre-of-mass energy in the photon-proton CMS. This leads to the following relation :

$$W^2 = \quad m_p^2 + 2m_p K = -Q^2 + 2m_p\nu + m_p^2$$

and yields the following formula for the flux factor of the virtual photons :

$$K_{Hand} = \nu - \frac{Q^2}{2m_p} \ . \tag{2.13}$$

With the above formulae for K_{Hand} and ϵ^{ν}_{λ} one obtains the following expressions for the total cross sections for polarised photons:

$$\sigma_T = \frac{1}{2}(\sigma^{tot}_{+} + \sigma^{tot}_{-}) = \frac{4\pi^2\alpha}{Km_p}F_1 \tag{2.14}$$

$$\sigma_L = \sigma^{tot}_0 = \frac{4\pi^2\alpha}{Km_p}\left(\frac{F_2}{2x} - F_1\right) \ . \tag{2.15}$$

After expressing Eq. 2.7 in terms of σ_L and σ_T one obtains the following formula relating the ep differential cross section and $\gamma^* p$ cross sections :

$$\frac{d^2\sigma}{dydQ^2} = \Gamma \cdot (\sigma_T + \epsilon\sigma_L) \ , \tag{2.16}$$

where photon polarisation ϵ and photon flux factor Γ are given by :

$$\epsilon = \frac{2(1-y)}{1+(1-y)^2} \tag{2.17}$$

$$\Gamma = \frac{\alpha(1+(1-y)^2)}{2\pi Q^2 y} \ . \tag{2.18}$$

Introducing the ratio R=σ_L/σ_T, Eq. 2.16 can be rewritten in the following form:

$$\frac{d^2\sigma}{dydQ^2} = \Gamma \cdot \sigma_T(1 + \epsilon R) \ . \tag{2.19}$$

In the presented analysis $\sigma_{tot} = \sigma_T + \sigma_L \approx \sigma_T + \epsilon\sigma_L$, since $< \epsilon > \approx 0.999$ in the data.

2.3 Theoretical overview

2.3.1 Vector Meson Dominance Model (VMD)

It was observed that at high energies photon-hadron interactions exhibit similar properties as hadron-hadron. It can be explained by photon fluctuations into a hadronic state with the same quantum numbers, $J^{PC} = 1^{--}$; Q=S=B=C=0, namely a vector meson. Thus the physical photon can be described as a superposition of a bare point-like photon and its hadronic component :

$$|\gamma >= |\gamma >_{bare} + |\gamma >_{hadronic} \quad . \tag{2.20}$$

The lifetime of the fluctuation into the hadronic state of mass M_V is given by [42]:

$$t_f = \frac{2\nu}{Q^2 + M_V^2} \quad . \tag{2.21}$$

Within the framework of the vector dominance model VMD [9] the hadronic component of the photon is assumed to be a superposition of contributions from the lightest vector mesons :

$$|\gamma >_{hadronic} = \sum_{V=\rho^0,\omega,\phi} \frac{M_V^2}{M_V^2 + Q^2} \frac{1}{f_V} |V > \tag{2.22}$$

where f_V is the γV coupling and $|V>$ denotes the wave function of the vector meson V. Using the above ansatz one can determine the following expressions for the total transversal $\sigma^T_{\gamma^* p}(W,Q^2)$ and longitudinal $\sigma^L_{\gamma^* p}(W,Q^2)$ cross sections :

$$\sigma_T^{\gamma^* p}(W,Q^2) = \sum_{V=\rho^0,\omega,\Phi} \frac{4\pi\alpha}{f_V^2} \left(\frac{M_V^2}{M_V^2 + Q^2} \right)^2 \sigma_T^{Vp\to Vp}(W) \tag{2.23}$$

$$\begin{aligned}
\sigma_L^{\gamma^* p}(W,Q^2) &= \sum_{V=\rho^0,\omega,\Phi} \frac{4\pi\alpha}{f_V^2} \left(\frac{M_V^2}{M_V^2 + Q^2} \right)^2 \frac{Q^2}{M_V^2} \sigma_L^{Vp\to Vp}(W) \\
&= \sum_{V=\rho^0,\omega,\Phi} \frac{4\pi\alpha}{f_V^2} \left(\frac{M_V^2}{M_V^2 + Q^2} \right)^2 \frac{Q^2}{M_V^2} \xi^2 \sigma_T^{Vp\to Vp}(W) \ ,
\end{aligned} \tag{2.24}$$

where $\sigma_L^{Vp\to Vp}$, $\sigma_T^{Vp\to Vp}$ are elastic $Vp \to Vp$ longitudinal and transverse cross sections, $\xi = \sigma_L^{Vp\to Vp} / \sigma_T^{Vp\to Vp}$.

Elastic Φ electroproduction cross section

The VMD relates the $\gamma p \to \Phi p$ to elastic $Vp \to Vp$ cross section as follows:

$$\sigma^{\gamma p\to Vp}(W) = \frac{4\pi\alpha}{f_V^2} \sigma^{Vp\to Vp}(W) \quad . \tag{2.25}$$

Using the above relation together with the Eq. 2.23 and 2.24 reduced to the Φ production case one obtains :

$$\sigma_T^{\gamma^* p\to\Phi p}(W,Q^2) = \left(\frac{M_\Phi^2}{M_\Phi^2+Q^2}\right)^2 \sigma_T^{\gamma^* p\to\Phi p}(W) \qquad (2.26)$$

$$\sigma_L^{\gamma^* p\to\Phi p}(W,Q^2) = \left(\frac{M_\Phi^2}{M_\Phi^2+Q^2}\right)^2 \xi^2 \frac{Q^2}{M_\Phi^2}\sigma_T^{\gamma^* p\to\Phi p}(W) \quad . \qquad (2.27)$$

From the above formulae it follows that within the VMD-model the previously defined ratio $R = \xi^2 \cdot Q^2/M_\Phi^2$. After substituting the above formulae into Eq. 2.16 the following formula is obtained :

$$\frac{d^2\sigma^{ep\to ep\Phi}}{dydQ^2} = \phi(y,Q^2)\cdot\sigma_T^{\gamma p\to\Phi p}(W) \quad , \qquad (2.28)$$

where $\phi(y,Q^2)$ is the effective photon flux :

$$\phi(y,Q^2) = \frac{\alpha}{2\pi}\frac{1}{Q^2}\left(\frac{1+(1-y)^2}{y} + \frac{2(1-y)}{y}\xi^2\frac{Q^2}{M_\Phi^2}\right)\left(\frac{M_\Phi^2}{M_\Phi^2+Q^2}\right)^2 \quad . \qquad (2.29)$$

Using the Eq. 2.17 and 2.18 one can express the *ep* cross section in the following form :

$$\frac{d^2\sigma^{ep\to ep\Phi}}{dydQ^2} = \Gamma(1+\epsilon R)\left(\frac{M_\Phi^2}{M_\Phi^2+Q^2}\right)^2 \sigma_T^{\gamma^* p\to\Phi^0 p}(W) \quad . \qquad (2.30)$$

2.3.2 Regge phenomenology

Long before formulation of QCD as a theory of strong interactions one of the most successive approaches were phenomenological models. Regge's idea of scattering amplitudes analytically continued in the complex momentum plane is the one which allows to estimate the characteristic features of the hadronic interactions behaviour in the high energy limit. Even now when the QCD is a well defined theory it is still very useful as perturbative QCD calculations are successful only at high energy scales. A detailed discussion of the Regge theory can be found elsewhere [8]. In this approach the t dependence of the differential cross section reads :

$$\frac{d\sigma}{d|t|} \propto e^{-b_0|t|}\left(\frac{W^2}{W_0^2}\right)^{2(\alpha(t)-1)} , \qquad (2.31)$$

where $\alpha(t) = \alpha(0) + \alpha' t$ is the Pomeron trajectory and W_0 is energy scale factor which was determined empirically to be comparable to the hadronic mass scale of 1 GeV [20, 21]. Fits to hadron-hadron and photoproduction data give $\alpha(0) = 1.085$ and $\alpha' = 0.25$ GeV^{-2} [22]. The cross section $d\sigma/dt$ can be rewritten in the form :

$$\frac{d\sigma}{d|t|} \propto e^{-b|t|}, \qquad (2.32)$$

where b=$b_0+2\alpha' ln\frac{W^2}{W_0^2}$. The W dependence of the cross section can obtained from Eq. 2.31 by integrating over t :

$$\sigma(W) \propto \frac{1}{b_0 + 2\alpha' ln\frac{W^2}{W_0^2}} \cdot \left(\frac{W}{W_0}\right)^{2(\alpha(0)-2)} \propto W^{0.22} \quad . \qquad (2.33)$$

In the case of light vector meson electroproduction, the above formula gives a good description of the W dependence of the cross sections in the small Q^2 region (Fig. 1.1).

2.3.3 Perturbative QCD Models for Vector Meson Production

The perturbative approach is justified when the energy scale, $\bar{q}^2$, involved in the process is large (i.e. $\alpha_s(\bar{q}^2)$ is small). The choice of the scale differs in various QCD models, e.g. in the model of Ryskin et al. [17] it is defined as :

$$\bar{q}^2 = \frac{M_V^2 + Q^2 + |t|}{2} \quad . \qquad (2.34)$$

In the case of the J/ψ electroproduction the hard scale is provided by its large mass ($M_{J/\psi} \simeq 3.1$ GeV). It is expected that the pQCD models should also describe the electroproduction of light vector mesons at higher values of Q^2 ($Q^2 \gtrsim 20$ GeV^2). The pQCD models predict a strong energy dependence of the cross sections, typically $\sigma(W) \propto W^{0.8-1.6}$.

2.3.4 Angular distributions

An important question is whether the helicity of the virtual photon is retained by the vector meson in the s-channel $\gamma^* p \to \Phi p$ (s-channel helicity conservation-SCHC). Analysis of angular distributions of the Φ meson decay products gives a possibility of testing SCHC hypothesis and determination of the ratio R=σ_L/σ_T. Fig. 2.3 shows the definitions of the angles used in the analysis. The angles are defined in the Φ meson rest frame where the axes are defined as follows : $\vec{x} - \vec{z}$ plane is the production plane, $\vec{y}$ is normal to this plane and $\vec{z}$ axis is opposite to the outgoing proton direction. In this frame the θ_h angle is defined as the angle between the momentum vector of the positive kaon from the Φ meson decay and the $\vec{z}$ axis, ϕ_h is the angle between the decay plane and the production plane and Φ_h is the angle between the electron scattering plane and the production plane. The general form of the angular distribution is given by the following formula [41]:

$$\begin{aligned} W(\cos\theta_h, \phi_h, \Phi_h) = & \\ & \frac{3}{4\pi}[\frac{1}{2}(1 - r_{00}^{04}) + \frac{1}{2}(3r_{00}^{04} - 1)\cos^2\theta_h \\ - & \sqrt{2}Re(r_{10}^{04})\sin 2\theta_h \cos\phi_h - r_{1-1}^{04}\sin^2\theta_h \cos 2\phi_h \end{aligned}$$

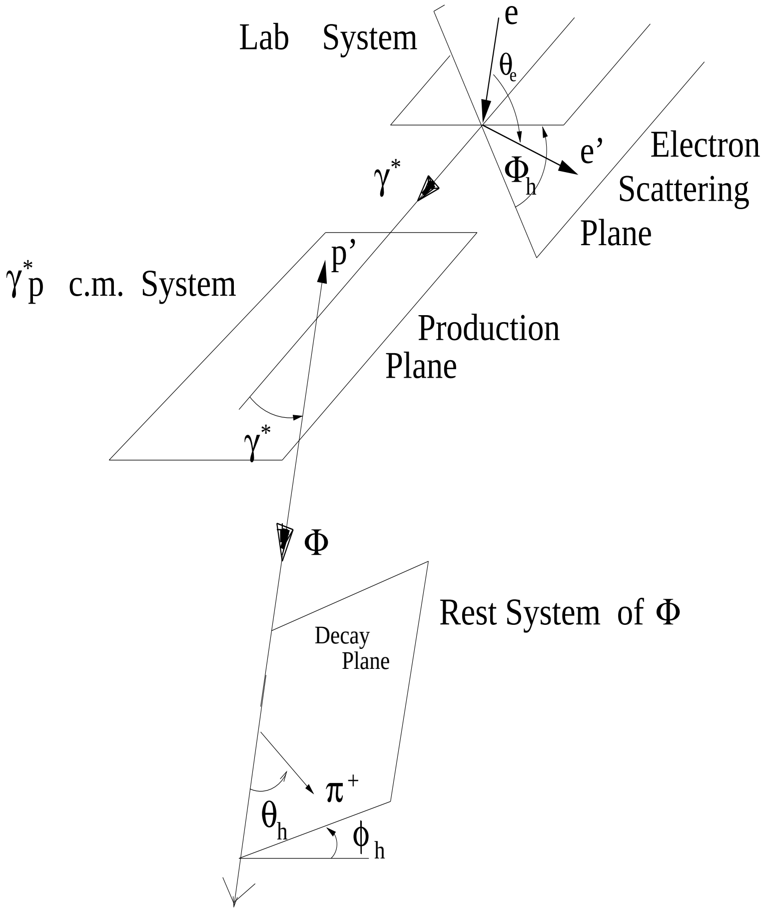

Figure 2.3: The Φ decay angles in the s-channel helicity system.

$$
\begin{aligned}
- &\quad \epsilon \cos 2\Phi_h \left(r^1_{11} \sin^2\theta_h + r^1_{00}\cos^2\theta_h - \sqrt{2}Re(r^1_{10}) \sin 2\theta_h \cos\phi_h - r^1_{1-1}\sin^2\theta_h \cos 2\phi_h \right) \\
- &\quad \epsilon \sin 2\Phi_h \left(\sqrt{2}Im(r^2_{10}) \sin 2\theta_h \sin\phi_h + Im(r^2_{1-1}) \sin^2\theta_h \sin 2\phi_h \right) \\
+ &\quad \sqrt{2\epsilon(1+\epsilon)} \cos\Phi_h \{ r^5_{11}\sin^2\theta_h + r^5_{00}\cos^2\theta_h - \sqrt{2}Re(r^5_{10}) \sin 2\theta_h \cos\phi_h \\
- &\quad r^5_{1-1} \sin^2\theta_h \cos 2\phi_h \} \\
+ &\quad \sqrt{2\epsilon(1+\epsilon)} \sin\Phi_h \{ \sqrt{2}Im(r^6_{10}) \sin 2\theta_h \sin\phi_h + Im(r^6_{1-1}) \sin^2\theta_h \sin 2\phi_h \}] \ , \qquad (2.35)
\end{aligned}
$$

where ϵ is given by Eq. 2.17 and the r^α_{ij} are parameters related to the spin density matrix elements. Assuming the SCHC only r^{04}_{00}, r^1_{1-1}, $Im(r^2_{1-1})$, $Re(r^5_{10})$ and $Im(r^6_{10})$ may be different from zero and additionally the following relations hold :

$$
Im(r^6_{1-1}) = -r^1_{1-1} \qquad Im(r^6_{10}) = -Re(r^5_{10}) \ .
$$

If in addition one assumes the natural parity exchange in the t channel ($P = (-1)^J$) Eq. 2.35 reduces to :

$$
\begin{aligned}
W(\cos\theta_h, \psi_h) &= \frac{1}{1+R\epsilon} \cdot \frac{3}{8\pi} [\sin^2\theta_h (1 + \epsilon\cos 2\psi_h) + 2\epsilon R\cos^2\theta_h \\
&- \sqrt{2\epsilon(1+\epsilon)R} \cos\delta \sin 2\theta_h \cos\psi_h] \ , \qquad (2.36)
\end{aligned}
$$

where $\psi_h = \Phi_h - \phi_h$ and δ is the relative phase between the two helicity amplitudes. Several matrix elements can be determined from distributions of $\cos\theta_h, \phi_h, \psi_h$ which can be obtained by integrating Eq. 2.36 :

$$\frac{1}{N}\frac{dN}{d\cos\theta_h} = \int_0^{2\pi} d\psi_h W(\cos\theta_h, \psi_h) = \frac{3}{4}\left[1 - r_{00}^{04} + \left(3r_{00}^{04} - 1\right)\cos^2\theta_h\right] \tag{2.37}$$

$$\frac{1}{N}\frac{dN}{d\psi_h} = \int_{-1}^{1} d(\cos\theta_h) W(\cos\theta_h, \psi_h) = \frac{1}{2\pi}\left(1 + 2\epsilon r_{1-1}^{1}\cos 2\psi_h\right) \quad . \tag{2.38}$$

Thus matrix elements r_{00}^{04}, r_{1-1}^{1} can be determined by fitting Eq. 2.37 and 2.38 to experimental distributions of $\cos\theta_h$ and ψ_h. Assuming SCHC, the ratio R is given by the formula :

$$R = \frac{1}{\epsilon}\cdot\frac{r_{00}^{04}}{1 - r_{00}^{04}} \ . \tag{2.39}$$