Bianchi I model in terms of loop geometry: Classical dynamics

Piotr Dzierżak

Theoretical Physics Department
Institute for Nuclear Studies
Warsaw, Poland

Based on:

[1] P. Dz., P. Małkiewicz and W. Piechocki,
'Turning Big Bang into Big Bounce: I. Classical dynamics',
Phys. Rev. D80, 104001 (2009) [arXiv:gr-qc/0907.3436].

[2] P. Dz., W. Piechocki,
'The Bianchi I model in terms of nonstandard loop quantum cosmology: Classical dynamics',
Phys. Rev. D80, 124033 (2009) [arXiv:gr-qc/0909.4211].

OUTLINE

Introduction

Two LQC methods:

- standard LQC: ‘first quantize, then impose constraints’
- non-standard LQC: ‘first solve constraints, then quantize’

Bianchi I with massless scalar field:

- standard LQC: classical Big Bang is replaced by quantum Big Bounce due to strong quantum effects at the Planck scales
- non-standard LQC: modification of GR by loop geometry is responsible for the resolution of the singularity

Introduction

Two LQC methods:

- standard LQC: ‘first quantize, then impose constraints’
- non-standard LQC: ‘first solve constraints, then quantize’

Bianchi I with massless scalar field:

- standard LQC: classical Big Bang is replaced by quantum Big Bounce due to strong quantum effects at the Planck scales
- non-standard LQC: modification of GR by loop geometry is responsible for the resolution of the singularity

Introduction

Two LQC methods:

- standard LQC: ‘first quantize, then impose constraints’
- non-standard LQC: ‘first solve constraints, then quantize’

Bianchi I with massless scalar field:

- standard LQC: classical Big Bang is replaced by quantum Big Bounce due to strong quantum effects at the Planck scales
- non-standard LQC: modification of GR by loop geometry is responsible for the resolution of the singularity

Bianchi I model

Bianchi I model describes homogeneous and anisotropic universe.
The metric reads:

$$ds^2 = -N^2\,dt^2 + \sum_{i=1}^{3} a_i^2(t)\,dx_i^2,$$

$$\sum_{i=1}^{3} \mathrm{k_i} = 1, \quad \sum_{i=1}^{3} \mathrm{k_i}^2 + \mathrm{k}_\phi^2 = 1 \tag{1}$$

where $a_i(t) = a_i(0)\left(\frac{\tau}{\tau_0}\right)^{\mathrm{k}_i}$, $d\tau = N\,dt$, k_ϕ describes matter density.
For $\mathrm{k}_\phi = 0$ (vacuum) we have Kasner model.

Modified Hamiltonian

We consider Bianchi I cosmology with massless scalar field in space with $\mathbb{R}^3$-topology.

- Hamiltonian reads:

$$H_g := \int_\Sigma d^3x(N^iC_i + N^aC_a + NC), \tag{2}$$

where: Σ is the space-like part of spacetime $\mathbb{R} \times \Sigma$; (N^i, N^a, N) Lagrange multipliers; (C_i, C_a, C) are Gauss, diffeomorphism and scalar constraints; $(a, b = 1, 2, 3)$, spatial indices; $(i, j, k = 1, 2, 3)$ internal $SU(2)$ indices. The constrains must satisfy specific algebra.

Modified Hamiltonian

We consider Bianchi I cosmology with massless scalar field in space with $\mathbb{R}^3$-topology.

- Hamiltonian reads:

$$H_g := \int_\Sigma d^3x (N^i C_i + N^a C_a + NC), \tag{2}$$

where: Σ is the space-like part of spacetime $\mathbb{R} \times \Sigma$; (N^i, N^a, N) Lagrange multipliers; (C_i, C_a, C) are Gauss, diffeomorphism and scalar constraints; $(a, b = 1, 2, 3)$, spatial indices; $(i, j, k = 1, 2, 3)$ internal $SU(2)$ indices. The constrains must satisfy specific algebra.

Modified Hamiltonian

Having fixed local gauge and diffeomorphism freedom we can rewrite the gravitational part of the classical Hamiltonian in the form:

$$H_g = -\gamma^{-2} \int_{\mathcal{V}} d^3x \; N e^{-1} \varepsilon_{ijk} E^{aj} E^{bk} F^i_{ab} \,, \tag{3}$$

where: γ is the Barbero-Immirzi parameter; $\mathcal{V} \subset \Sigma$ elementary cell; N lapse function; ε_{ijk} alternating tensor; E^a_i density weighted triad; $F^k_{ab} = \partial_a A^k_b - \partial_b A^k_a + \epsilon^k_{ij} A^i_a A^j_b$ curvature of $SU(2)$ connection A^i_a; $e := \sqrt{|\det E|}$;

Modified Hamiltonian

Rewriting the curvature in terms of holonomies:

$$F_{ab}^{k} = -2 \lim_{Ar\,\Box_{ij} \to 0} Tr \left(\frac{h_{\Box_{ij}}^{(\mu)} - 1}{Ar\,\Box_{ij}} \right) \tau^{k}\ {}^{o}\omega_{a}^{i}\ {}^{o}\omega_{a}^{j}, \tag{4}$$

where:

$$h_{\Box_{ij}} = h_{i}^{(\mu_i)} h_{j}^{(\mu_j)} (h_{i}^{(\mu_i)})^{-1} (h_{j}^{(\mu_j)})^{-1} \tag{5}$$

is the holonomy of the gravitational connection around the square loop $\Box_{ij}$, considered over a face of the elementary cell, each of whose sides has length $\mu_j L_j$ (and $V_o := L_1 L_2 L_3$) with respect to the flat fiducial metric ${}^{o}q_{ab} := \delta_{ij}\, {}^{o}\omega_{a}^{i}\, {}^{o}\omega_{a}^{j}$.

In the fundamental, $j = 1/2$, representation of $SU(2)$, reads

$$h_{i}^{(\mu_i)} = \cos(\mu_i c_i/2)\, \mathbb{I} + 2\sin(\mu_i c_i/2)\, \tau_i \tag{6}$$

where $\tau_i = -i\sigma_i/2$ (σ_i are the Pauli spin matrices)

Modified Hamiltonian

Rewriting the curvature in terms of holonomies:

$$F^k_{ab} = -2 \lim_{Ar\,\Box_{ij} \to 0} Tr\left(\frac{h^{(\mu)}_{\Box_{ij}} - 1}{Ar\,\Box_{ij}}\right) \tau^k \, {}^o\omega^i_a \, {}^o\omega^j_a, \tag{4}$$

where:

$$h_{\Box_{ij}} = h^{(\mu_i)}_i h^{(\mu_j)}_j (h^{(\mu_i)}_i)^{-1} (h^{(\mu_j)}_j)^{-1} \tag{5}$$

is the holonomy of the gravitational connection around the square loop $\Box_{ij}$, considered over a face of the elementary cell, each of whose sides has length $\mu_j L_j$ (and $V_o := L_1 L_2 L_3$) with respect to the flat fiducial metric ${}^o q_{ab} := \delta_{ij} \, {}^o\omega^i_a \, {}^o\omega^j_a$.
In the fundamental, $j = 1/2$, representation of $SU(2)$, reads

$$h^{(\mu_i)}_i = \cos(\mu_i c_i/2) \, \mathbb{I} + 2 \sin(\mu_i c_i/2) \, \tau_i \tag{6}$$

where $\tau_i = -i\sigma_i/2$ (σ_i are the Pauli spin matrices)

Modified Hamiltonian

Making use of (3), (4) and the so-called Thiemann identity leads to H_g in the form:

$$H_g = \lim_{\mu_1,\mu_2,\mu_3 \to 0} H_g^{(\mu_1\,\mu_2\,\mu_3)}, \tag{7}$$

where:

$$\begin{aligned} H_g^{(\mu_1\,\mu_2\,\mu_3)} &= -\frac{\mathrm{sgn}(p_1 p_2 p_3)}{2\pi G \gamma^3 \mu_1 \mu_2 \mu_3} \sum_{ijk} N\,\varepsilon^{ijk}\, Tr\Big(h_i^{(\mu_i)} h_j^{(\mu_j)} \times \\ &\times \quad (h_i^{(\mu_i)})^{-1} (h_j^{(\mu_j)})^{-1} h_k^{(\mu_k)} \{(h_k^{(\mu_k)})^{-1}, V\}\Big) \end{aligned} \tag{8}$$

and is the $V = a_1\, a_2\, a_3 V_0$ volume of the elementary cell.

Modified Hamiltonian

In considered model the hamiltonian reads:

$$H = H_g + H_\phi \approx 0, \tag{9}$$

where H_g is defined by (7) and $H_\phi = N\, p_\phi^2 |p|^{-\frac{3}{2}}/2$. The relation $H \approx 0$ defines the physical phase space of considered gravitational system with constraints.

Making use of (6) we calculate (8) and get the modified total Hamiltonian. The Hamiltonian modified by loop geometry (in the gauge $N = \sqrt{|p_1\, p_2\, p_3|}$) reads:

$$H^{(\lambda)} = -\frac{1}{8\pi G \gamma^2 \mu_1 \mu_2} \left[|p_1 p_2| \sin(c_1 \mu_1) \sin(c_2 \mu_2) + \text{cyclic} \right] + \frac{p_\phi^2}{2} \tag{10}$$

Modified Hamiltonian

We use so called $\overline{\mu}$ scheme:

$\mu_i := \sqrt{\frac{1}{|p_i|}}\,\lambda$ and λ is a free parameter.

Now we introduce following canonical variables:

$\beta_i := \frac{c_i}{\sqrt{|p_i|}}, \quad v_i := |p_i|^{3/2}\,\mathrm{sgn}(p_i),$

and the Poisson bracket:

$\{\cdot,\cdot\} := 12\pi G\gamma \sum_{k=1}^{3}\left[\frac{\partial\cdot}{\partial\beta_k}\frac{\partial\cdot}{\partial v_k} - \frac{\partial\cdot}{\partial v_k}\frac{\partial\cdot}{\partial\beta_k}\right] + \frac{\partial\cdot}{\partial\phi}\frac{\partial\cdot}{\partial p_\phi} - \frac{\partial\cdot}{\partial p_\phi}\frac{\partial\cdot}{\partial\phi}$

The Hamiltonian in new variables reads:

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\Bigg(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2}v_1 v_2 + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2}v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2}v_2 v_3\Bigg) \quad (11)$$

It is classical modified Hamiltonian.

Modified Hamiltonian

We use so called $\overline{\mu}$ scheme:

$\mu_i := \sqrt{\frac{1}{|p_i|}}\,\lambda$ and λ is a free parameter.

Now we introduce following canonical variables:

$\beta_i := \frac{c_i}{\sqrt{|p_i|}}, \quad v_i := |p_i|^{3/2}\,\mathrm{sgn}(p_i),$

and the Poisson bracket:

$\{\cdot,\cdot\} := 12\pi G\gamma \sum_{k=1}^{3}\left[\frac{\partial\cdot}{\partial\beta_k}\frac{\partial\cdot}{\partial v_k} - \frac{\partial\cdot}{\partial v_k}\frac{\partial\cdot}{\partial\beta_k}\right] + \frac{\partial\cdot}{\partial\phi}\frac{\partial\cdot}{\partial p_\phi} - \frac{\partial\cdot}{\partial p_\phi}\frac{\partial\cdot}{\partial\phi}$

The Hamiltonian in new variables reads:

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\left(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2}v_1 v_2 + \right. \tag{11}$$

$$\left. + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2}v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2}v_2 v_3\right)$$

It is classical modified Hamiltonian.

Modified Hamiltonian

We use so called $\overline{\mu}$ scheme:

$\mu_i := \sqrt{\frac{1}{|p_i|}}\,\lambda$ and λ is a free parameter.

Now we introduce following canonical variables:

$\beta_i := \frac{c_i}{\sqrt{|p_i|}}, \quad v_i := |p_i|^{3/2}\,\mathrm{sgn}(p_i),$

and the Poisson bracket:

$\{\cdot,\cdot\} := 12\pi G\gamma \sum_{k=1}^{3} \left[\frac{\partial\cdot}{\partial\beta_k}\frac{\partial\cdot}{\partial v_k} - \frac{\partial\cdot}{\partial v_k}\frac{\partial\cdot}{\partial\beta_k} \right] + \frac{\partial\cdot}{\partial\phi}\frac{\partial\cdot}{\partial p_\phi} - \frac{\partial\cdot}{\partial p_\phi}\frac{\partial\cdot}{\partial\phi}$

The Hamiltonian in new variables reads:

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\left(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2} v_1 v_2 + \right. \tag{11}$$

$$\left. + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2} v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2} v_2 v_3 \right)$$

It is classical modified Hamiltonian.

Modified Hamiltonian

We use so called $\overline{\mu}$ scheme:

$\mu_i := \sqrt{\frac{1}{|p_i|}}\,\lambda$ and λ is a free parameter.

Now we introduce following canonical variables:

$\beta_i := \frac{c_i}{\sqrt{|p_i|}}, \quad v_i := |p_i|^{3/2}\,\mathrm{sgn}(p_i),$

and the Poisson bracket:

$\{\cdot,\cdot\} := 12\pi G\gamma \sum_{k=1}^{3} \left[\frac{\partial\cdot}{\partial\beta_k}\frac{\partial\cdot}{\partial v_k} - \frac{\partial\cdot}{\partial v_k}\frac{\partial\cdot}{\partial\beta_k}\right] + \frac{\partial\cdot}{\partial\phi}\frac{\partial\cdot}{\partial p_\phi} - \frac{\partial\cdot}{\partial p_\phi}\frac{\partial\cdot}{\partial\phi}$

The Hamiltonian in new variables reads:

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\left(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2}v_1 v_2 + \right.$$
$$\left. + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2}v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2}v_2 v_3\right) \tag{11}$$

It is classical modified Hamiltonian.

Equations of motion

Equations of motion plus the constraint reads:

$$\dot{\beta}_i = -18\pi G \frac{\sin(\lambda\beta_i)}{\lambda} (\mathrm{O}_j + \mathrm{O}_k) \quad (12)$$

$$\dot{v}_i = 18\pi G\, v_i \cos(\lambda\beta_i) (\mathrm{O}_j + \mathrm{O}_k) \quad (13)$$

$$\dot{p}_\phi = 0 \quad (14)$$

$$\dot{\phi} = p_\phi \quad (15)$$

$$H^{(\lambda)} \approx 0, \quad (16)$$

where $\mathrm{O}_i := \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda}$, O_i are constants of motion.
Worth of noting: ϕ plays the role of time.

Equations of motion

Equations of motion plus the constraint reads:

$$\dot{\beta}_i = -18\pi G \frac{\sin(\lambda\beta_i)}{\lambda} (\mathrm{O}_j + \mathrm{O}_k) \quad (12)$$

$$\dot{v}_i = 18\pi G\, v_i \cos(\lambda\beta_i) (\mathrm{O}_j + \mathrm{O}_k) \quad (13)$$

$$\dot{p}_\phi = 0 \quad (14)$$

$$\dot{\phi} = p_\phi \quad (15)$$

$$H^{(\lambda)} \approx 0, \quad (16)$$

where $\mathrm{O}_i := \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda}$, O_i are constants of motion.
Worth of noting: ϕ plays the role of time.

Equations of motion

Solution of equation, for example (13):

$$
\begin{aligned}
2\, v_i(\phi) &= \exp\left(\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0)\right) + \\
&+ (12\pi G\gamma\lambda\, \mathrm{O}_i)^2 \cdot \exp\left(-\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0)\right) \quad (17)
\end{aligned}
$$

We can write this in a different form:

$$
v_i = 12\pi G\gamma\lambda \mathrm{O}_i \cosh\left(\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0) - \ln\left|12\pi G\gamma\lambda \mathrm{O}_i\right|\right) \quad (18)
$$

Because $V = (v_1 v_2 v_3)^{1/3}$ it is clear that for nonzero λ there is no singularity for any value of ϕ.

Big Bounce already at classical level!

Equations of motion

Solution of equation, for example (13):

$$
\begin{aligned}
2\, v_i(\phi) &= \exp\left(\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0)\right) + \\
&+ (12\pi G\gamma\lambda\, \mathrm{O}_i)^2 \cdot \exp\left(-\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0)\right) \quad (17)
\end{aligned}
$$

We can write this in a different form:

$$
v_i = 12\pi G\gamma\lambda\mathrm{O}_i \cosh\left(\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0) - \ln\left|12\pi G\gamma\lambda\mathrm{O}_i\right|\right) \quad (18)
$$

Because $V = (v_1 v_2 v_3)^{1/3}$ it is clear that for nonzero λ there is no singularity for any value of ϕ.

Big Bounce already at classical level!

Equations of motion

Solution of equation, for example (13):

$$
\begin{aligned}
2\, v_i(\phi) &= \exp\left(\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0)\right) + \\
&+ (12\pi G\gamma\lambda\, \mathrm{O}_i)^2 \cdot \exp\left(-\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0)\right) \quad (17)
\end{aligned}
$$

We can write this in a different form:

$$
v_i = 12\pi G\gamma\lambda\mathrm{O}_i \cosh\left(\frac{18\pi G}{p_\phi}\,(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0) - \ln\left|12\pi G\gamma\lambda\mathrm{O}_i\right|\right) \quad (18)
$$

Because $V = (v_1 v_2 v_3)^{1/3}$ it is clear that for nonzero λ there is no singularity for any value of ϕ.

Big Bounce already at classical level!

Equations of motion

Solution of equation, for example (13):

$$
\begin{aligned}
2\, v_i(\phi) &= \exp\left(\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0)\right) + \\
&+ (12\pi G\gamma\lambda\, \mathrm{O}_i)^2 \cdot \exp\left(-\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0)\right) \quad (17)
\end{aligned}
$$

We can write this in a different form:

$$
v_i = 12\pi G\gamma\lambda \mathrm{O}_i \cosh\left(\frac{18\pi G}{p_\phi}(\mathrm{O}_j + \mathrm{O}_k)\,(\phi - \phi_i^0) - \ln\left|12\pi G\gamma\lambda \mathrm{O}_i\right|\right) \quad (18)
$$

Because $V = (v_1 v_2 v_3)^{1/3}$ it is clear that for nonzero λ there is no singularity for any value of ϕ.

Big Bounce already at classical level!

Algebra of observables

Our aim : Singularity aspects of the model in terms of observables.

A function F defined on phase space is a Dirac observable if it is the solution to the equation

$$\left\{F, H^{(\lambda)}\right\} = 0 \tag{19}$$

We found that:

$$\mathrm{A}_i^{\mathrm{dyn}} = \ln\left|\tan\left(\frac{\lambda\beta_i}{2}\right)\right| + \frac{3\sqrt{\pi G}\,\mathrm{sgn}(p_\phi)(\mathrm{O}_j + \mathrm{O}_k)\,\phi}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}} \tag{20}$$

$$\mathrm{O}_i = \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda} \tag{21}$$

These are dynamical (satisfy the constraint) observables.
We can express by them any interesting physical function.

Algebra of observables

Our aim : Singularity aspects of the model in terms of observables.

A function F defined on phase space is a Dirac observable if it is the solution to the equation

$$\left\{F, H^{(\lambda)}\right\} = 0 \tag{19}$$

We found that:

$$\mathrm{A}_i^{\mathrm{dyn}} = \ln\left|\tan\left(\frac{\lambda\beta_i}{2}\right)\right| + \frac{3\sqrt{\pi G}\,\mathrm{sgn}(p_\phi)\left(\mathrm{O}_j + \mathrm{O}_k\right)\phi}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}} \tag{20}$$

$$\mathrm{O}_i = \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda} \tag{21}$$

These are dynamical (satisfy the constraint) observables.
We can express by them any interesting physical function.

Algebra of observables

Our aim : Singularity aspects of the model in terms of observables.

A function F defined on phase space is a Dirac observable if it is the solution to the equation

$$\left\{F, H^{(\lambda)}\right\} = 0 \tag{19}$$

We found that:

$$\mathrm{A}_i^{\mathrm{dyn}} = \ln\left|\tan\left(\frac{\lambda\beta_i}{2}\right)\right| + \frac{3\sqrt{\pi G}\,\mathrm{sgn}(p_\phi)\left(\mathrm{O}_j + \mathrm{O}_k\right)\phi}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}} \tag{20}$$

$$\mathrm{O}_i = \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda} \tag{21}$$

These are dynamical (satisfy the constraint) observables.
We can express by them any interesting physical function.

Algebra of observables

Our aim : Singularity aspects of the model in terms of observables.

A function F defined on phase space is a Dirac observable if it is the solution to the equation

$$\left\{F, H^{(\lambda)}\right\} = 0 \tag{19}$$

We found that:

$$\mathrm{A}_i^{\mathrm{dyn}} = \ln\left|\tan\left(\frac{\lambda\beta_i}{2}\right)\right| + \frac{3\sqrt{\pi G}\,\mathrm{sgn}(p_\phi)\left(\mathrm{O}_j + \mathrm{O}_k\right)\phi}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}} \tag{20}$$

$$\mathrm{O}_i = \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda} \tag{21}$$

These are dynamical (satisfy the constraint) observables.
We can express by them any interesting physical function.

Algebra of observables

These observables satisfy the Lie algebra:

$$\{\mathrm{O}_i, \mathrm{O}_j\}_{\mathrm{dyn}} = 0, \quad \{\mathrm{A}_i^{\mathrm{dyn}}, \mathrm{O}_i\}_{\mathrm{dyn}} = 1,$$
$$\{\mathrm{A}_i^{\mathrm{dyn}}, \mathrm{O}_j\}_{\mathrm{dyn}} = 0, \quad \{\mathrm{A}_i^{\mathrm{dyn}}, \mathrm{A}_j^{\mathrm{dyn}}\}_{\mathrm{dyn}} = 0. \tag{22}$$

where:

$$\{\cdot, \cdot\}_{\mathrm{dyn}} := \sum_{i=1}^{3} \left(\frac{\partial \cdot}{\partial \mathrm{A}_i^{\mathrm{dyn}}} \frac{\partial \cdot}{\partial \mathrm{O}_i} - \frac{\partial \cdot}{\partial \mathrm{O}_i} \frac{\partial \cdot}{\partial \mathrm{A}_i^{\mathrm{dyn}}} \right) \tag{23}$$

Functions on the physical space

- Directional energy density:

$$\rho_i(\lambda, \phi) := \frac{p_\phi^2}{2\, v_i^2} \tag{24}$$

The bounce in i-th direction occurs when ρ_i approaches its maximum value[1].

We would like to express above function in terms of observables and an evolution parameter ϕ:

$$\rho_i(\lambda, \phi) = \frac{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}{8\pi G\gamma^2\, \mathrm{O}_i^2\, \cosh^2\left(\frac{3\sqrt{\pi G}\ \mathrm{sgn}(p_\phi)\left(\mathrm{O}_j+\mathrm{O}_k\right)}{\sqrt{\mathrm{O}_1\mathrm{O}_2+\mathrm{O}_1\mathrm{O}_3+\mathrm{O}_2\mathrm{O}_3}}\phi - \mathrm{A}_i^{\mathrm{dyn}}\right)} \tag{25}$$

It will be an observable for each fixed value of ϕ, since in such case it will be function of observables only.

---

[1] D.W. Chiou PRD (**75**), 24029 (2007)

Functions on the physical space

- Directional energy density:

$$\rho_i(\lambda, \phi) := \frac{p_\phi^2}{2\, v_i^2} \tag{24}$$

The bounce in i-th direction occurs when ρ_i approaches its maximum value[1].

We would like to express above function in terms of observables and an evolution parameter ϕ:

$$\rho_i(\lambda, \phi) = \frac{O_1 O_2 + O_1 O_3 + O_2 O_3}{8\pi G \gamma^2 O_i^2 \cosh^2\left(\frac{3\sqrt{\pi G}\,\mathrm{sgn}(p_\phi)\left(O_j + O_k\right)}{\sqrt{O_1 O_2 + O_1 O_3 + O_2 O_3}}\phi - A_i^{\mathrm{dyn}}\right)} \tag{25}$$

It will be an observable for each fixed value of ϕ, since in such case it will be function of observables only.

[1] D.W. Chiou PRD (**75**), 24029 (2007)

Functions on the physical space

- Directional energy density:

$$\rho_i(\lambda, \phi) := \frac{p_\phi^2}{2\, v_i^2} \tag{24}$$

The bounce in i-th direction occurs when ρ_i approaches its maximum value[1].

We would like to express above function in terms of observables and an evolution parameter ϕ:

$$\rho_i(\lambda, \phi) = \frac{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}{8\pi G\gamma^2\, \mathrm{O}_i^2\, \cosh^2\left(\frac{3\sqrt{\pi G}\ \mathrm{sgn}(p_\phi)\left(\mathrm{O}_j + \mathrm{O}_k\right)}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}}\phi - \mathrm{A}_i^{\mathrm{dyn}}\right)} \tag{25}$$

It will be an observable for each fixed value of ϕ, since in such case it will be function of observables only.

[1] D.W. Chiou PRD (**75**), 24029 (2007)

Functions on the physical space

- Directional energy density:

$$\rho_i(\lambda, \phi) := \frac{p_\phi^2}{2\, v_i^2} \tag{24}$$

The bounce in i-th direction occurs when ρ_i approaches its maximum value[1].

We would like to express above function in terms of observables and an evolution parameter ϕ:

$$\rho_i(\lambda, \phi) = \frac{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}{8\pi G\gamma^2\, \mathrm{O}_i^2\, \cosh^2\left(\frac{3\sqrt{\pi G}\ \mathrm{sgn}(p_\phi)\left(\mathrm{O}_j + \mathrm{O}_k\right)}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}}\phi - \mathrm{A}_i^{\mathrm{dyn}}\right)} \tag{25}$$

It will be an observable for each fixed value of ϕ, since in such case it will be function of observables only.

[1] D.W. Chiou PRD (**75**), 24029 (2007)

Functions on the physical space

Big bounce happens when v_i takes its minimal value: $v_i^{\min} = 12\pi G\gamma\lambda\mathrm{O}_i$. Thus the maximum value of this density:

$$\rho_i^{\max} = \frac{1}{16\pi G\gamma^2\lambda^2}\left(\frac{k_\phi}{k_i}\right)^2 \tag{26}$$

where k_ϕ and k_i taken from the metric.

λ is a free parameter of this formalism.
Let assume that $\lambda = l_{Pl}$. We get:

$$\rho_i^{\max} \simeq 0,35\left(\frac{k_\phi}{k_i}\right)^2 \rho_{Pl} \tag{27}$$

So we fit the Planck scale.

Functions on the physical space

Big bounce happens when v_i takes its minimal value: $v_i^{\min} = 12\pi G\gamma\lambda \mathrm{O}_i$. Thus the maximum value of this density:

$$\rho_i^{\max} = \frac{1}{16\pi G\gamma^2\lambda^2}\left(\frac{k_\phi}{k_i}\right)^2 \quad (26)$$

where k_ϕ and k_i taken from the metric.

λ is a free parameter of this formalism.
Let assume that $\lambda = l_{Pl}$. We get:

$$\rho_i^{\max} \simeq 0,35\left(\frac{k_\phi}{k_i}\right)^2 \rho_{Pl} \quad (27)$$

So we fit the Planck scale.

Functions on the physical space

Big bounce happens when v_i takes its minimal value: $v_i^{\mathrm{min}} = 12\pi G\gamma\lambda \mathrm{O}_i$. Thus the maximum value of this density:

$$\rho_i^{\mathrm{max}} = \frac{1}{16\pi G\gamma^2\lambda^2}\left(\frac{k_\phi}{k_i}\right)^2 \tag{26}$$

where k_ϕ and k_i taken from the metric.

λ is a free parameter of this formalism.
Let assume that $\lambda = l_{Pl}$. We get:

$$\rho_i^{\mathrm{max}} \simeq 0,35\left(\frac{k_\phi}{k_i}\right)^2 \rho_{Pl} \tag{27}$$

So we fit the Planck scale.

Functions on the physical space

Big bounce happens when v_i takes its minimal value: $v_i^{\min} = 12\pi G\gamma\lambda \mathrm{O}_i$. Thus the maximum value of this density:

$$\rho_i^{\max} = \frac{1}{16\pi G\gamma^2\lambda^2}\left(\frac{k_\phi}{k_i}\right)^2 \qquad (26)$$

where k_ϕ and k_i taken from the metric.

λ is a free parameter of this formalism.
Let assume that $\lambda = l_{Pl}$. We get:

$$\rho_i^{\max} \simeq 0,35\left(\frac{k_\phi}{k_i}\right)^2 \rho_{Pl} \qquad (27)$$

So we fit the Planck scale.

Conclusions

- Modification of gravitational part of classical Hamiltonian, realized by using the loop geometry turns big-bang into big-bounce.
- λ is a free parameter of the theory as in the case of FRW model.
- Next step: quantization of classical dynamics, examine spectra of physical observables.

Conclusions

- Modification of gravitational part of classical Hamiltonian, realized by using the loop geometry turns big-bang into big-bounce.
- λ is a free parameter of the theory as in the case of FRW model.
- Next step: quantization of classical dynamics, examine spectra of physical observables.

Conclusions

- Modification of gravitational part of classical Hamiltonian, realized by using the loop geometry turns big-bang into big-bounce.
- λ is a free parameter of the theory as in the case of FRW model.
- Next step: quantization of classical dynamics, examine spectra of physical observables.

Conclusions

- Modification of gravitational part of classical Hamiltonian, realized by using the loop geometry turns big-bang into big-bounce.
- λ is a free parameter of the theory as in the case of FRW model.
- Next step: quantization of classical dynamics, examine spectra of physical observables.