Math 2210 - Multivariable calculus

Practice Exam

04. 25. 2011

Directions: For full credit show all your work and clearly mark your answers. After solving a problem check if your solution is reasonable! You have 110 minutes to complete this exam. Good luck.

Instructor: **Pawel Nurowski**

Name: ....

Score: ..../100

Problem 1. (15 points) Show that the function

$$f(x, y) = \frac{7x^3 + 3y}{y}$$

has no limit as $(x, y) \to (0, 0)$.

Problem 2. (17 points) Find all critical points of the function

$$f(x,y) = 3 + 8x - 3x^2 - 6y - 8xy + 3y^2.$$

State clearly if at the critical point there is a local minimum, local maximum or saddle point.

Problem 3. (17 points) Find the volume of the region bounded above by the paraboloid $z = 12 - x^2 - y^2$ and below by the paraboloid $z = \frac{1}{2}(x^2 + y^2)$.

Problem 4. (17 points) Consider a vector filed

$$\vec{H} = (y^2 + 3x^2 + 4xy - 2x)\vec{\mathbf{i}} + 2(x^2 + xy + y + z)\vec{\mathbf{j}} + (2y - 3z^2)\vec{\mathbf{k}}.$$

Check if $\vec{H}$ is conservative. If yes find its potential, and evaluate the integral

$$\int_A^B \vec{H}\vec{T}\mathrm{d}l,$$

where $A = (1, 1, 1)$ and $B = (2, 2, 2)$.

Problem 5. (17 points) Calculate

$$\oint (6y + x)\mathrm{d}x + (y + 2x)\mathrm{d}y$$

along the counterclockwise oriented circle $(x-2)^2 + (y-3)^2 = 4$.

Problem 6. (17 points) Find the area of the portion of the cone $z^2 = x^2 + y^2$ that lies over the region between the circle $x^2 + y^2 = 1$ and the circle $x^2 + y^2 = 4$.