Math 2270 - Linear algebra

Practice Exam

9 grudnia 2011

Problem 1. (15 points) Find all numbers c for which the vector

$$\mathbf{u} = \begin{pmatrix} 19 \\ c \\ -19 \end{pmatrix}$$

is a linear combination of the two vectors

$$\mathbf{v} = \begin{pmatrix} -4 \\ 2 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{w} = \begin{pmatrix} -3 \\ 1 \\ -4 \end{pmatrix}$$

For each such number find the linear combination of $\mathbf{v}$ and $\mathbf{w}$ making $\mathbf{u}$.

Problem 2. (15 points) Invert the matrix

$$A = \begin{pmatrix} -1 & 5 & 1 \\ -3 & 2 & -1 \\ 3 & 1 & 2 \end{pmatrix}.$$

Problem 3. (15 points) Find the complete solution to the following system of linear equations:

$$\begin{aligned} x_1 + x_3 + 5x_4 &= 1 \\ 2x_1 + 2x_2 + 13x_4 &= 2 \\ x_1 - 3x_2 + 4x_3 &= 1. \end{aligned}$$

Problem 4. (20 points) Find the characteristic polynomial, eigenvalues, eigenvectors, and eigenspaces for the following matrix:

$$A = \begin{pmatrix} -2 & -3 & 2 \\ -1 & -4 & 2 \\ -2 & -6 & 3 \end{pmatrix}.$$

What are the dimensions of the eigenspaces? Write down a basis for each eigenspace.

Problem 5. (20 points) Write the system of differential equations

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= 10x - 7y + 3z \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= 15x - 10y + 3z \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= 3x - y - 2z, \end{aligned}$$

in a matrix form. Find its general solution, and a solution which at $t = 0$ has the following values of x, y and z:

$$x(0) = 1, \quad y(0) = 2, \quad z(0) = 3.$$

Problem 6. (15 points) Find a solution $x = x(t)$ to the differential equation

$$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 0,$$

satisfying initial condition $\frac{\mathrm{d}x}{\mathrm{d}t}(t = 0) = 0$, $x(t = 0) = 1$.