Quantum Bianchi I model: an attempt to understand very early Universe

Piotr Dzierżak

Theoretical Physics Department
Institute for Nuclear Studies
Warsaw, Poland

OUTLINE

Introduction

As you probably know:

- Cosmology is devoted to Universe as a whole.
- Gravitational field plays crucial role in large distances.
- Einstein's General Relativity (GR) is a theory of gravity.

In fact we can say that cosmology bases on GR.
Unfortunatelly GR is a very difficult theory (due to non-linearity).
It is possible to consider only some simple models of space (metrics).
How simple?
Friedmann-Robertson-Walker (FRW) model which is of a primary importance today describes space of the whole Universe by one function $a = a(t)$.

Introduction

As you probably know:

- Cosmology is devoted to Universe as a whole.
- Gravitational field plays crucial role in large distances.
- Einstein's General Relativity (GR) is a theory of gravity.

In fact we can say that cosmology bases on GR.
Unfortunatelly GR is a very difficult theory (due to non-linearity).
It is possible to consider only some simple models of space (metrics).
How simple?
Friedmann-Robertson-Walker (FRW) model which is of a primary importance today describes space of the whole Universe by one function $a = a(t)$.

Introduction

As you probably know:

- Cosmology is devoted to Universe as a whole.
- Gravitational field plays crucial role in large distances.
- Einstein's General Relativity (GR) is a theory of gravity.

In fact we can say that cosmology bases on GR.
Unfortunatelly GR is a very difficult theory (due to non-linearity).
It is possible to consider only some simple models of space (metrics).
How simple?
Friedmann-Robertson-Walker (FRW) model which is of a primary importance today describes space of the whole Universe by one function $a = a(t)$.

Introduction

FRW metric:

$$ds^2 = -N^2\, dt^2 + a(t)\left(dx^2 + dy^2 + dz^2\right) \tag{1}$$

$a(t)$ is scale factor
FRW describes space which is homogeneous and isotropic.

homogeneous - space has the same metric in all points

isotropic - space has the same metric in all directions

In other words: homogenity and isotropicity are symmetries of space.

Introduction

FRW metric:

$$ds^2 = -N^2\, dt^2 + a(t)\left(dx^2 + dy^2 + dz^2\right) \qquad (1)$$

$a(t)$ is scale factor
FRW describes space which is homogeneous and isotropic.

homogeneous - space has the same metric in all points

isotropic - space has the same metric in all directions

In other words: homogenity and isotropicity are symmetries of space.

Introduction

A bit more complicated classes of metrics:

- homogeneous and anisotropic - Bianchi models
- nonhomogeneous and isotropic - cannot happen
- nonhomogeneous and anisotropic - generic metric form known

We are interested in the first case:

All possible homogeneous and anisotropic spaces classified according to Luigi Bianchi in 9 classes from Bianchi I to Bianchi IX.

Each of them describes space with different group of symmetry e.g. for Bianchi I group of symmetry is $\mathbb{R}^3$ (translation group) and for Bianchi IX it is $\mathbf{SO}(3)$ (rotations in 3D) .

Introduction

A bit more complicated classes of metrics:

- homogeneous and anisotropic - Bianchi models
- nonhomogeneous and isotropic - cannot happen
- nonhomogeneous and anisotropic - generic metric form known

We are interested in the first case:
All possible homogeneous and anisotropic spaces classified according to Luigi Bianchi in 9 classes from Bianchi I to Bianchi IX.
Each of them describes space with different group of symmetry e.g. for Bianchi I group of symmetry is $\mathbb{R}^3$ (translation group) and for Bianchi IX it is $\mathbf{SO}(3)$ (rotations in 3D) .

Introduction

A bit more complicated classes of metrics:

- homogeneous and anisotropic - Bianchi models
- nonhomogeneous and isotropic - cannot happen
- nonhomogeneous and anisotropic - generic metric form known

We are interested in the first case:
All possible homogeneous and anisotropic spaces classified according to Luigi Bianchi in 9 classes from Bianchi I to Bianchi IX.
Each of them describes space with different group of symmetry e.g. for Bianchi I group of symmetry is $\mathbb{R}^3$ (translation group) and for Bianchi IX it is $\mathbf{SO}(3)$ (rotations in 3D) .

Introduction

Bianchi I metric:

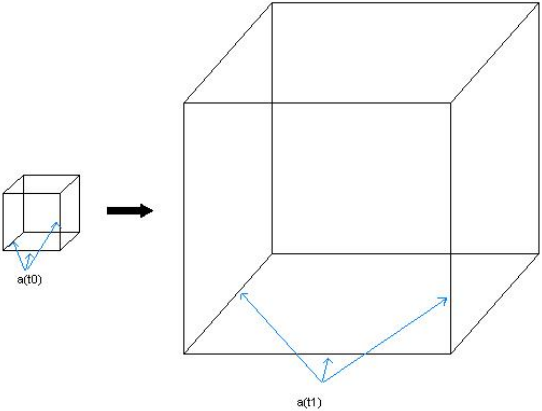
$$ds^2 = -N^2\,dt^2 + a_1^2(t)\,dx^2 + a_2^2(t)\,dy^2 + a_3^2(t)\,dz^2, \quad (2)$$

$$\sum_{i=1}^{3} \mathrm{k_i} = 1, \quad \sum_{i=1}^{3} \mathrm{k_i^2} = 1 \quad (3)$$

where $a_i(t) = a_i(0)\left(\frac{\tau}{\tau_0}\right)^{\mathrm{k}_i},\ d\tau = N\,dt$.
There are three different scale factors - this means that evolution is different in each direction!
In vacuum case (as above) called Kasner metric.

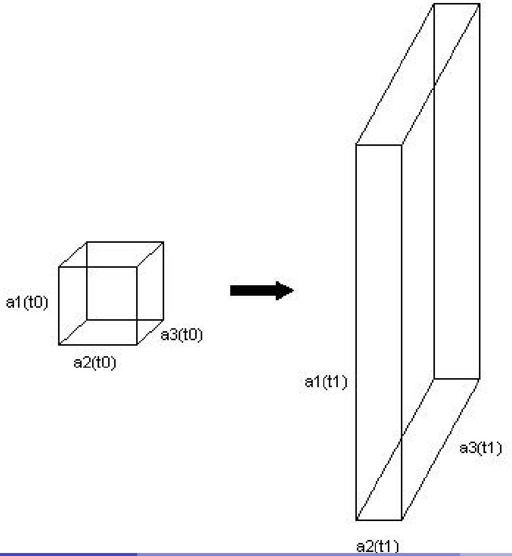
Introduction

Consider: Evolution of some imagined cube in space.
Expansion in FRW:



Introduction

Expansion in Bianchi I:



Can we believe in Bianchi models?

Question: Why we are interested in such strange models when it is clear from present observations that the Universe is almost perfectly homogeneous and isotropic in large scales?
Answer: Symmetries of space in very early Universe could be very different.
From observational point of view: Today's observations cannot tell us almost anything about symmetries of space near initial singularity.
Checking the assumption of nonisotropicity (in the future):

- more precise data from Cosmic Microwave Background (CMB) radiation
- spectrum of gravity waves created before Cosmic Inflation

(Cosmic Inflation - exponensial expansion of the Universe about 10^{-36} seconds after the Big Bang.)

Can we believe in Bianchi models?

Question: Why we are interested in such strange models when it is clear from present observations that the Universe is almost perfectly homogeneous and isotropic in large scales?
Answer: Symmetries of space in very early Universe could be very different.
From observational point of view: Today's observations cannot tell us almost anything about symmetries of space near initial singularity.
Checking the assumption of nonisotropicity (in the future):

- more precise data from Cosmic Microwave Background (CMB) radiation
- spectrum of gravity waves created before Cosmic Inflation

(Cosmic Inflation - exponensial expansion of the Universe about 10^{-36} seconds after the Big Bang.)

Can we believe in Bianchi models?

Question: Why we are interested in such strange models when it is clear from present observations that the Universe is almost perfectly homogeneous and isotropic in large scales?
Answer: Symmetries of space in very early Universe could be very different.
From observational point of view: Today's observations cannot tell us almost anything about symmetries of space near initial singularity.
Checking the assumption of nonisotropicity (in the future):

- more precise data from Cosmic Microwave Background (CMB) radiation
- spectrum of gravity waves created before Cosmic Inflation

(Cosmic Inflation - exponensial expansion of the Universe about 10^{-36} seconds after the Big Bang.)

Can we believe in Bianchi models?

Question: Why we are interested in such strange models when it is clear from present observations that the Universe is almost perfectly homogeneous and isotropic in large scales?
Answer: Symmetries of space in very early Universe could be very different.
From observational point of view: Today's observations cannot tell us almost anything about symmetries of space near initial singularity.
Checking the assumption of nonisotropicity (in the future):

- more precise data from Cosmic Microwave Background (CMB) radiation
- spectrum of gravity waves created before Cosmic Inflation

(Cosmic Inflation - exponensial expansion of the Universe about 10^{-36} seconds after the Big Bang.)

Can we believe in Bianchi models?

From theoretical point of view:

1. Cosmic Inflation scenario offers an efficient isotropisation mechanism, able to smooth out Bianchi Universe!
2. It was shown (numerical calculations) that FRW model is backward unstable.
3. Anisotropic Universe predicted by Belinskii-Khalatnikov-Lifshitz (BKL) scenario.

Can we believe in Bianchi models?

From theoretical point of view:

1. Cosmic Inflation scenario offers an efficient isotropisation mechanism, able to smooth out Bianchi Universe!
2. It was shown (numerical calculations) that FRW model is backward unstable.
3. Anisotropic Universe predicted by Belinskii-Khalatnikov-Lifshitz (BKL) scenario.

Can we believe in Bianchi models?

From theoretical point of view:

1. Cosmic Inflation scenario offers an efficient isotropisation mechanism, able to smooth out Bianchi Universe!
2. It was shown (numerical calculations) that FRW model is backward unstable.
3. Anisotropic Universe predicted by Belinskii-Khalatnikov-Lifshitz (BKL) scenario.

BKL scenario

In late 60-ties V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz were trying to investigate possibly general solution of GR near cosmic singularity.

As a starting point authors chose Bianchi class of models, but their ultimate goal: nonhomogeneous and anisotropic model.

The authors investigated Bianchi VIII and Bianchi IX (called Mixmaster Universe (MU)).

Analytical but approximated calculations.

BKL scenario

The results of BKL approach:

- Singularity is a necessary property of the solution and will appear in the exact solution.
- In neighbourhood of the singularity evolution of the Universe has oscillatory nature. The details depend on what specific Bianchi model is chosen.
- There are reasons to believe that singularities in the most general (nonhomogeneous, anisotropic) solution of Einstein equations have the same characteristics. BKL may be generic!

BKL scenario

The results of BKL approach:

- Singularity is a necessary property of the solution and will appear in the exact solution.
- In neighbourhood of the singularity evolution of the Universe has oscillatory nature. The details depend on what specific Bianchi model is chosen.
- There are reasons to believe that singularities in the most general (nonhomogeneous, anisotropic) solution of Einstein equations have the same characteristics. BKL may be generic!

BKL scenario

The results of BKL approach:

- Singularity is a necessary property of the solution and will appear in the exact solution.
- In neighbourhood of the singularity evolution of the Universe has oscillatory nature. The details depend on what specific Bianchi model is chosen.
- There are reasons to believe that singularities in the most general (nonhomogeneous, anisotropic) solution of Einstein equations have the same characteristics. BKL may be generic!

BKL scenario

Few words about Mixmaster Universe:

- The most promising and interesting from all BKL models, based on Bianchi IX.
- Evolution is described by an infinite alternation of Kasner (vacuum Bianchi I) epochs. This makes Bianchi I very important - it plays the role of a "brick" used to construct the evolution!
- Mixmaster Universe contains isotropisation machanism - you don't have to believe in Cosmic Inflation.

BKL scenario

Few words about Mixmaster Universe:

- The most promising and interesting from all BKL models, based on Bianchi IX.
- Evolution is described by an infinite alternation of Kasner (vacuum Bianchi I) epochs. This makes Bianchi I very important - it plays the role of a "brick" used to construct the evolution!
- Mixmaster Universe contains isotropisation machanism - you don't have to believe in Cosmic Inflation.

BKL scenario

Few words about Mixmaster Universe:

- The most promising and interesting from all BKL models, based on Bianchi IX.
- Evolution is described by an infinite alternation of Kasner (vacuum Bianchi I) epochs. This makes Bianchi I very important - it plays the role of a "brick" used to construct the evolution!
- Mixmaster Universe contains isotropisation machanism - you don’t have to believe in Cosmic Inflation.

Quantum Cosmology in general

1. Many people believe that gravity has quantum nature. Probably it is true near the Planck scale $= 10^{35}$GeV (very near to Big Bang).
2. From previous sections it is clear that the first step in analysing gravity near the Big Bang is Bianchi I model.

Conclusion: Bianchi I should be quantised!

Quantum Cosmology in general

1. Many people believe that gravity has quantum nature. Probably it is true near the Planck scale $= 10^{35}$GeV (very near to Big Bang).
2. From previous sections it is clear that the first step in analysing gravity near the Big Bang is Bianchi I model.

Conclusion: Bianchi I should be quantised!

Quantum Cosmology in general

1. Many people believe that gravity has quantum nature. Probably it is true near the Planck scale $= 10^{35}$GeV (very near to Big Bang).
2. From previous sections it is clear that the first step in analysing gravity near the Big Bang is Bianchi I model.

Conclusion: Bianchi I should be quantised!

Quantum Cosmology in general

There are many attempts to quantising gravity.
One of the most promising is Loop Quantum Gravity (LQG) (theory of field, inifinite number of degrees of freedom).
In cosmological regime, because of assumed symmetries (only few degrees of freedom) - Loop Quantum Cosmology (LQC).
LQG and LQC - very complicated theories, but based on very simple idea: discretisation of the space by loops.
Quantum of space exists! (of unknown size...)
Two LQC methods:

- standard LQC: ‘first quantise, then impose constraints’
- non-standard LQC: ‘first solve constraints, then quantise’

(by constraint we mean Hamiltonian)

Quantum Cosmology in general

There are many attempts to quantising gravity.
One of the most promising is Loop Quantum Gravity (LQG) (theory of field, inifinite number of degrees of freedom).
In cosmological regime, because of assumed symmetries (only few degrees of freedom) - Loop Quantum Cosmology (LQC).
LQG and LQC - very complicated theories, but based on very simple idea: discretisation of the space by loops.
Quantum of space exists! (of unknown size...)
Two LQC methods:

- standard LQC: 'first quantise, then impose constraints'
- non-standard LQC: 'first solve constraints, then quantise'

(by constraint we mean Hamiltonian)

Quantum Cosmology in general

There are many attempts to quantising gravity.
One of the most promising is Loop Quantum Gravity (LQG) (theory of field, inifinite number of degrees of freedom).
In cosmological regime, because of assumed symmetries (only few degrees of freedom) - Loop Quantum Cosmology (LQC).
LQG and LQC - very complicated theories, but based on very simple idea: discretisation of the space by loops.
Quantum of space exists! (of unknown size...)
Two LQC methods:

- standard LQC: 'first quantise, then impose constraints'
- non-standard LQC: 'first solve constraints, then quantise'

(by constraint we mean Hamiltonian)

Quantum Cosmology in general

There are many attempts to quantising gravity.
One of the most promising is Loop Quantum Gravity (LQG) (theory of field, inifinite number of degrees of freedom).
In cosmological regime, because of assumed symmetries (only few degrees of freedom) - Loop Quantum Cosmology (LQC).
LQG and LQC - very complicated theories, but based on very simple idea: discretisation of the space by loops.
Quantum of space exists! (of unknown size...)
Two LQC methods:

- standard LQC: 'first quantise, then impose constraints'
- non-standard LQC: 'first solve constraints, then quantise'

(by constraint we mean Hamiltonian)

Bianchi I non-standard LQC

Few words about non-standard LQC:

- Invented here in INS about two years ago by prof. W. Piechocki and dr P. Małkiewicz
- Revolutionary method, makes calculations extremely easy: Bianchi I solved analitically in few months!
- Spectra of operators in physical phase space obtained - No comparable results using standard LQC!

Bianchi I non-standard LQC

Few words about non-standard LQC:

- Invented here in INS about two years ago by prof. W. Piechocki and dr P. Małkiewicz
- Revolutionary method, makes calculations extremely easy: Bianchi I solved analitically in few months!
- Spectra of operators in physical phase space obtained - No comparable results using standard LQC!

Bianchi I non-standard LQC

Few words about non-standard LQC:

- Invented here in INS about two years ago by prof. W. Piechocki and dr P. Małkiewicz
- Revolutionary method, makes calculations extremely easy: Bianchi I solved analitically in few months!
- Spectra of operators in physical phase space obtained - No comparable results using standard LQC!

Bianchi I non-standard LQC

Few words about non-standard LQC:

- Invented here in INS about two years ago by prof. W. Piechocki and dr P. Małkiewicz
- Revolutionary method, makes calculations extremely easy: Bianchi I solved analitically in few months!
- Spectra of operators in physical phase space obtained - No comparable results using standard LQC!

Bianchi I non-standard LQC

How it looks like in practise?

- Discretise Hamiltonian using loops

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\left(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2}v_1 v_2 + \right. \quad (4)$$

$$\left. + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2}v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2}v_2 v_3\right)$$

where: $\beta_i := \frac{\gamma\,\dot{a}_i}{\sqrt{|a_j\,a_k|}}$, $\quad v_i := |a_j\,a_k|^{3/2}\,\mathrm{sgn}(a_j\,a_k)$;
p_ϕ -momentum of scalar field which plays the role of time (value of scalar field ϕ is assumed to grow with time);
λ is a regularisation parameter connected to size of the quantum of space.

Bianchi I non-standard LQC

How it looks like in practise?

- Discretise Hamiltonian using loops

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\Big(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2}v_1 v_2 + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2}v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2}v_2 v_3\Big) \tag{4}$$

where: $\beta_i := \frac{\gamma\,\dot{a}_i}{\sqrt{|a_j\,a_k|}}$, $\quad v_i := |a_j\,a_k|^{3/2}\,\mathrm{sgn}(a_j\,a_k)$;

p_ϕ -momentum of scalar field which plays the role of time (value of scalar field ϕ is assumed to grow with time);

λ is a regularisation parameter connected to size of the quantum of space.

Bianchi I non-standard LQC

How it looks like in practise?

- Discretise Hamiltonian using loops

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\left(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2} v_1 v_2 + \right. \qquad (4)$$
$$\left. + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2} v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2} v_2 v_3\right)$$

where: $\beta_i := \frac{\gamma \dot{a}_i}{\sqrt{|a_j a_k|}}$, $\quad v_i := |a_j a_k|^{3/2} \operatorname{sgn}(a_j a_k)$;

p_ϕ -momentum of scalar field which plays the role of time (value of scalar field ϕ is assumed to grow with time);

λ is a regularisation parameter connected to size of the quantum of space.

Bianchi I non-standard LQC

How it looks like in practise?

- Discretise Hamiltonian using loops

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\left(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2}v_1 v_2 + \right. \tag{4}$$
$$\left. + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2}v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2}v_2 v_3\right)$$

where: $\beta_i := \frac{\gamma\,\dot{a}_i}{\sqrt{|a_j\,a_k|}}$, $\quad v_i := |a_j\,a_k|^{3/2}\,\mathrm{sgn}(a_j\,a_k)$;

p_ϕ -momentum of scalar field which plays the role of time (value of scalar field ϕ is assumed to grow with time);

λ is a regularisation parameter connected to size of the quantum of space.

Bianchi I non-standard LQC

How it looks like in practise?

- Discretise Hamiltonian using loops

$$H^{(\lambda)} = \frac{p_\phi^2}{2} - \frac{1}{8\pi G\gamma^2}\left(\frac{\sin(\lambda\beta_1)\sin(\lambda\beta_2)}{\lambda^2}v_1 v_2 + \right. \tag{4}$$
$$\left. + \frac{\sin(\lambda\beta_1)\sin(\lambda\beta_3)}{\lambda^2}v_1 v_3 + \frac{\sin(\lambda\beta_2)\sin(\lambda\beta_3)}{\lambda^2}v_2 v_3\right)$$

where: $\beta_i := \frac{\gamma\,\dot{a}_i}{\sqrt{|a_j\,a_k|}}$, $\quad v_i := |a_j\,a_k|^{3/2}\,\mathrm{sgn}(a_j\,a_k)$;
p_ϕ -momentum of scalar field which plays the role of time (value of scalar field ϕ is assumed to grow with time);
λ is a regularisation parameter connected to size of the quantum of space.

Bianchi I non-standard LQC

- Find functions F satisfying $\{F, H^{(\lambda)}\} = 0$ so they are constants of motion. We called them elementary observables.

$$\mathrm{A}_i = \ln\left|\tan\left(\frac{\lambda\beta_i}{2}\right)\right| + \frac{3\sqrt{\pi G}\,\mathrm{sgn}(p_\phi)(\mathrm{O}_j + \mathrm{O}_k)\,\phi}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}} \quad (5)$$

$$\mathrm{O}_i = \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda} \quad (6)$$

Find their Poisson algebra:

$$\{\mathrm{O}_i, \mathrm{O}_j\} = 0, \quad \{\mathrm{A}_i, \mathrm{O}_i\} = 1,$$
$$\{\mathrm{A}_i, \mathrm{O}_j\} = 0, \quad \{\mathrm{A}_i, \mathrm{A}_j\} = 0. \quad (7)$$

Bianchi I non-standard LQC

- Find functions F satisfying $\{F, H^{(\lambda)}\} = 0$ so they are constants of motion. We called them elementary observables.

$$\mathrm{A}_i = \ln\left|\tan\left(\frac{\lambda\beta_i}{2}\right)\right| + \frac{3\sqrt{\pi G}\,\mathrm{sgn}(p_\phi)\left(\mathrm{O}_j + \mathrm{O}_k\right)\phi}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}} \quad (5)$$

$$\mathrm{O}_i = \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda} \quad (6)$$

Find their Poisson algebra:

$$\{\mathrm{O}_i, \mathrm{O}_j\} = 0, \quad \{\mathrm{A}_i, \mathrm{O}_i\} = 1,$$
$$\{\mathrm{A}_i, \mathrm{O}_j\} = 0, \quad \{\mathrm{A}_i, \mathrm{A}_j\} = 0. \quad (7)$$

Bianchi I non-standard LQC

- Find functions F satisfying $\{F, H^{(\lambda)}\} = 0$ so they are constants of motion. We called them elementary observables.

$$\mathrm{A}_i = \ln\left|\tan\left(\frac{\lambda\beta_i}{2}\right)\right| + \frac{3\sqrt{\pi G}\,\mathrm{sgn}(p_\phi)\left(\mathrm{O}_j + \mathrm{O}_k\right)\phi}{\sqrt{\mathrm{O}_1\mathrm{O}_2 + \mathrm{O}_1\mathrm{O}_3 + \mathrm{O}_2\mathrm{O}_3}} \quad (5)$$

$$\mathrm{O}_i = \frac{v_i \sin(\lambda\beta_i)}{12\pi G\gamma\lambda} \quad (6)$$

Find their Poisson algebra:

$$\{\mathrm{O}_i, \mathrm{O}_j\} = 0, \quad \{\mathrm{A}_i, \mathrm{O}_i\} = 1,$$
$$\{\mathrm{A}_i, \mathrm{O}_j\} = 0, \quad \{\mathrm{A}_i, \mathrm{A}_j\} = 0. \quad (7)$$

Bianchi I non-standard LQC

- Use elementary observables to express interesting physical functions i.e. volume of the Universe, density of matter... . These are called compound observables and are functions of time.

  For example volume of the Universe $V = (w_1 w_2 w_3)^{1/3}$, where

  $$w_i = \kappa\gamma\lambda \mathrm{O}_i \cosh 3\kappa\big((\mathrm{O}_j + \mathrm{O}_k)\,\phi - \mathrm{A}_i\big) \tag{8}$$

  and where $\kappa := \sqrt{4\pi G/3}$

Bianchi I non-standard LQC

- Use elementary observables to express interesting physical functions i.e. volume of the Universe, density of matter... . These are called compound observables and are functions of time.
  For example volume of the Universe $V = (w_1 w_2 w_3)^{1/3}$, where

$$w_i = \kappa\gamma\lambda \mathrm{O}_i \cosh 3\kappa\big((\mathrm{O}_j + \mathrm{O}_k)\,\phi - \mathrm{A}_i\big) \tag{8}$$

  and where $\kappa := \sqrt{4\pi G/3}$

Bianchi I non-standard LQC

- Find self-adjoint representation for elementary observables.

We use the Schrödinger representation defined on some dense subspaces of a Hilbert space:

$$\mathrm{O}_1 \to \widehat{\mathrm{O}}_1\, f_1(x) := -i\hbar\,\partial_x f_1(x), \;\; \mathrm{O}_2 \to \widehat{\mathrm{O}}_2\, f_2(y) := -i\hbar\,\partial_y f_2(y), \;\; \mathrm{O}_3 \to \widehat{\mathrm{O}}_3\, f_3(z) := -i\hbar\,\partial_z f_3(z), \quad (9)$$

and

$$\mathrm{A}_1 \to \widehat{\mathrm{A}}_1\, f_1(x) := x\, f_1(x), \;\; \mathrm{A}_2 \to \widehat{\mathrm{A}}_2\, f_2(y) := y\, f_2(y), \;\; \mathrm{A}_3 \to \widehat{\mathrm{A}}_3\, f_3(z) := z\, f_3(z). \quad (10)$$

One may verify that

$$[\widehat{\mathrm{O}}_i, \widehat{\mathrm{O}}_j] = 0, \;\; [\widehat{\mathrm{A}}_i, \widehat{\mathrm{A}}_j] = 0, \;\; [\widehat{\mathrm{A}}_i, \widehat{\mathrm{O}}_j] = i\hbar\,\delta_{ij}. \quad (11)$$

Bianchi I non-standard LQC

- Find self-adjoint representation for elementary observables. We use the Schrödinger representation defined on some dense subspaces of a Hilbert space:

$$\mathrm{O}_1 \to \widehat{\mathrm{O}}_1\, f_1(x) := -i\hbar\,\partial_x f_1(x), \;\; \mathrm{O}_2 \to \widehat{\mathrm{O}}_2\, f_2(y) := -i\hbar\,\partial_y\, f_2(y),$$
$$\mathrm{O}_3 \to \widehat{\mathrm{O}}_3\, f_3(z) := -i\hbar\,\partial_z\, f_3(z), \quad (9)$$

and

$$\mathrm{A}_1 \to \widehat{\mathrm{A}}_1\, f_1(x) := x\, f_1(x), \quad \mathrm{A}_2 \to \widehat{\mathrm{A}}_2\, f_2(y) := y\, f_2(y),$$
$$\mathrm{A}_3 \to \widehat{\mathrm{A}}_3\, f_3(z) := z\, f_3(z). \qquad (10)$$

One may verify that

$$[\widehat{\mathrm{O}}_i, \widehat{\mathrm{O}}_j] = 0, \;\; [\widehat{\mathrm{A}}_i, \widehat{\mathrm{A}}_j] = 0, \;\; [\widehat{\mathrm{A}}_i, \widehat{\mathrm{O}}_j] = i\hbar\,\delta_{ij}. \qquad (11)$$

Bianchi I non-standard LQC

- Find self-adjoint representation for elementary observables. We use the Schrödinger representation defined on some dense subspaces of a Hilbert space:

$$\mathrm{O}_1 \rightarrow \widehat{\mathrm{O}}_1\, f_1(x) := -i\hbar\,\partial_x f_1(x), \;\; \mathrm{O}_2 \rightarrow \widehat{\mathrm{O}}_2\, f_2(y) := -i\hbar\,\partial_y\, f_2(y), \;\; \mathrm{O}_3 \rightarrow \widehat{\mathrm{O}}_3\, f_3(z) := -i\hbar\,\partial_z\, f_3(z), \tag{9}$$

and

$$\mathrm{A}_1 \rightarrow \widehat{\mathrm{A}}_1\, f_1(x) := x\, f_1(x), \quad \mathrm{A}_2 \rightarrow \widehat{\mathrm{A}}_2\, f_2(y) := y\, f_2(y), \quad \mathrm{A}_3 \rightarrow \widehat{\mathrm{A}}_3\, f_3(z) := z\, f_3(z). \tag{10}$$

One may verify that

$$[\widehat{\mathrm{O}}_i, \widehat{\mathrm{O}}_j] = 0, \;\; [\widehat{\mathrm{A}}_i, \widehat{\mathrm{A}}_j] = 0, \;\; [\widehat{\mathrm{A}}_i, \widehat{\mathrm{O}}_j] = i\hbar\,\delta_{ij}. \tag{11}$$

Bianchi I non-standard LQC

- Using that representation build operators corresponding to compound observables. Examine these operators.

  Finally obtained:

$$\widehat{V}\Psi_{n_1,n_2,n_3}(x,y,z) := \Box\,\Psi_{n_1,n_2,n_3}(x,y,z). \tag{12}$$

  The eigenfunctions are $\Psi_{n_1,n_2,n_3}(x,y,z) := \psi_{n_1}(x)\psi_{n_2}(y)\psi_{n_3}(z)$ where $\psi_{n_1}(x) := \frac{\exp\left(i\,4n_1\,\mathrm{arctg}(e^{bx})\right)}{\cosh^{1/2}(bx)}$.

  The eigenvalues:

$$\Box := (n_1 n_2 n_3)^{1/3}\,\triangle, \tag{13}$$

  where $\triangle$ is quantum of space in LQC obtained for FRW metric.

Bianchi I non-standard LQC

- Using that representation build operators corresponding to compound observables. Examine these operators.
  Finally obtained:

$$\widehat{V}\Psi_{n_1,n_2,n_3}(x,y,z) := \Box\,\Psi_{n_1,n_2,n_3}(x,y,z). \tag{12}$$

The eigenfunctions are $\Psi_{n_1,n_2,n_3}(x,y,z) := \psi_{n_1}(x)\psi_{n_2}(y)\psi_{n_3}(z)$
where $\psi_{n_1}(x) := \frac{\exp\left(i\,4n_1\,\mathrm{arctg}(e^{bx})\right)}{\cosh^{1/2}(bx)}$.
The eigenvalues:

$$\Box := (n_1 n_2 n_3)^{1/3}\,\triangle, \tag{13}$$

where $\triangle$ is quantum of space in LQC obtained for FRW metric.

Bianchi I non-standard LQC

- Using that representation build operators corresponding to compound observables. Examine these operators.
  Finally obtained:

$$\widehat{V}\Psi_{n_1,n_2,n_3}(x,y,z) := \Box\,\Psi_{n_1,n_2,n_3}(x,y,z). \tag{12}$$

  The eigenfunctions are $\Psi_{n_1,n_2,n_3}(x,y,z) := \psi_{n_1}(x)\psi_{n_2}(y)\psi_{n_3}(z)$ where $\psi_{n_1}(x) := \frac{\exp\left(i\,4n_1\,\mathrm{arctg}(e^{bx})\right)}{\cosh^{1/2}(bx)}$.
  The eigenvalues:

$$\Box := (n_1 n_2 n_3)^{1/3}\,\triangle, \tag{13}$$

  where $\triangle$ is quantum of space in LQC obtained for FRW metric.

Bianchi I non-standard LQC

- Using that representation build operators corresponding to compound observables. Examine these operators.
  Finally obtained:

$$\widehat{V}\Psi_{n_1,n_2,n_3}(x,y,z) := \square\,\Psi_{n_1,n_2,n_3}(x,y,z). \tag{12}$$

  The eigenfunctions are $\Psi_{n_1,n_2,n_3}(x,y,z) := \psi_{n_1}(x)\psi_{n_2}(y)\psi_{n_3}(z)$
  where $\psi_{n_1}(x) := \frac{\exp\left(i\,4n_1\,\mathrm{arctg}(e^{bx})\right)}{\cosh^{1/2}(bx)}$.
  The eigenvalues:

$$\square := (n_1 n_2 n_3)^{1/3}\,\triangle, \tag{13}$$

  where $\triangle$ is quantum of space in LQC obtained for FRW metric.

Conclusions

1. Bianchi I is (among other Bianchi model) is extremely important in modern cosmology.
2. Non-standard LQC is very promising and effective method of quantising cosmological models as it was shown in Bianchi I case.
3. Lot to do...

Conclusions

1. Bianchi I is (among other Bianchi model) is extremely important in modern cosmology.
2. Non-standard LQC is very promising and effective method of quantising cosmological models as it was shown in Bianchi I case.
3. Lot to do...

Conclusions

1. Bianchi I is (among other Bianchi model) is extremely important in modern cosmology.
2. Non-standard LQC is very promising and effective method of quantising cosmological models as it was shown in Bianchi I case.
3. Lot to do...

Conclusions

1. Bianchi I is (among other Bianchi model) is extremely important in modern cosmology.
2. Non-standard LQC is very promising and effective method of quantising cosmological models as it was shown in Bianchi I case.
3. Lot to do...

Conclusions

Next steps:

- Examine other operators. For example operator of directional density of matter $\widehat{\rho_i}$ which tells us when the bounce occurs in i-th direction.
- Make this model more physical by e.g. adding some potential to scalar field.
- Consider different Bianchi models. The ultimate goal: understand evolution of Mixmaster Universe.

Conclusions

Next steps:

- Examine other operators. For example operator of directional density of matter $\widehat{\rho}_i$ which tells us when the bounce occurs in i-th direction.
- Make this model more physical by e.g. adding some potential to scalar field.
- Consider different Bianchi models. The ultimate goal: understand evolution of Mixmaster Universe.

Conclusions

Next steps:

- Examine other operators. For example operator of directional density of matter $\widehat{\rho}_i$ which tells us when the bounce occurs in i-th direction.
- Make this model more physical by e.g. adding some potential to scalar field.
- Consider different Bianchi models. The ultimate goal: understand evolution of Mixmaster Universe.

Conclusions

Next steps:

- Examine other operators. For example operator of directional density of matter $\widehat{\rho}_i$ which tells us when the bounce occurs in i-th direction.
- Make this model more physical by e.g. adding some potential to scalar field.
- Consider different Bianchi models. The ultimate goal: understand evolution of Mixmaster Universe.

Based on:

[1] P. Dz., P. Małkiewicz and W. Piechocki,
'Turning Big Bang into Big Bounce: I. Classical dynamics',
Phys. Rev. D80, 104001 (2009) [arXiv:gr-qc/0907.3436].

[2] P. Dz., W. Piechocki,
'The Bianchi I model in terms of nonstandard loop quantum cosmology: Classical dynamics',
Phys. Rev. D80, 124033 (2009) [arXiv:gr-qc/0909.4211].

Thank you for your attention!