Math 2270 - Linear algebra

Exam III

12. 02. 2011

Directions:
Print these pages (one side!), and write your solutions on the print out.
For full credit show all your work with as much explanation of what you are doing as possible.
Clearly mark your answers.
After solving a problem check if your solution is reasonable!
YOU MAY USE THE TEXTBOOK, BUT NO OTHER AIDS (calculators, computers, internet, chats with other students, ...) ARE ALLOWED.
Good luck.

Name: .....

Score: ...../100

Problem 1. (25 points) Find the characteristic polynomial, eigenvalues, eigenvectors, and eigenspaces for the following matrix:

$$A = \begin{pmatrix} 0 & 3 & -5 \\ 1 & -2 & 5 \\ 1 & -3 & 6 \end{pmatrix}.$$

What are the dimensions of the eigenspaces? Write down a basis for each eigenspace.

Problem 2. (25 points) Is the matrix

$$A = \begin{pmatrix} -3 & 6 & -10 \\ 2 & -7 & 10 \\ 2 & -6 & 9 \end{pmatrix}$$

diagonalizable? If yes, diagonalize it, i.e. bring it to the form

$$A = SDS^{-1},$$

where S is an invertible 3×3 matrix, and D is diagonal 3×3 matrix.

Problem 3. (25 points) Write the system of differential equations

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= -x + 4y - 6z\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= x - y + 3z\\ \frac{\mathrm{d}z}{\mathrm{d}t} &= x - 2y + 4z,\end{aligned}$$

in a matrix form. Find its general solution, and a solution which at $t = 1$ has the following values of x, y and z:

$$x(1) = 0, \quad y(1) = 0, \quad z(1) = e.$$

Here e is the number $\exp(1) = 2.718...$

Problem 4. (25 points) Find a solution $x = x(t)$ to the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0,$$

satisfying initial condition $\frac{\mathrm{d}x}{\mathrm{d}t}(t=0) = 0$, $x(t=0) = 1$.