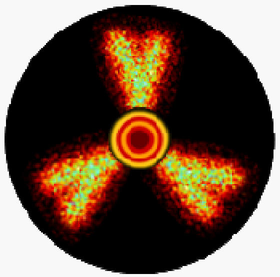
Two-proton radioactivity

Lecture 2



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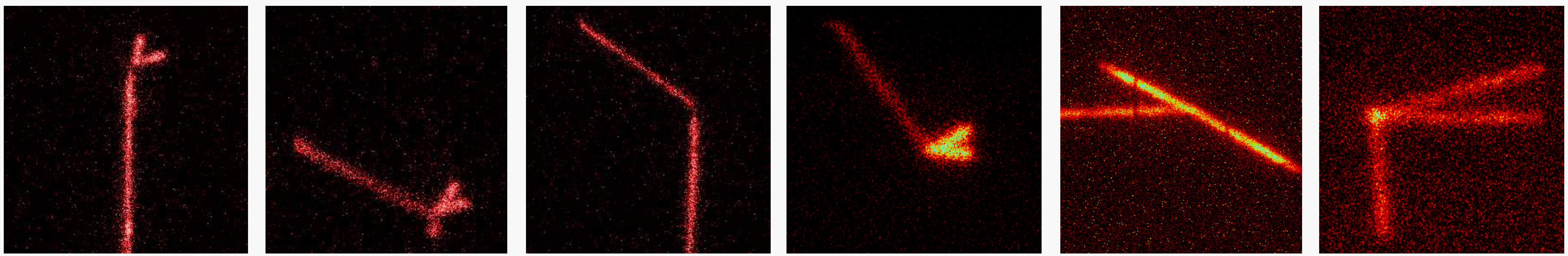
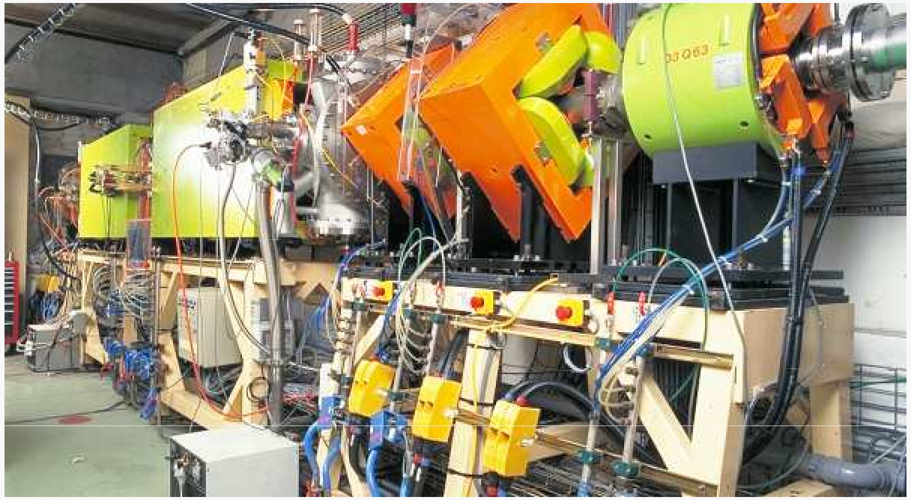
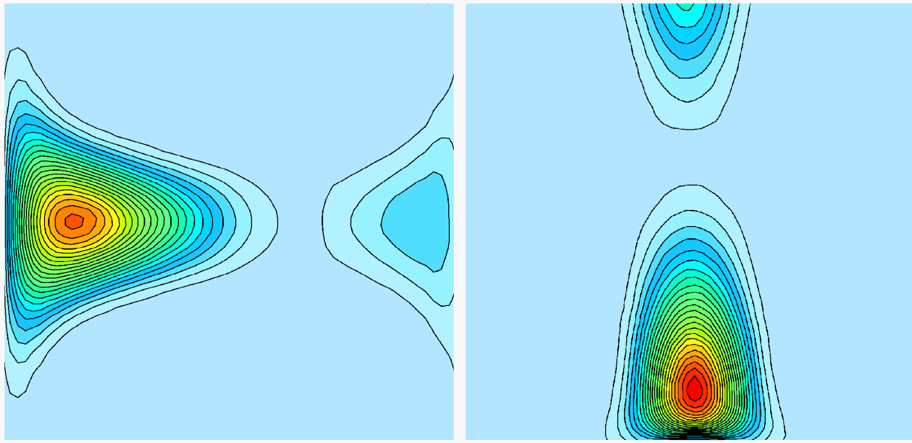


Outline



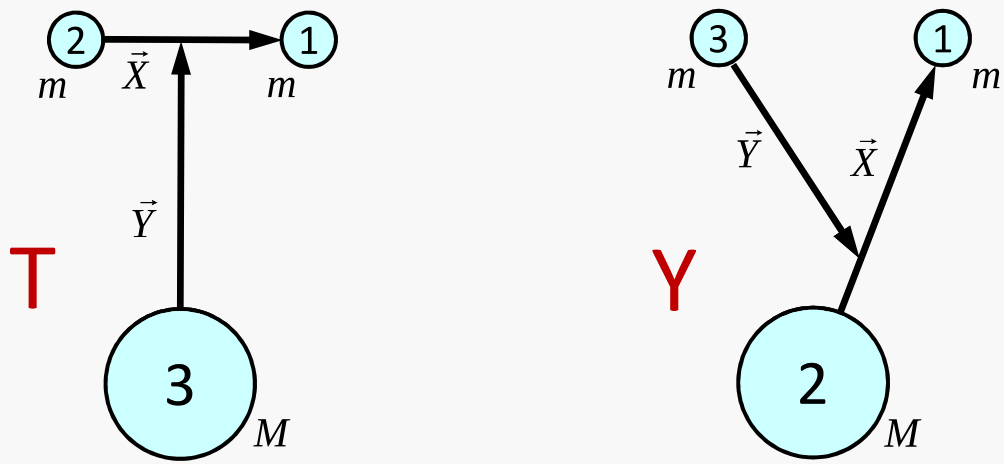


- Basic introduction
- The story of ^{45}Fe
  - mass predictions
  - production method
  - discovery of 2p decay
- Quest for p-p correlations
  - OTPC detector
  - images of ^{45}Fe decay

- Introduction to theory
  - Jacobi coordinates
  - Simplified models
- Momentum correlations
- Decays of ^{6}Be, ^{19}Mg, ^{48}Ni and ^{54}Zn
- Predictions of heavier emitters and the full 2p landscape
- Summary

Jacobi coordinates, positions

- Three-body kinematics is simpler in Jacobi coordinates



- In place of the radius and solid angle of one particle, the three particles are described by the *hyperradius* and *hyper solid angle*:

$$r,\ \Omega \ \rightarrow\ \rho, \Omega_5$$

$$\Omega_5 = \left\{\theta_\rho, \Omega_X, \Omega_Y\right\} \qquad \rho^2 = \frac{A_1 A_2 A_3}{A_1 + A_2 + A_3}\left(\frac{\vec{r}_{12}^{\,2}}{A_3} + \frac{\vec{r}_{23}^{\,2}}{A_1} + \frac{\vec{r}_{31}^{\,2}}{A_2}\right)$$

$$\tan\left(\theta_\rho\right) = \sqrt{M_x / M_y}\, X/Y \qquad M_x = \frac{A_1 A_2}{A_1 + A_2} u\ ,\ \ M_y = \frac{\left(A_1 + A_2\right) A_3}{A_1 + A_2 + A_3} u$$

Jacobi coordinates, momenta

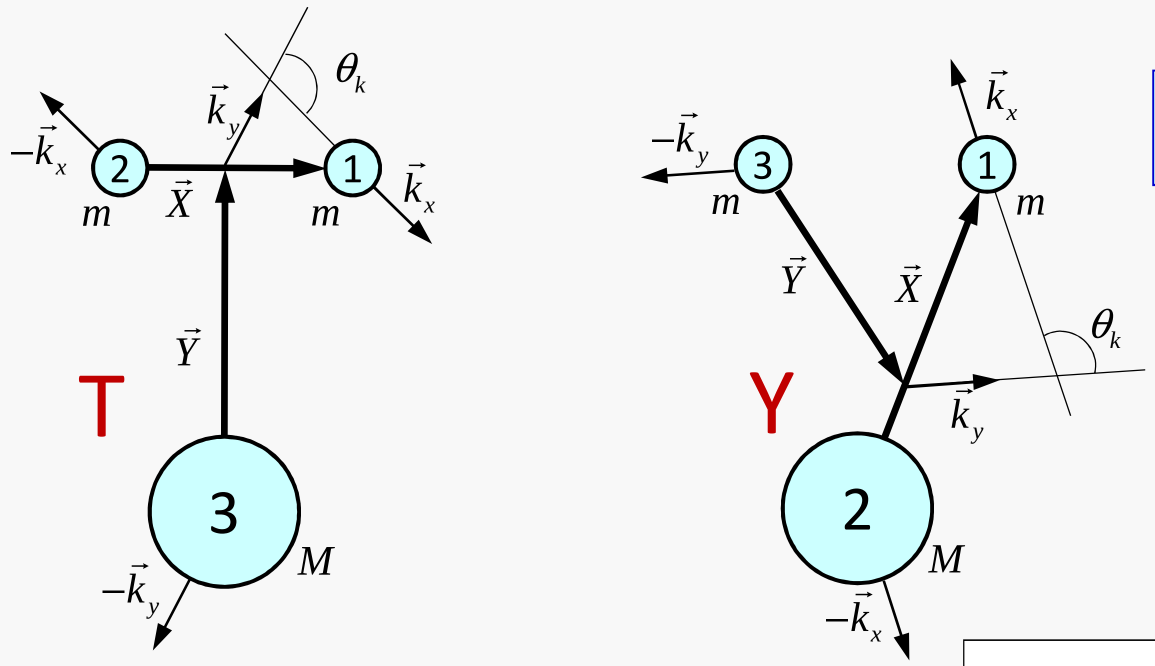


Illustration
⇨ *Mathematica_Adds-on*

- Complete correlation picture is given by two parameters: $\varepsilon = E_x / E_T\,, \quad \cos(\theta_k) = \dfrac{\vec{k}_x \cdot \vec{k}_y}{k_x\, k_y}$

E_T is the total decay energy: $E_T = E_x + E_y = \dfrac{k_x^2}{2M_x} + \dfrac{k_y^2}{2M_y}$

$$M_x = \frac{A_1 A_2}{A_1 + A_2} u$$

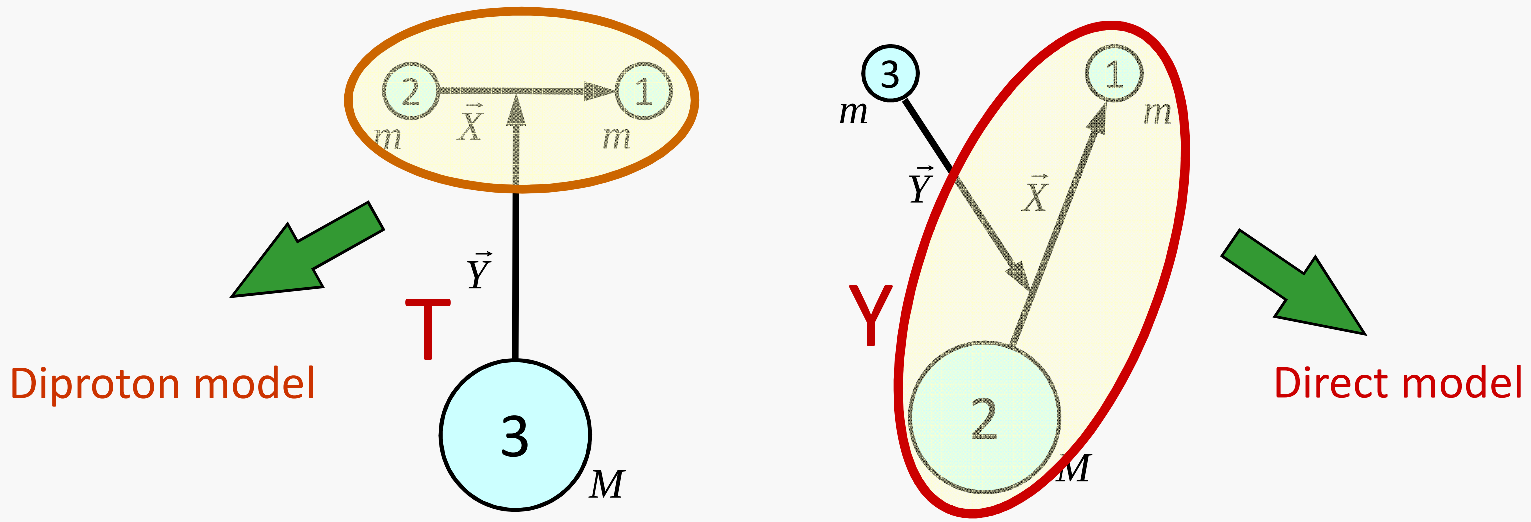
If the decay occurs at rest: $\vec{k}_x = \dfrac{A_2 \vec{k}_1 - A_1 \vec{k}_2}{A_1 + A_2}\,, \quad \vec{k}_y = -\vec{k}_3$

$$M_y = \frac{(A_1 + A_2) A_3}{A_1 + A_2 + A_3} u$$

Simplified models

- By simplifying interactions describing the *core+p+p* system, the three-body decay can be reduced to the combination of two-body processes. With the simplified Hamiltonian, the problem can be solved exactly.

→ Two types of approximations are considered:



- The simplified models are very useful to estimate decay rates and to verify numerical procedures used in the full three-body model.

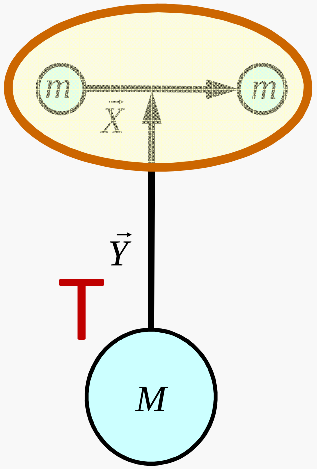
Diproton model

- Jacobi T system → **diproton model**

The WKB approximation

$$\Gamma_{2p,dipr} = \theta^2_{dipr} \mathcal{N} \frac{\hbar^2}{4\mu} \exp\left[-2\int_{r_2}^{r_3} k(r)\, dr \right]$$

$$T_{1/2} = \frac{\ln 2\, \hbar}{\Gamma}$$



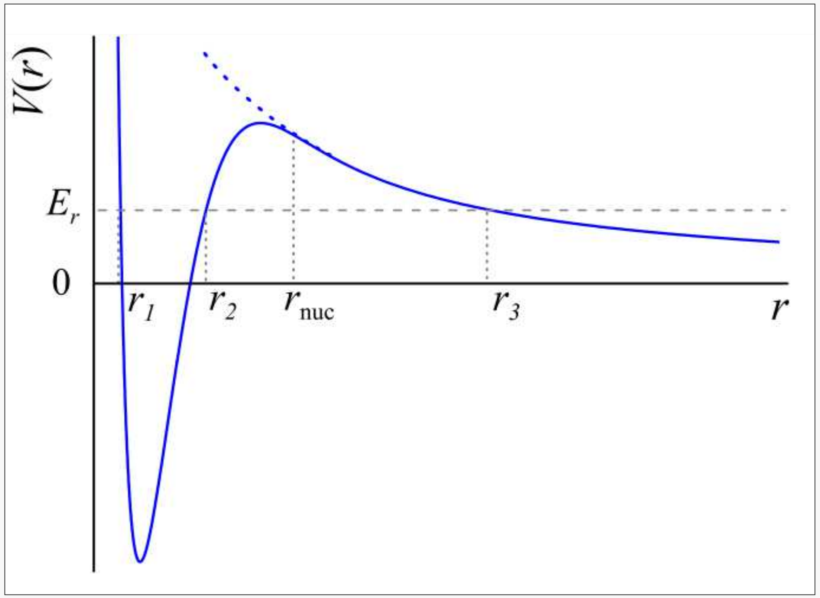
$$\mathcal{N} \int_{r_1}^{r_2} \frac{dr}{2k(r)} = 1$$

$$k(r) = \sqrt{2\mu \left| E_T - 2V_p(r) \right|} \Big/ \hbar$$

$$\theta^2_{\mathrm{dipr}} = \frac{(2n)!}{2^{2n}(n!)^2} \left[\frac{A}{A-2} \right]^{2n} \mathcal{O}^2$$

$$n \approx (3Z)^{1/3} - 1$$

cluster overlap: $\mathcal{O}^2 = \left| \left\langle \psi_f \middle| \psi_{2p} \middle| \psi_i \right\rangle \right|^2$



Brown, PRC 43 (91) R1513
Nazarewicz et al., PRC 53 (1996) 740

Direct model

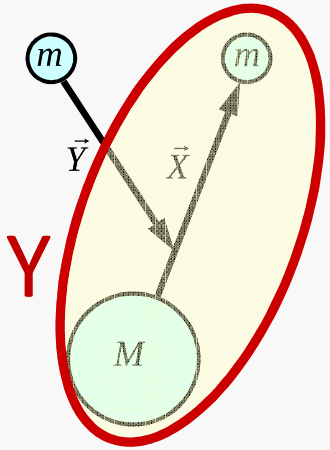
➤ In the Y Jacobi system: $\vec{k}_x = \vec{k}_1 + \frac{\mu}{M}\vec{k}_3, \quad \vec{k}_y = -\vec{k}_3, \quad E_x = \frac{k_x^2}{2\mu}$ $\qquad \mu = \frac{mM}{m+M}$

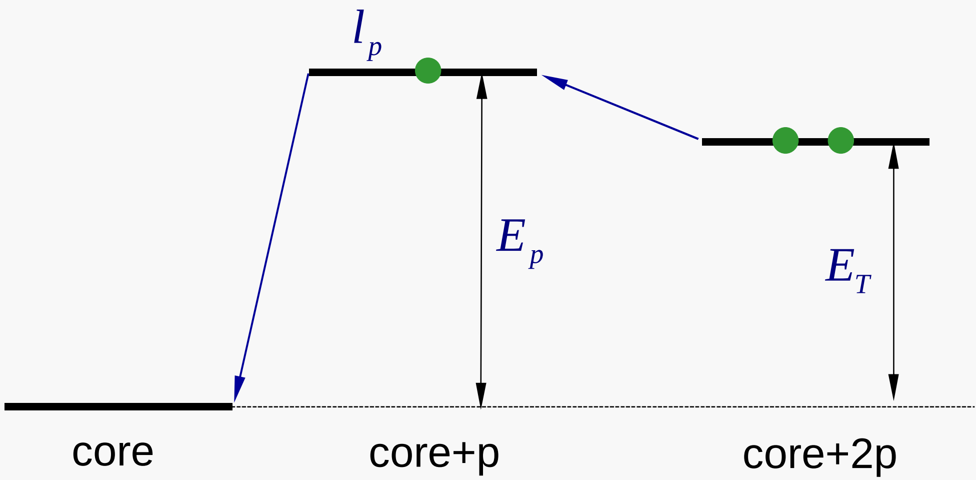
❶ Assume: $M >> m \;\rightarrow \mu \cong m \;\rightarrow \vec{k}_x \cong \vec{k}_1, \; E_x = \frac{k_1^2}{2m}$

➔ Then ε is the fraction of the decay energy taken by one proton and

θ_k is the angle between momenta of both protons + π

❷ Assume: both protons occupy the same orbital with angular momentum l_p





Direct model

- The *2p* decay width in the direct model is given by:

$$\Gamma_{2p,dir} = \frac{E_T}{2\pi}\left(E_T - 2E_p\right)^2 \int_0^1 d\varepsilon \frac{\Gamma_x(\varepsilon E_T)}{\left(\varepsilon E_T - E_p\right)^2 + \Gamma_x(\varepsilon E_T)^2/4} \times \frac{\Gamma_y((1-\varepsilon)E_T)}{\left((1-\varepsilon)E_T - E_p\right)^2 + \Gamma_y((1-\varepsilon)E_T)^2/4}$$

$$\Gamma_{2p,dir} \cong \frac{8E_T}{\pi\left(E_T - 2E_p\right)^2}\int_0^1 d\varepsilon\, \Gamma_x(\varepsilon E_T)\,\Gamma_y((1-\varepsilon)E_T)$$

where Γ_i is the width of the two-body subsystem: $\Gamma_i(E) = 2\gamma_i^2 P_{l_p}(E,R,Z_i)$

reduced width: $\gamma_i^2 = \frac{\hbar^2}{2\mu_i R^2}\theta_i^2$

$Z_x = Z_{\text{core}}$

penetrability: $P_{l_p}(E,R,Z_i) = \frac{kR}{F_{l_p}^2(\eta,kR) + G_{l_p}^2(\eta,kR)}$

$Z_y = Z_{\text{core}} + 1$

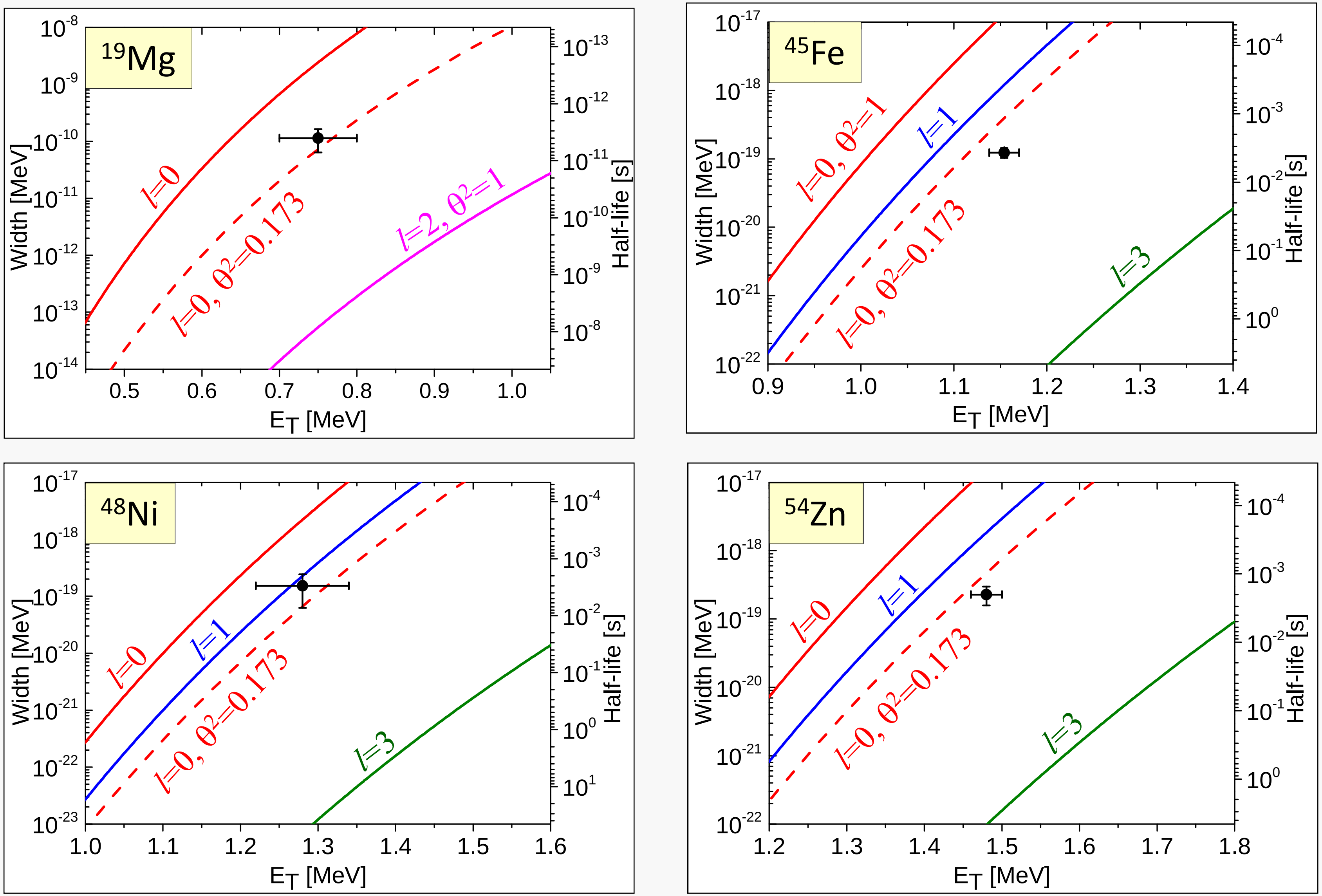
radius: $R = 1.4\left(A_{\text{core}} + 1\right)^{1/3}$ fm

Sommerfeld parameter: $\eta = \mu Z e^2/\hbar^2 k$

wave number: $k = \sqrt{2\mu E}/\hbar$

Grigorenko and Zhukov, PRC 76 (07) 014009
M.P. et al, RMP (2012) 567

Direct model for known 2p emitters



Direct model

- The direct model is a very convenient tool to estimate the half-life of the 2p decay.
- With one set of parameters, $l = 0$ and $\theta^2 = 0.173$, the model reproduces the four measured half-lives within a factor of 2-3
- The model yields the realistic shape of the proton's energy spectrum

$$\frac{d\Gamma_{dir}}{d\varepsilon}(\varepsilon) \sim \frac{\Gamma_x(\varepsilon E_T)}{(\varepsilon E_T - E_p)^2 + \Gamma_x(\varepsilon E_T)^2/4} \times \frac{\Gamma_y((1-\varepsilon)E_T)}{((1-\varepsilon)E_T - E_p)^2 + \Gamma_y((1-\varepsilon)E_T)^2/4}$$

- This feature allows to inspect how the position of the intermediate state influences the shape of the spectrum. In particular, one can study the transition from the simultaneous to the sequential emission

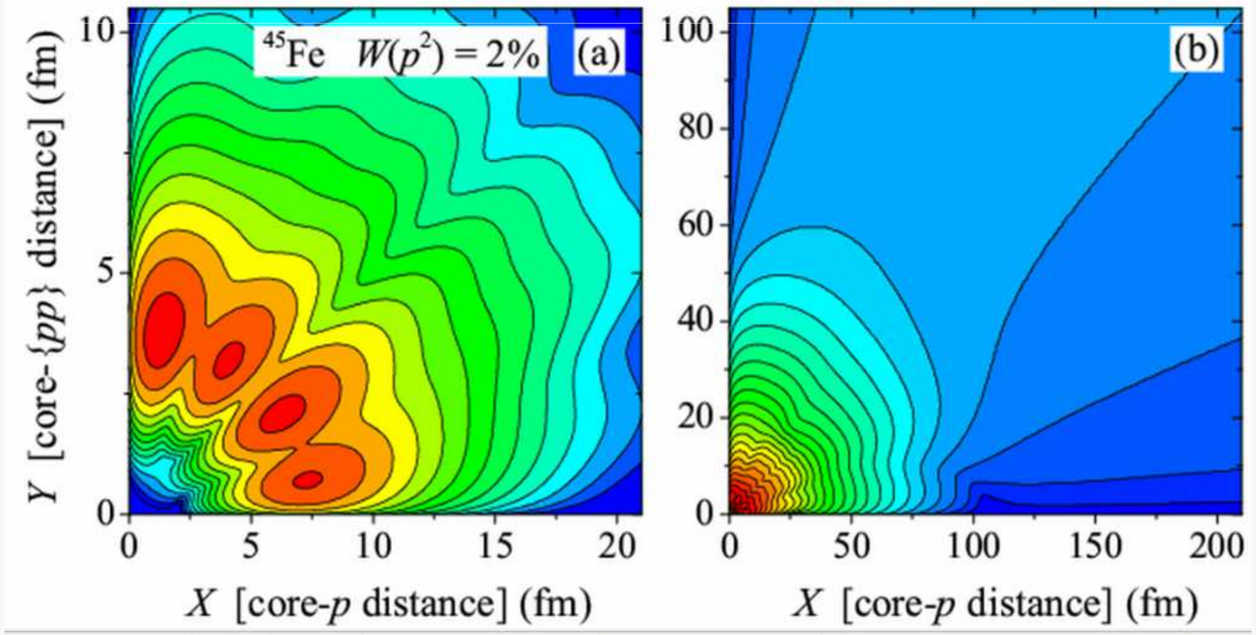
Illustration ⇨ *Mathematica_Adds-on*

3-body model

- The full three-body model of the 2p radioactivity has been constructed by Grigorenko and Zhukov. Presently it is the only model which predicts the momentum correlations between emitted protons.

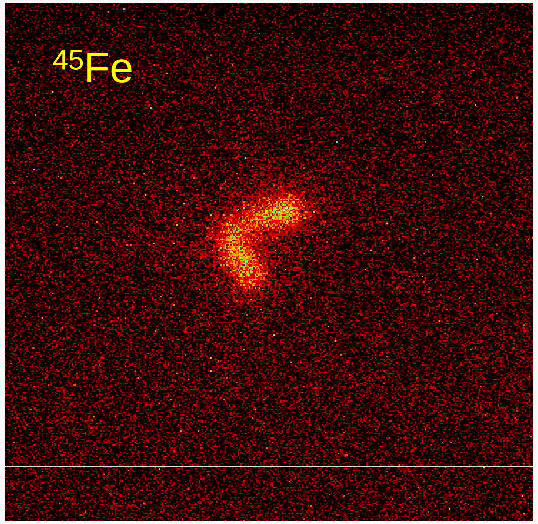
See the next two lectures by Leonid Grigorenko!

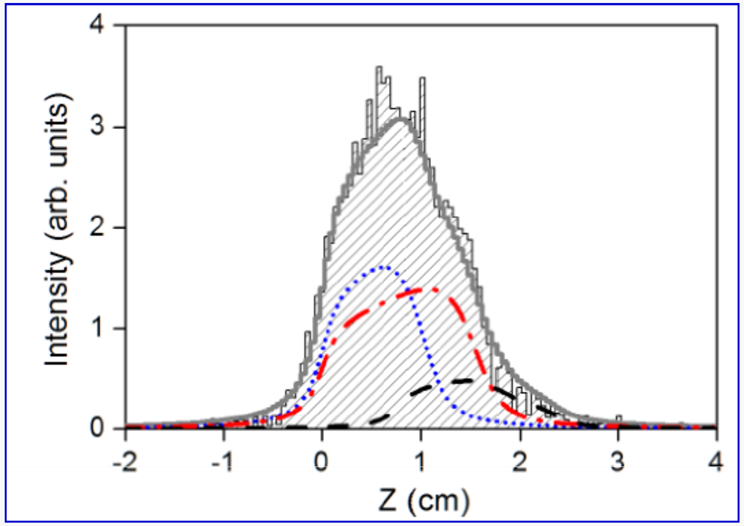
The ^{45}Fe wave function density in the T system for the configuration 98% f^2 + 2% p^2

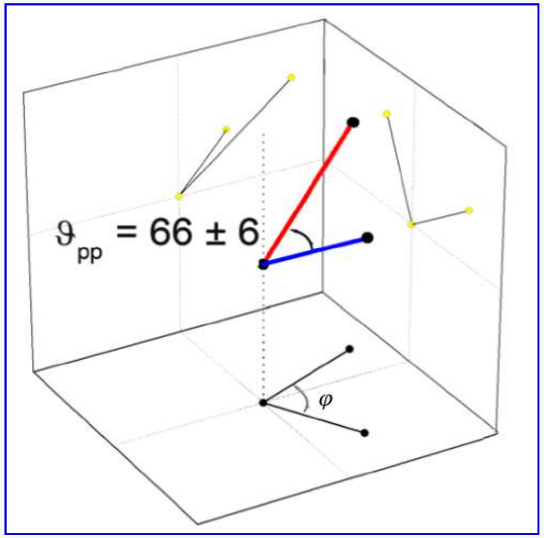


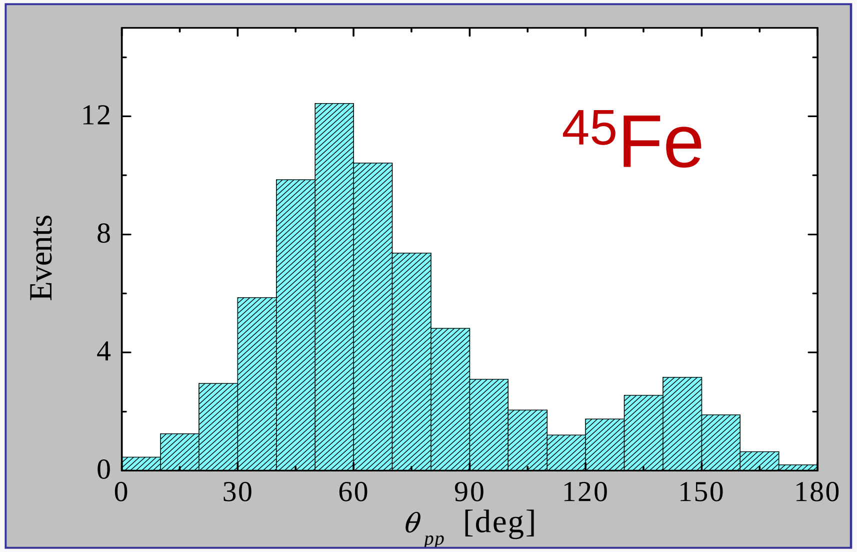
Grigorenko and Zhukov, PRC 68 (03) 054005
M.P. et al, RMP (2012) 567

Back to ^{45}Fe



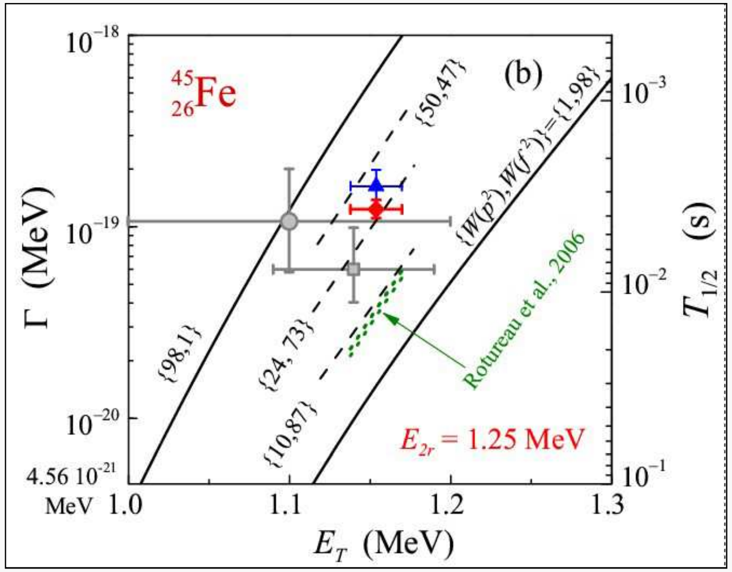


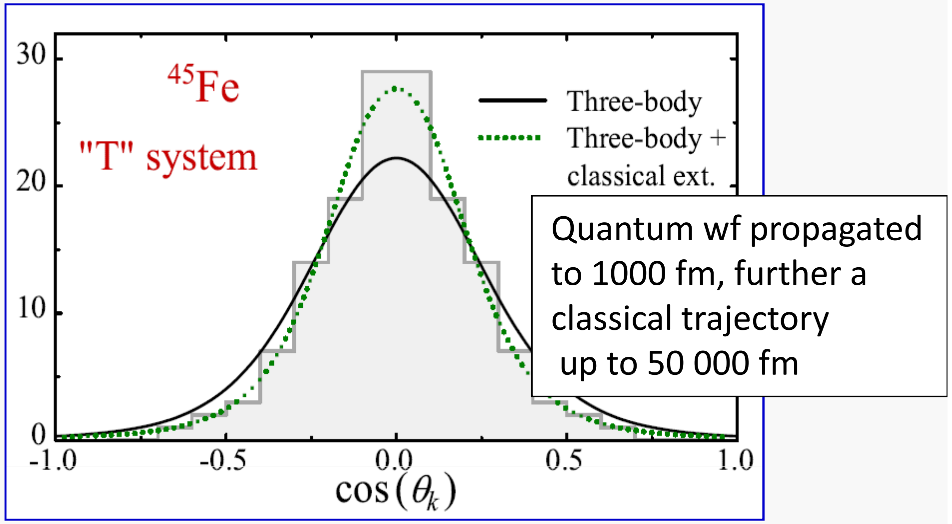


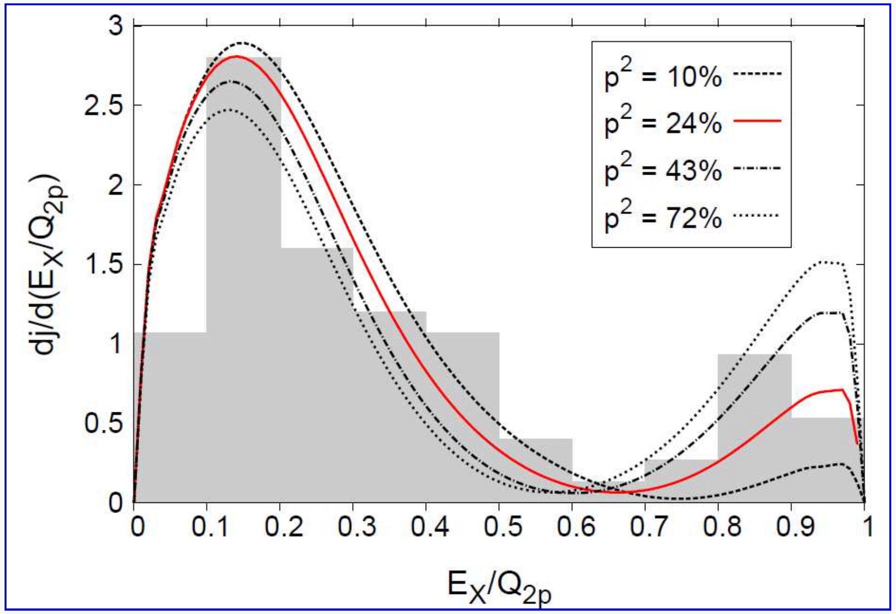


K. Miernik et al., PRL 99 (07) 192501

3-body model and ^{45}Fe





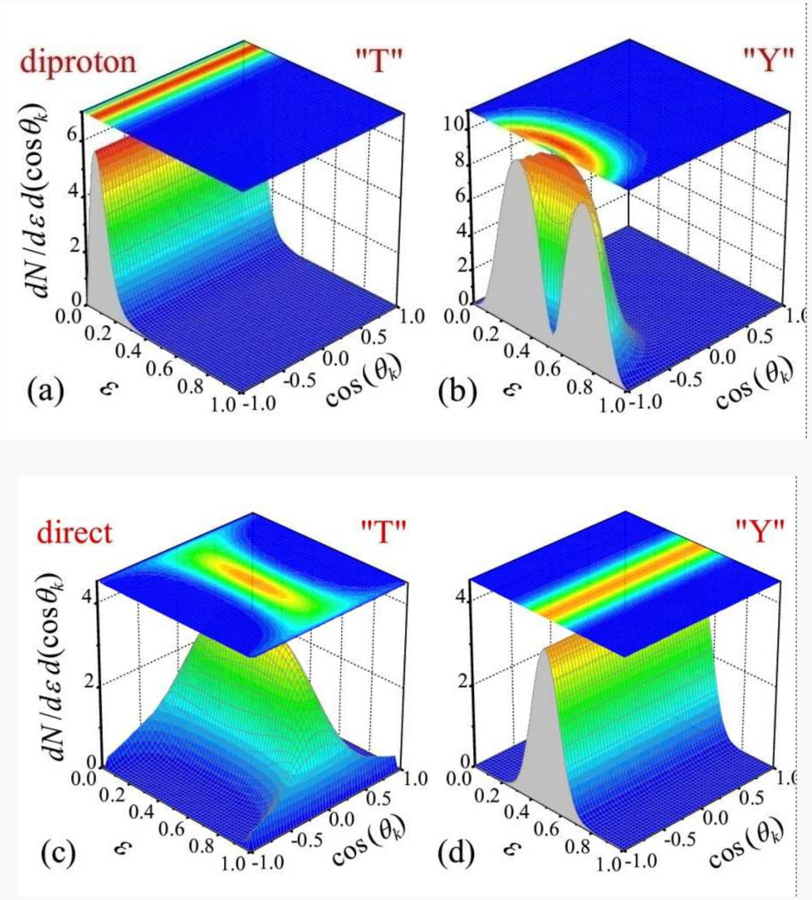


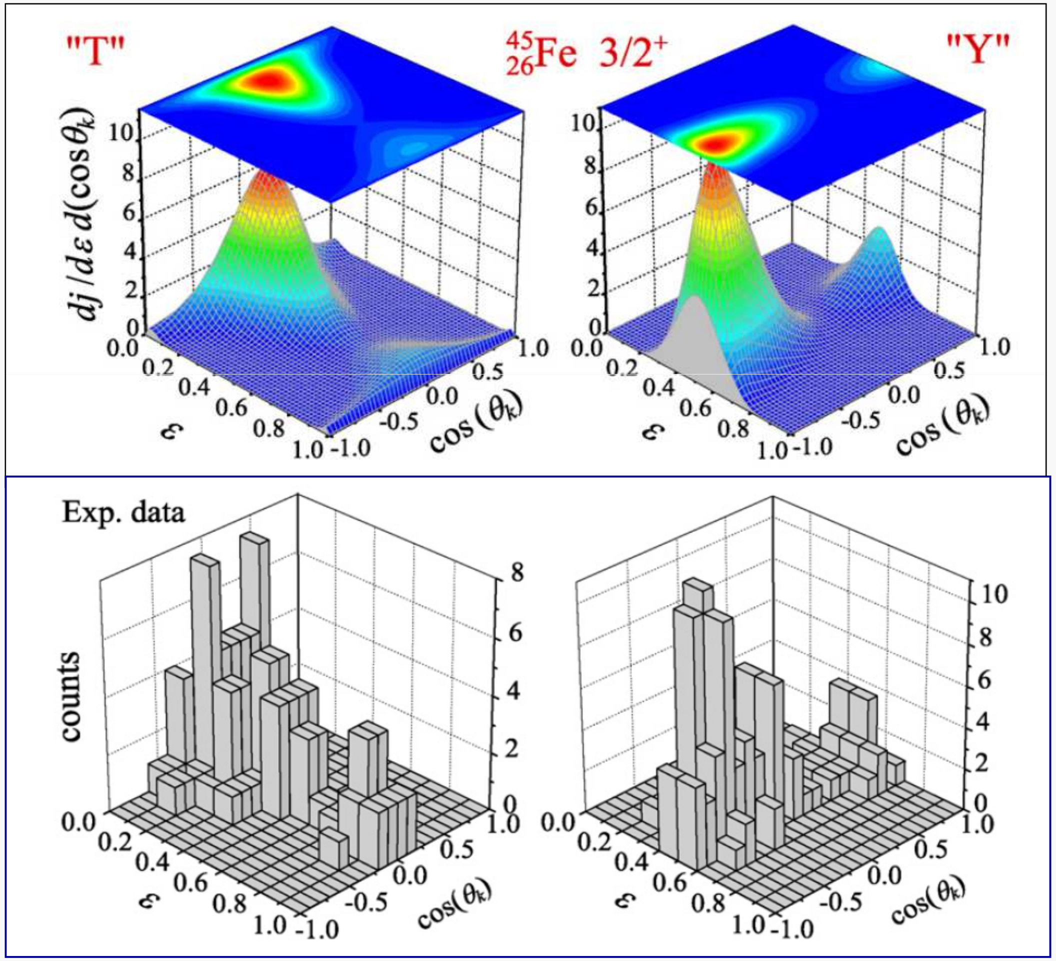
- 3-body model consistently reproduces all observables for certain composition of an initial wave function

K. Miernik *et al.*, EPJA 42 (09) 431

Full correlation picture for ^{45}Fe

Three-body model $W(p^2) = 24\%$





Grigorenko and Zhukov, Phys. Rev. C 68 (2003) 054005
Grigorenko *et al.*, PLB 677 (09) 30

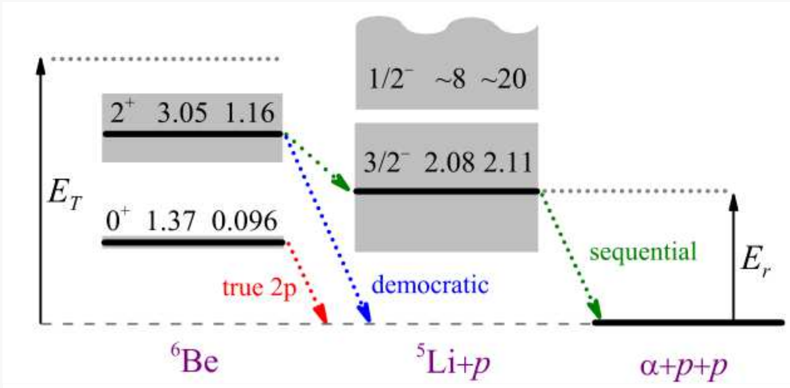
Decay of ^{6}Be

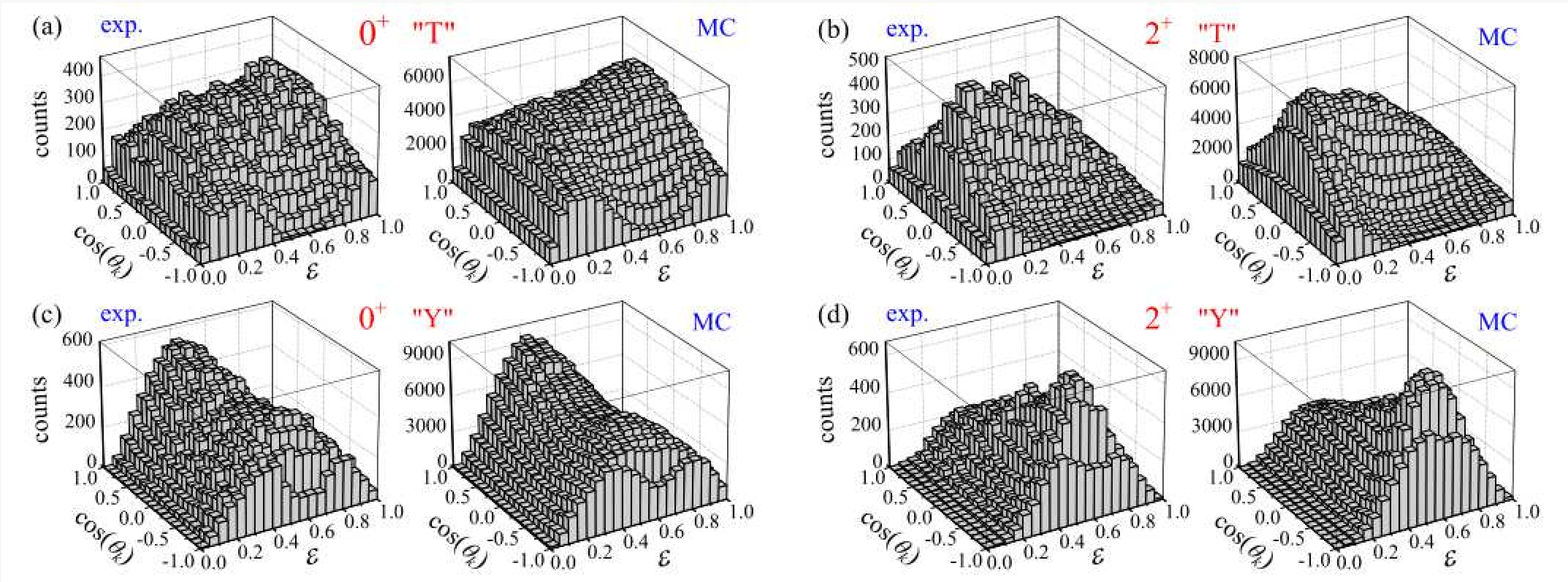
- The high resolution study the unbound ^{6}Be was recently done at NSCL/MSU

^{16}O @ 150 MeV/u + ^{9}Be → ^{7}Be

^{7}Be + ^{9}Be → ^{6}Be

Decay products detected by the High Resolution Array (HiRA) of ΔE-E (Si-CsI(Tl) telescopes





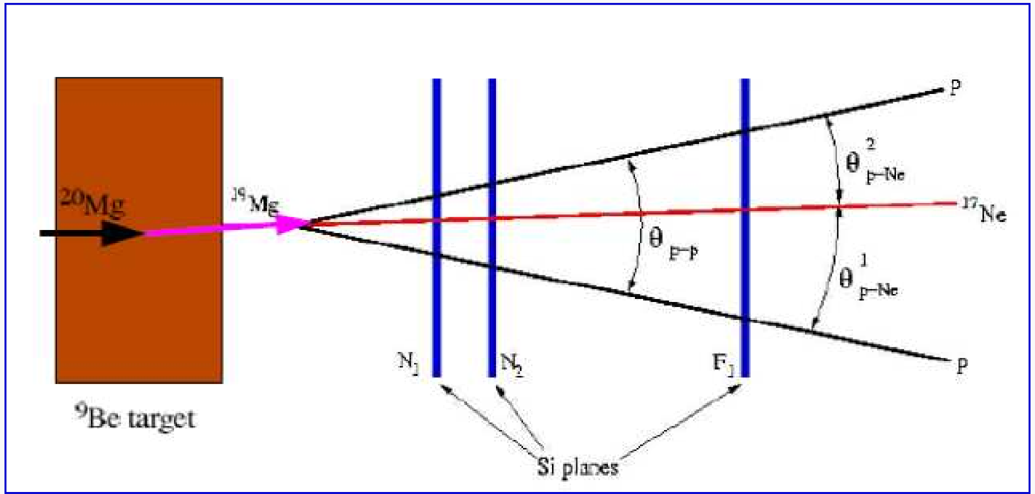
Egorova *et al.*, PRL 109 (12) 202502

A short-lived case of ^{19}Mg

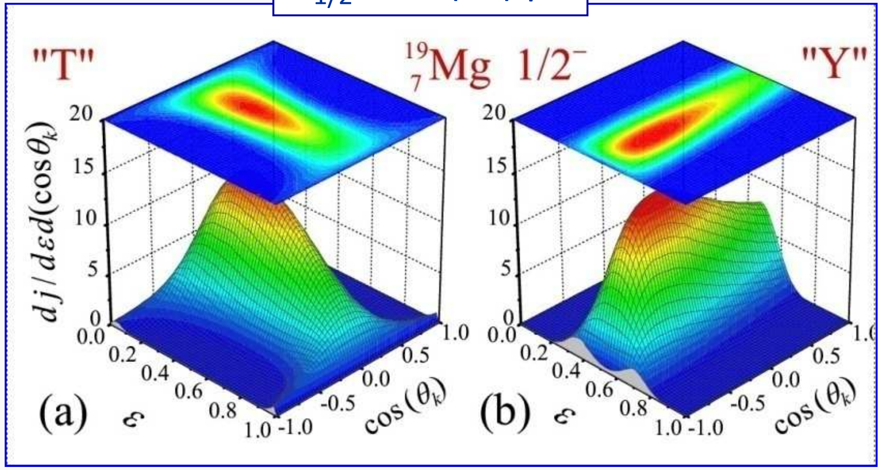
- The tracking technique for very short-lived *2p* decays was pioneered at GSI

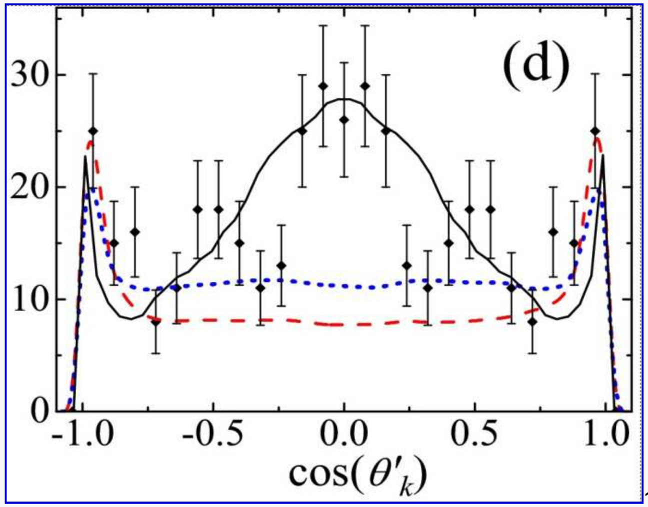
^{24}Mg @ 600 MeV/u + Be → ^{20}Mg

^{20}Mg + Be → ^{19}Mg



$T_{1/2}$ = 4.0(15) ps





I. Mukha et al., PRL. 99 (2007) 182501

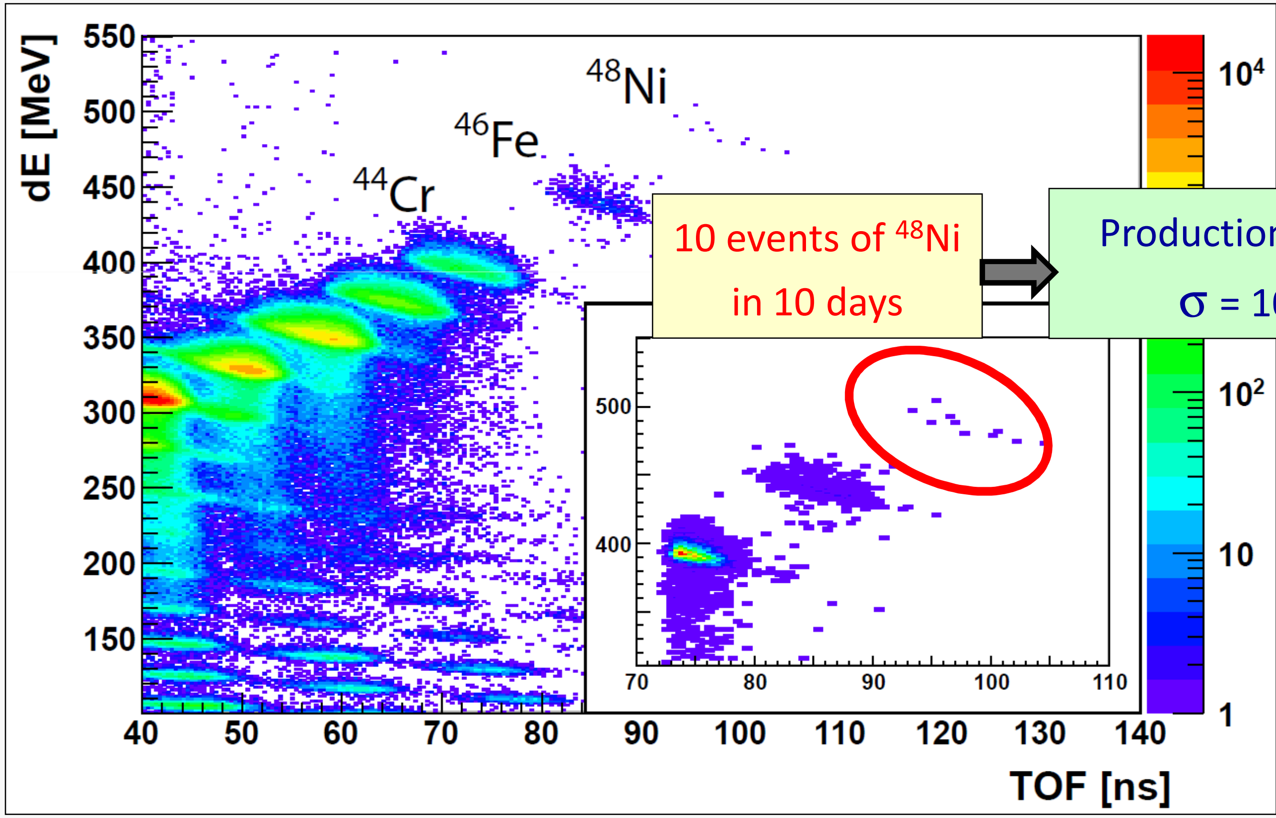
I. Mukha et al., PR C 77 (2008) 061303(R)

I. Mukha et al., EPJA 42 (2009) 421

Study of ^{48}Ni

Experiment in March 2011, A1900 fragment separator at NSCL/MSU

^{58}Ni @ 160 MeV/u + ^{nat}Ni → ^{48}Ni



10 events of ^{48}Ni in 10 days

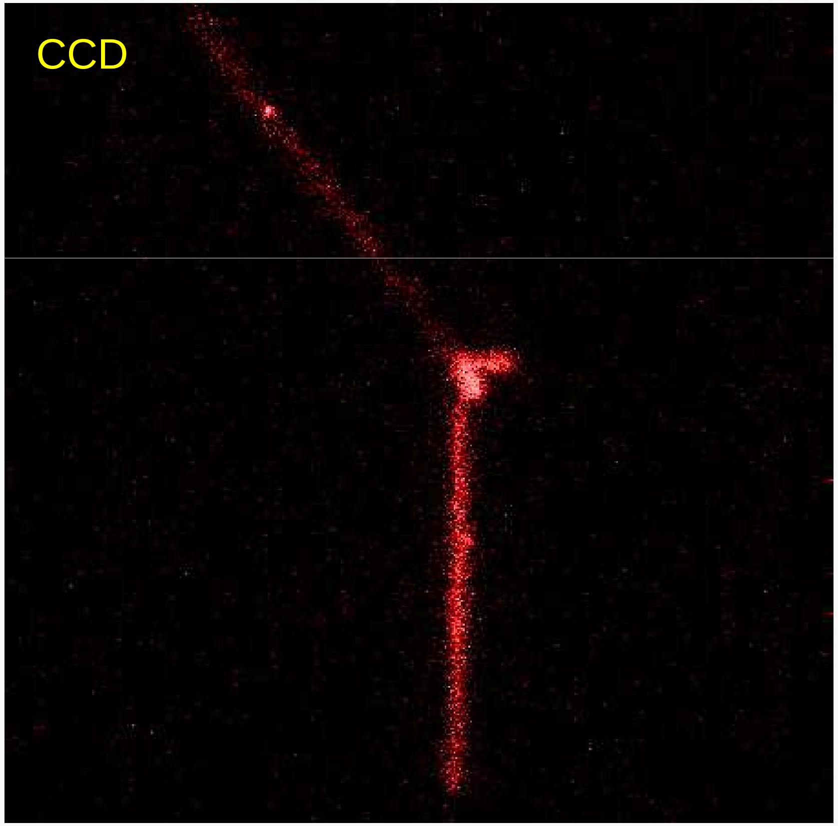
Production cross section: σ = 100(30) fb !

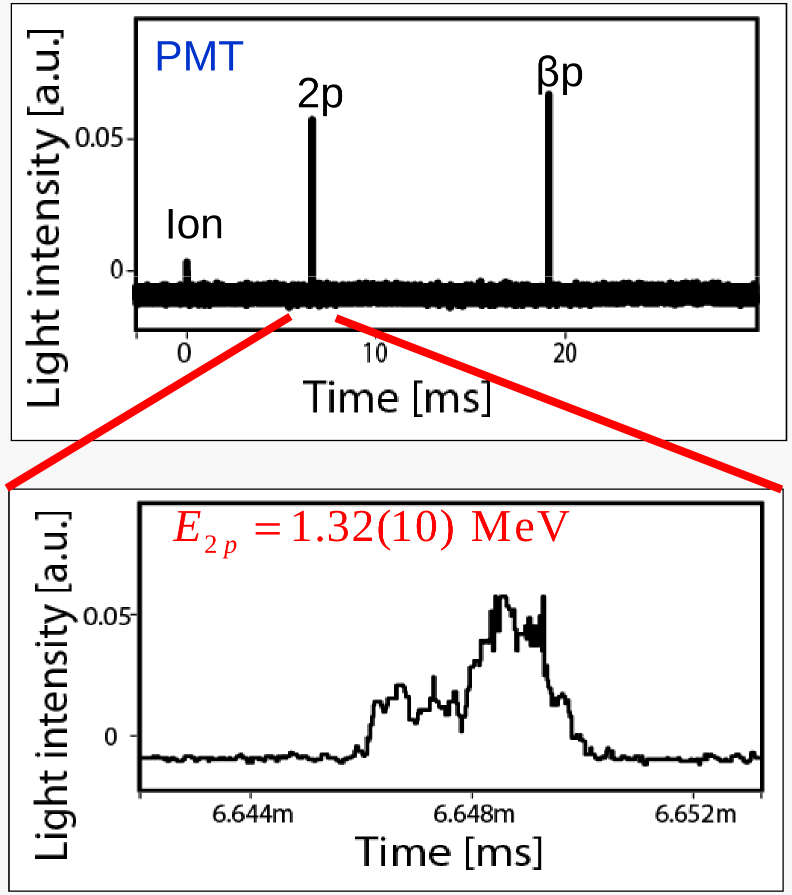
In superheavies 100 fb corresponds to 5 events in 1 year! (yesterday lecture of A. Popeko)

Pomorski et al., PRC 83 (11) 061303(R)

2p radioactivity of ^{48}Ni

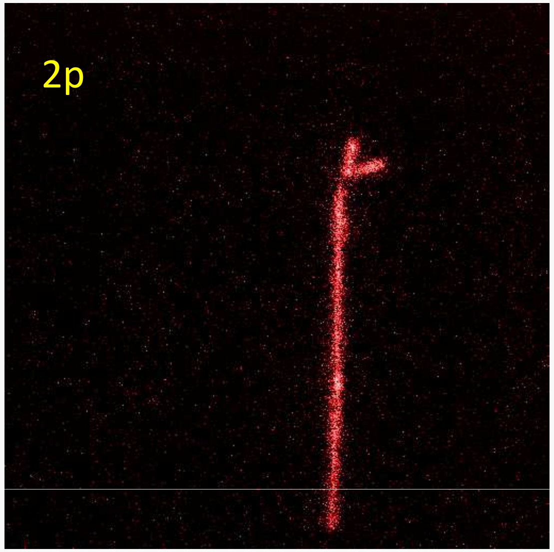
- The first direct observation of 2p radioactivity of ^{48}Ni

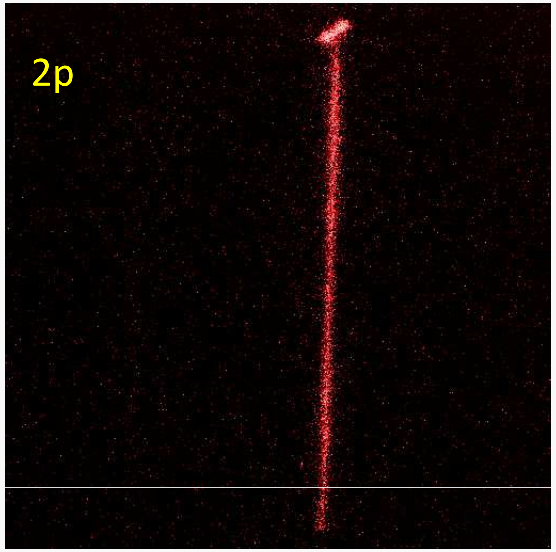


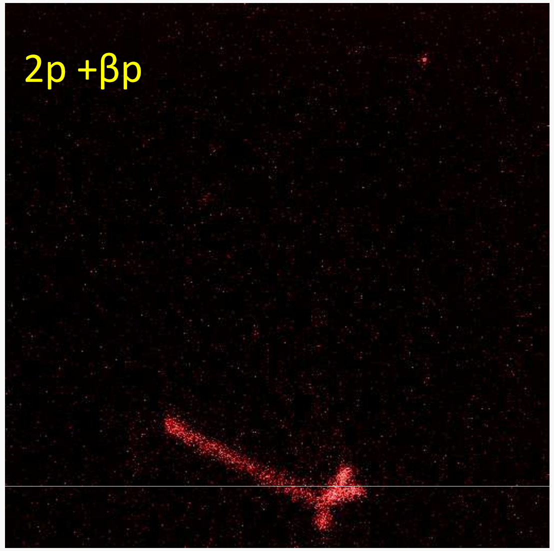


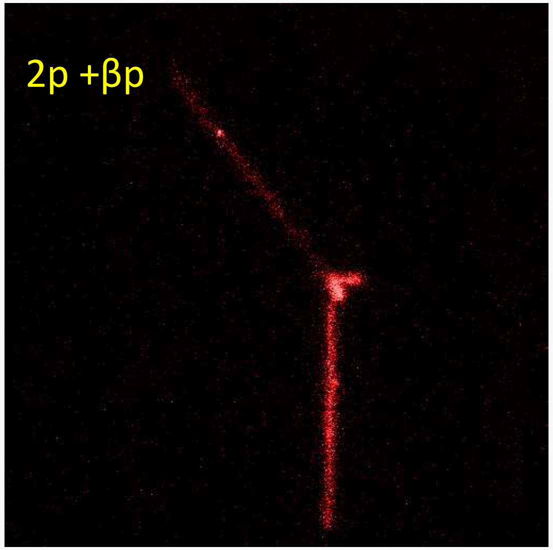
Pomorski et al., PRC 83 (2011) 061303(R)

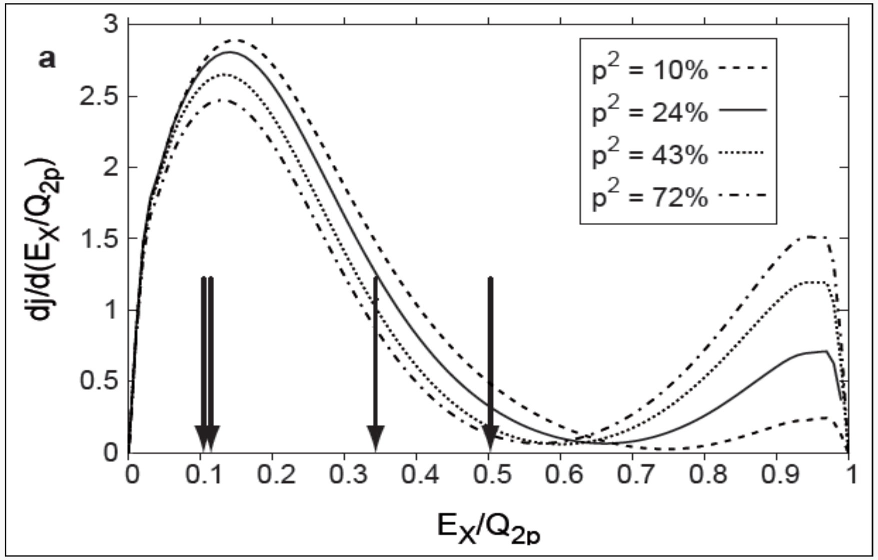
2p decay events of ^{48}Ni





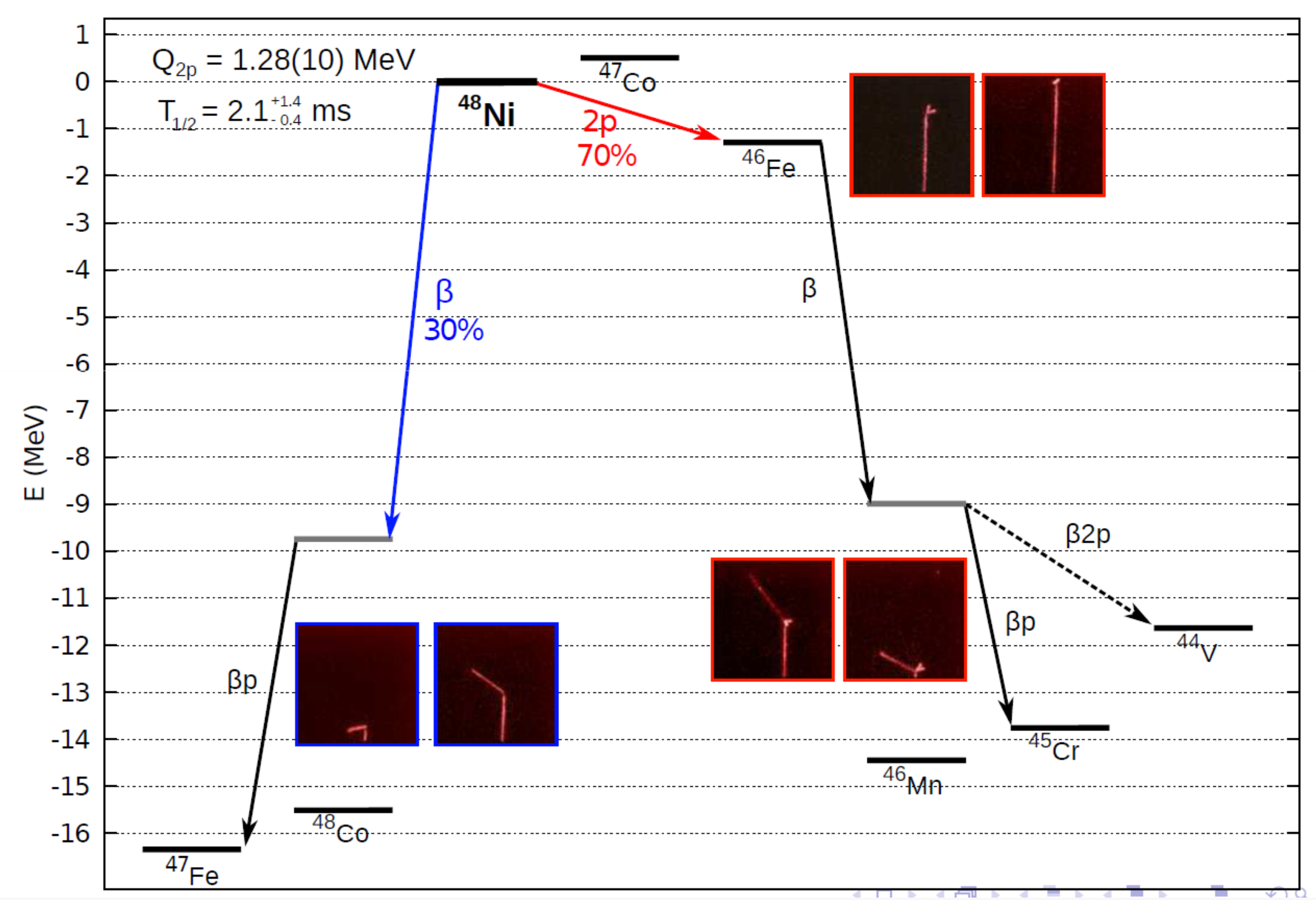






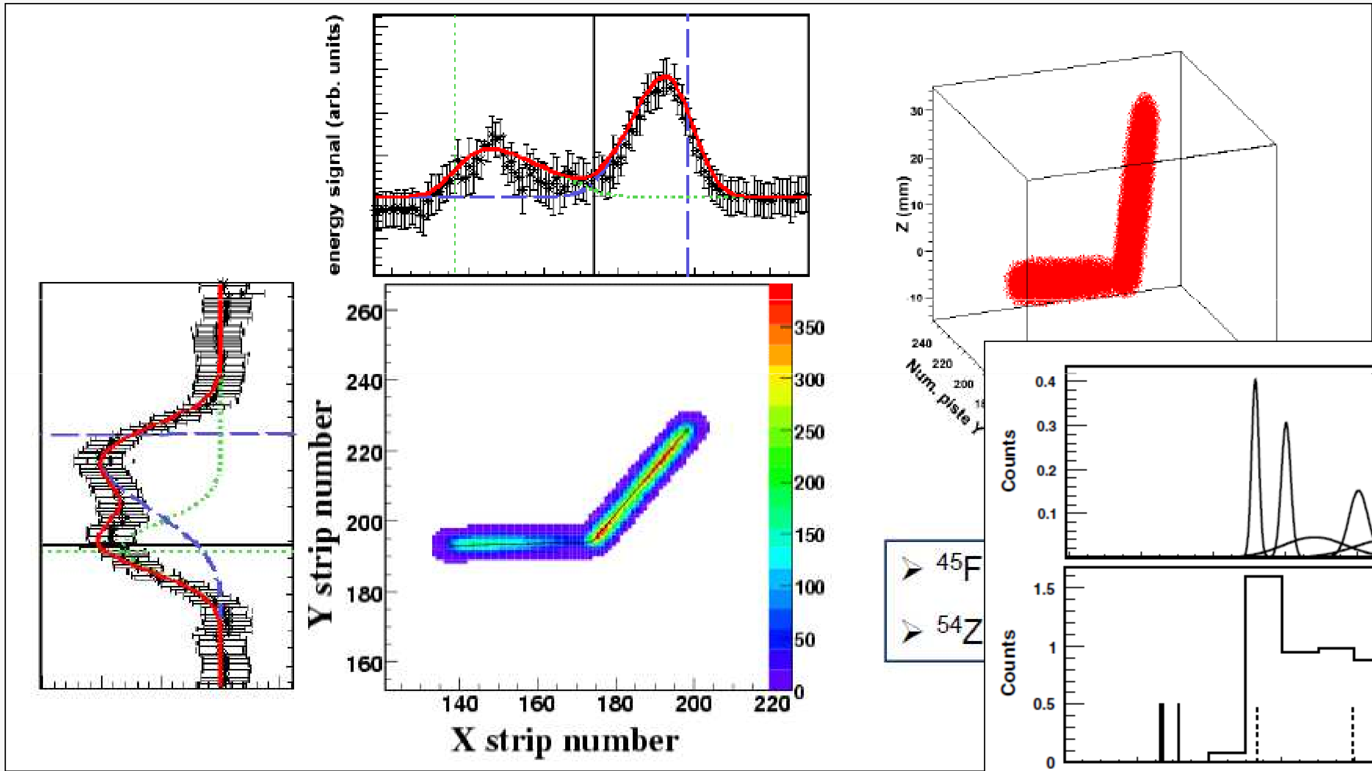
Pomorski et al., Acta Phys. Pol. B 43 (2012) 267

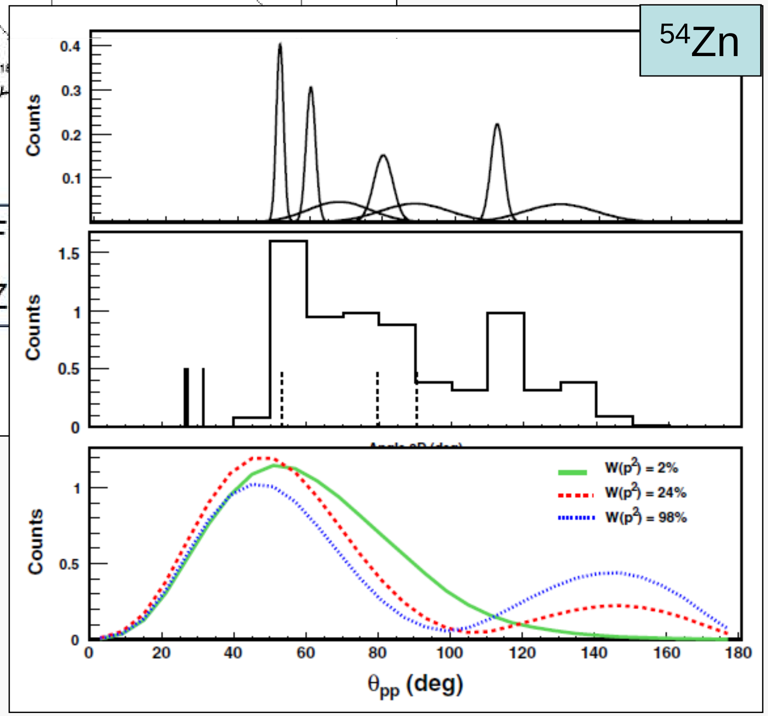
Decay scheme of ^{48}Ni



p-p correlations in ^{54}Zn

- ^{54}Zn studied at GANIL with the Bordeaux TPC. Seven events reconstructed in 3D

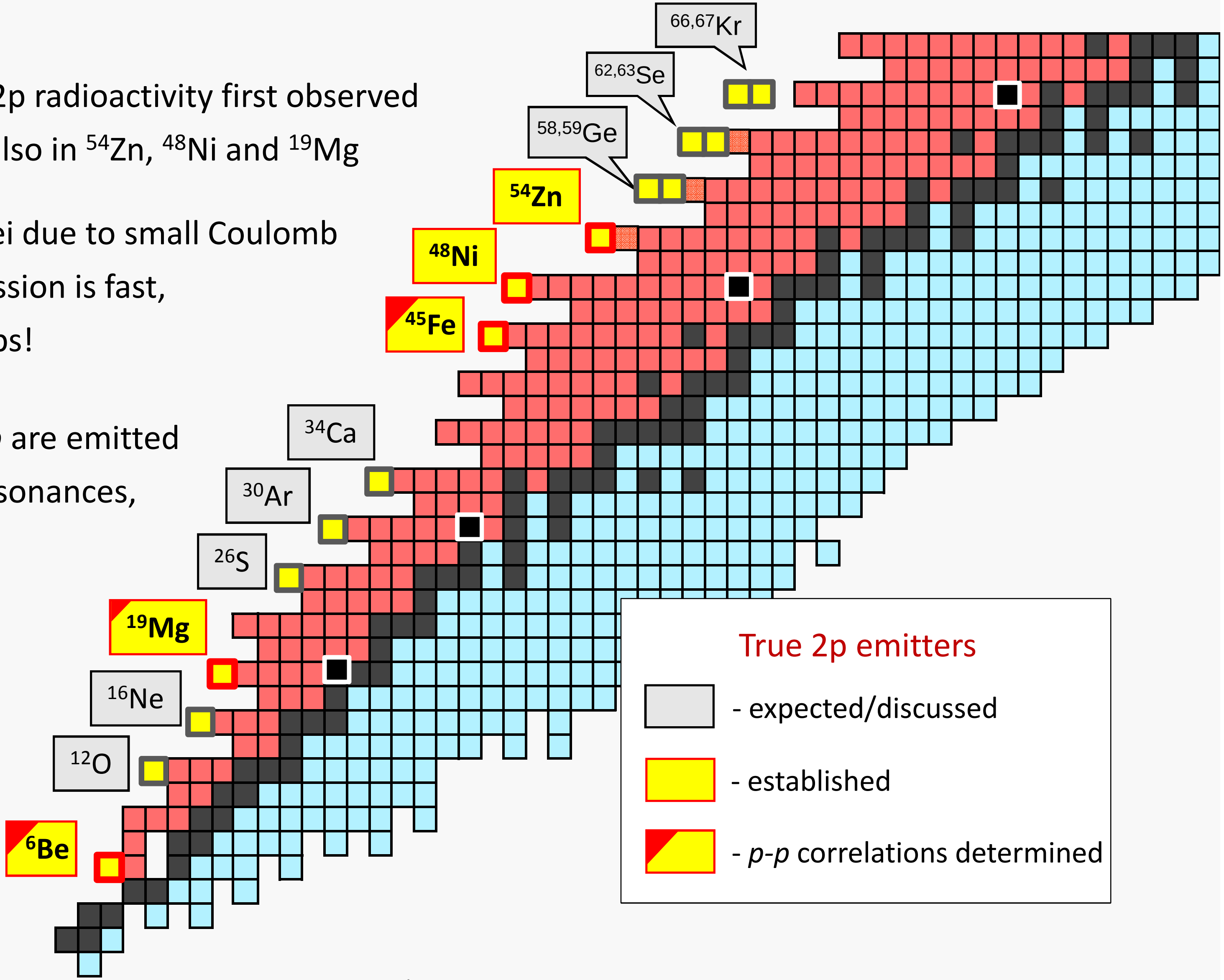




Ascher et al., PRL 107 (2011) 102502

True 2p emitters

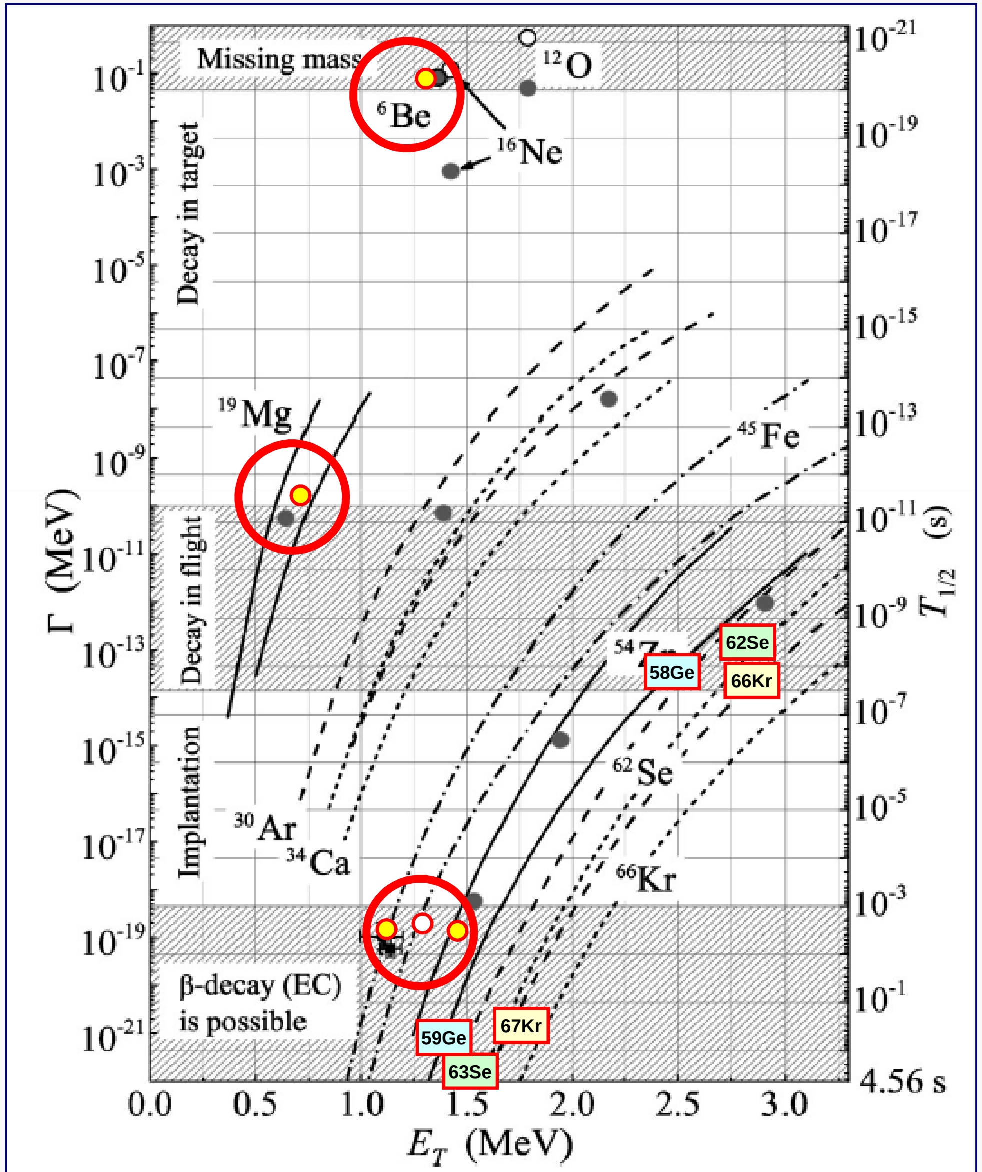
- Ground-state 2p radioactivity first observed in ^{45}Fe. Later also in ^{54}Zn, ^{48}Ni and ^{19}Mg
- In lighter nuclei due to small Coulomb barrier *2p* emission is fast, $T_{1/2}(^{19}Mg)$ = 4 ps!
- Below ^{19}Mg *2p* are emitted from broad resonances, like ^{6}Be



Range of lifetimes

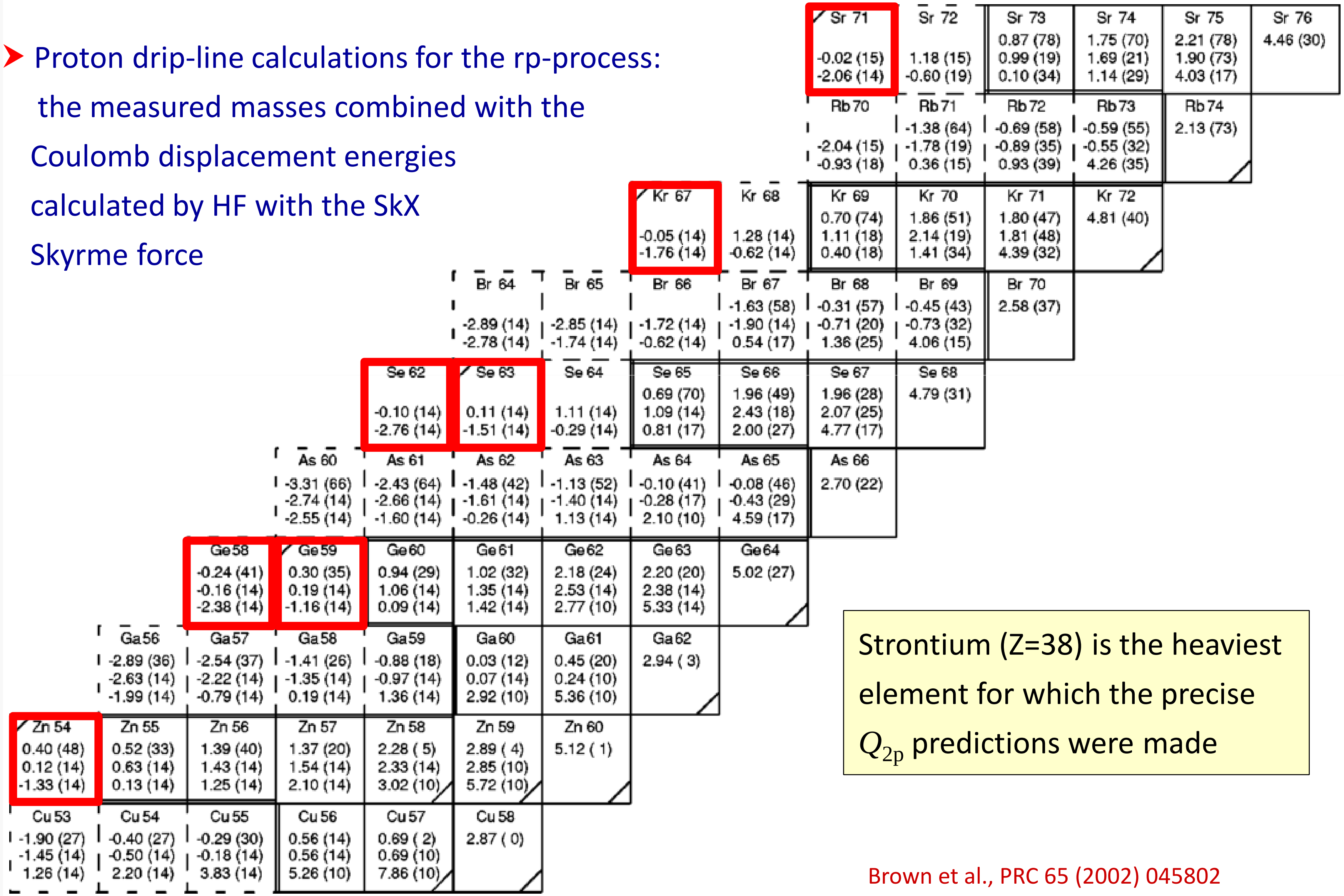
- The three-body model seems to work in the range of half-lives covering 18 orders of magnitude!

  - Invariant mass method for broad resonanses
    $T_{1/2} \leq 10^{-19}$ s
  - In-flight decays
    $T_{1/2}$ = 1 ps – 50 ns
  - Implantation method
    $T_{1/2}$ > 50 ns



Heavier 2p candidates

- Proton drip-line calculations for the rp-process: the measured masses combined with the Coulomb displacement energies calculated by HF with the SkX Skyrme force

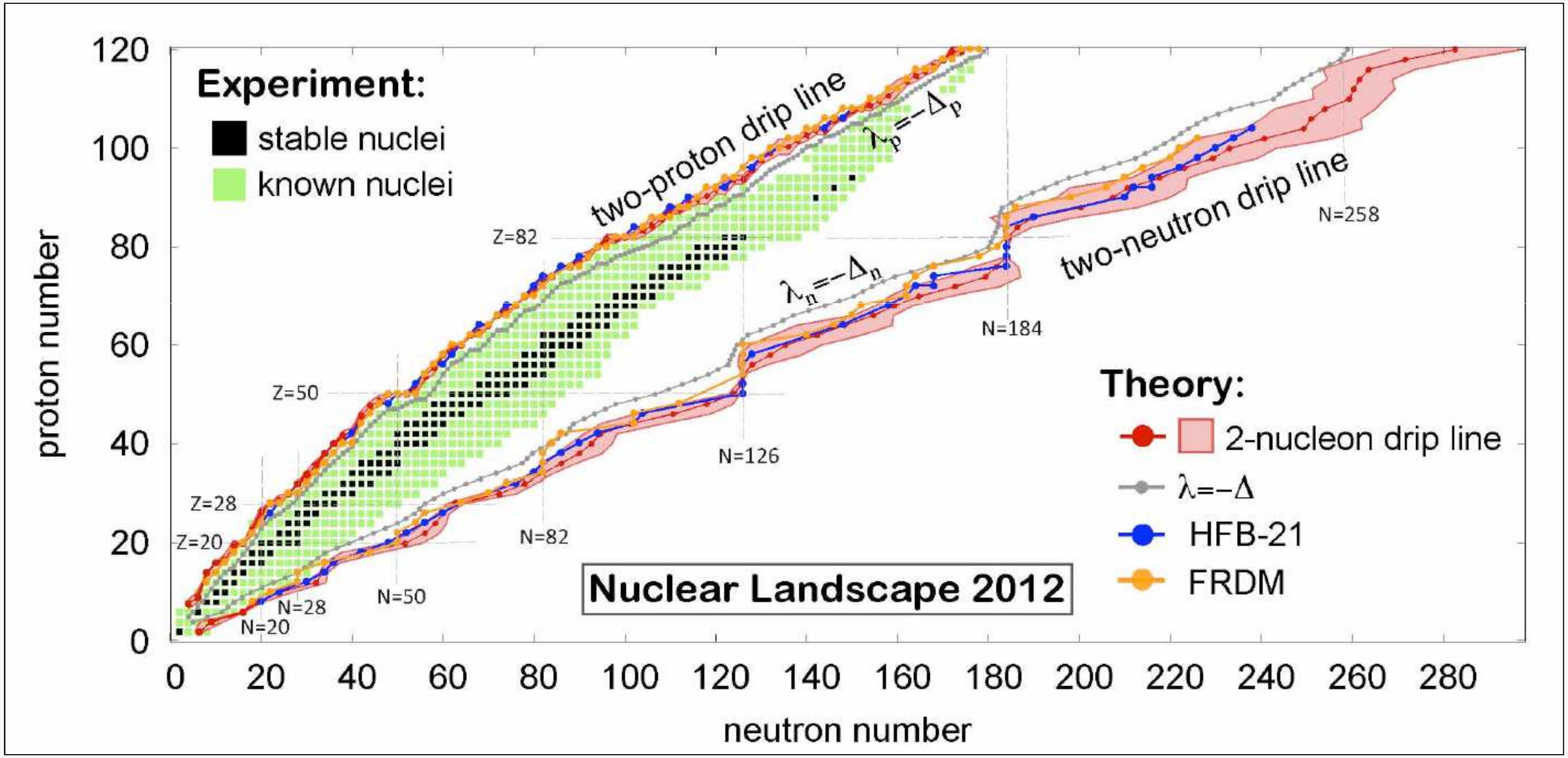


Strontium (Z=38) is the heaviest element for which the precise Q_{2p} predictions were made

Brown et al., PRC 65 (2002) 045802

Nuclear landscape

- Global mass predictions using density functional theory with **6 different Skyrme interactions**



→ There are 6900 ± 500 nuclei bound with Z ≤ 120

Erler et al., Nature 486 (2012) 509

2p-emission models

Direct model

$$\Gamma_{2p,dir} \cong \frac{8E_T}{\pi\left(E_T-2E_p\right)^2}\int_0^1 d\varepsilon\,\Gamma_x\left(\varepsilon E_T\right)\Gamma_y\left(\left(1-\varepsilon\right)E_T\right)$$

$$l_p = 0$$
$$\theta_x^2 = \theta_y^2 = 0.173$$

Diproton model

$$\Gamma_{2p,dipr} = \theta_{dipr}^2 \mathcal{N} \frac{\hbar^2}{4\mu} \exp\left[-2\int_{r_2}^{r_3} k(r)\,dr\right]$$

$$\mathcal{O}^2 = 0.015$$

- The spectroscopic factors determined from the experimental half-lives of known 2p emitters: ^{19}Mg, ^{45}Fe, ^{48}Ni, and ^{54}Zn

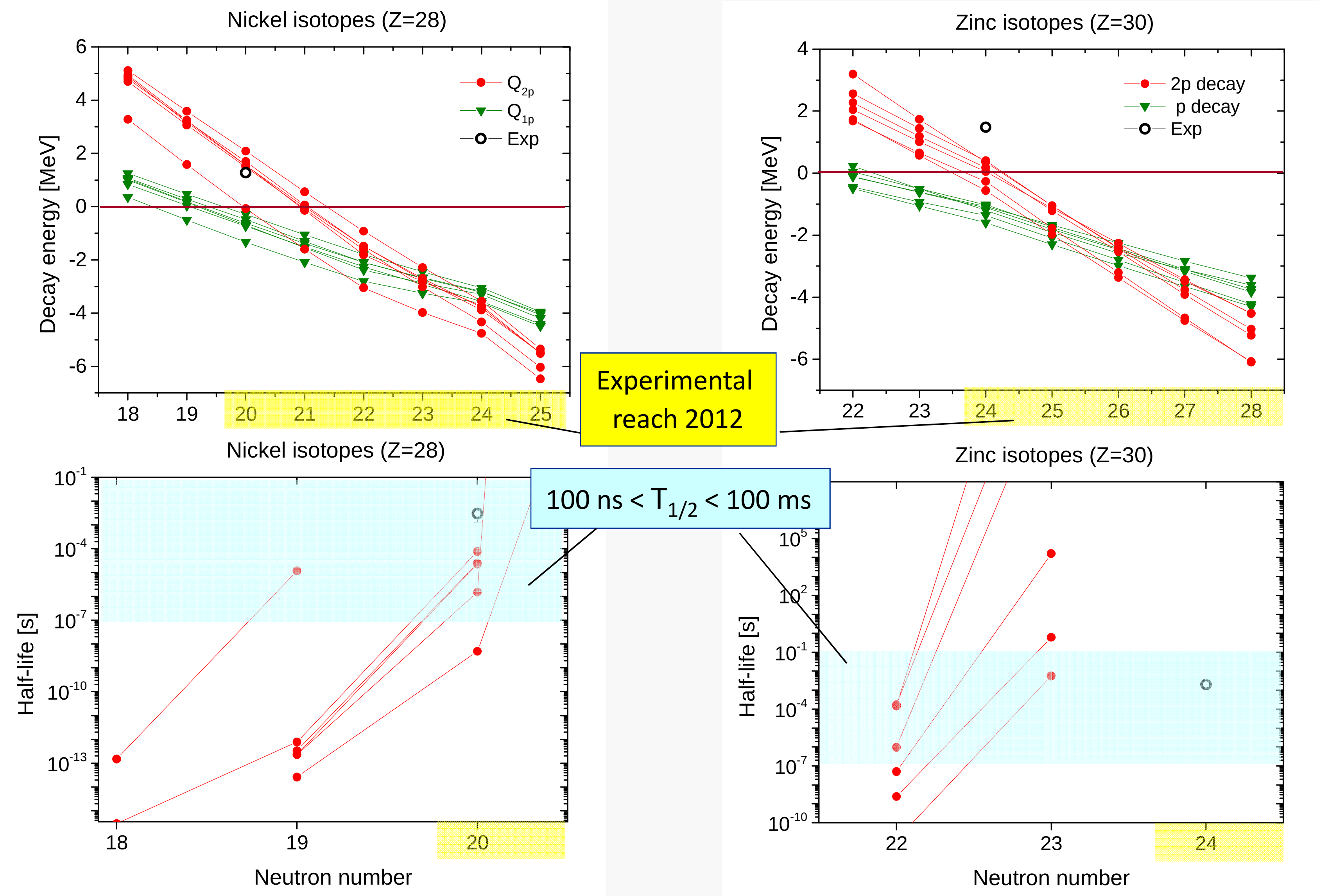
The comparison of predicted half-lives with experiment

Nucleus	Experiment	Direct	Diproton
^{19}Mg [7]	4.0(15) ps	6.2 ps	12.3 ps
^{45}Fe [10]	3.7(4) ms	1.1 ms	8.7 ms
^{48}Ni [8]	$3.0^{+2.2}_{-1.2}$ ms	6.8 ms	5.3 ms
^{54}Zn [9]	$1.98^{+0.73}_{-0.41}$ ms	1.0 ms	0.8 ms

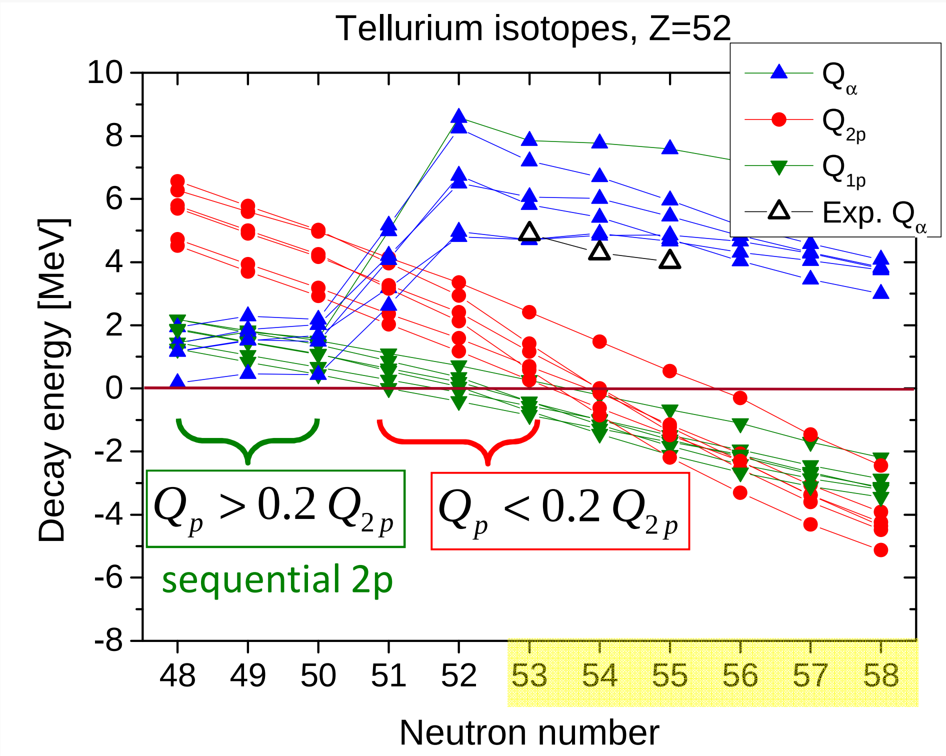
$$T_{1/2} = \frac{\ln 2\,\hbar}{\Gamma}$$

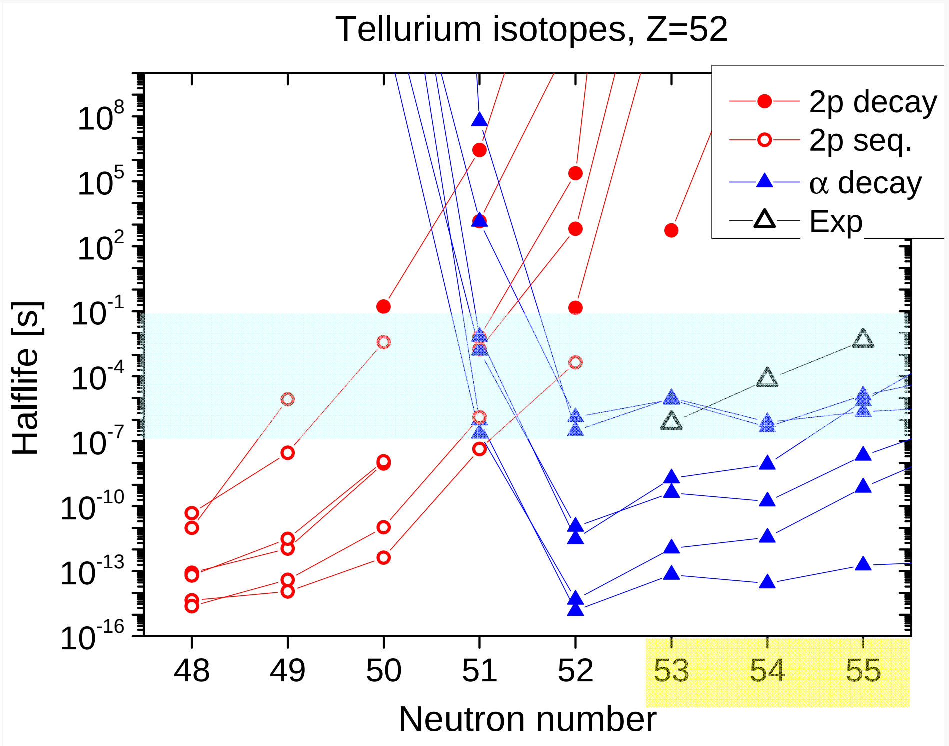
Olsen et al., PRL 110 (2013) 222501

Predictions for nickel and zinc



Tellurium

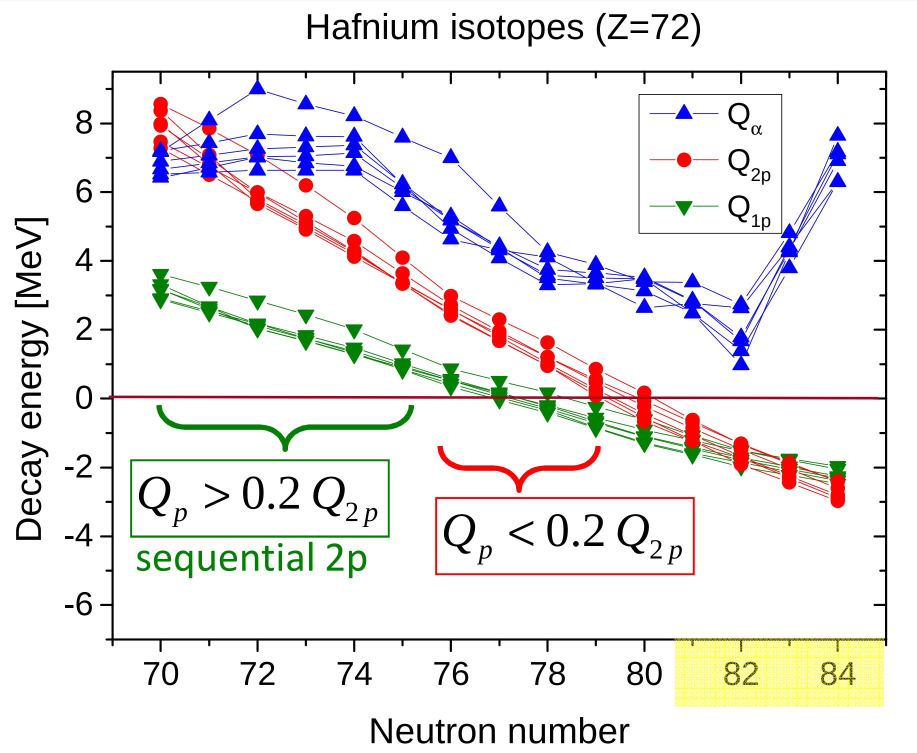


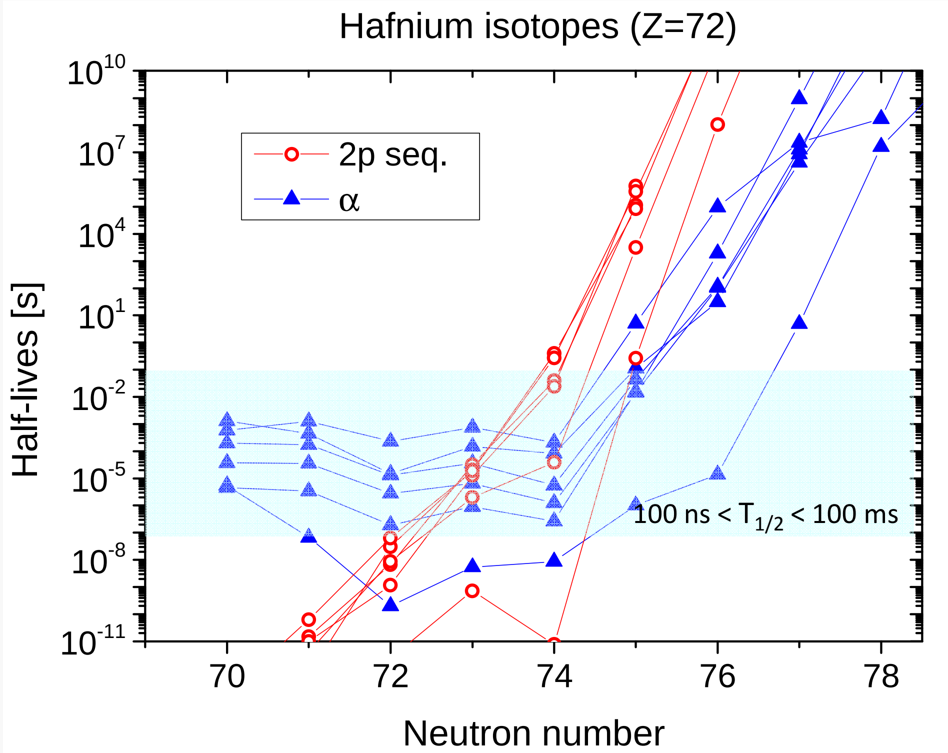


α decay half-lives from the global, fenomenological formula by Koura, J. Nucl. Science and Tech. 49 (2012) 816

- At ^{103}Te a transition from the simultaneous 2p to the sequential emission occurs
- In addition, in ^{103}Te both decays, α and 2p may be observable!

No 2p above tellurium!?



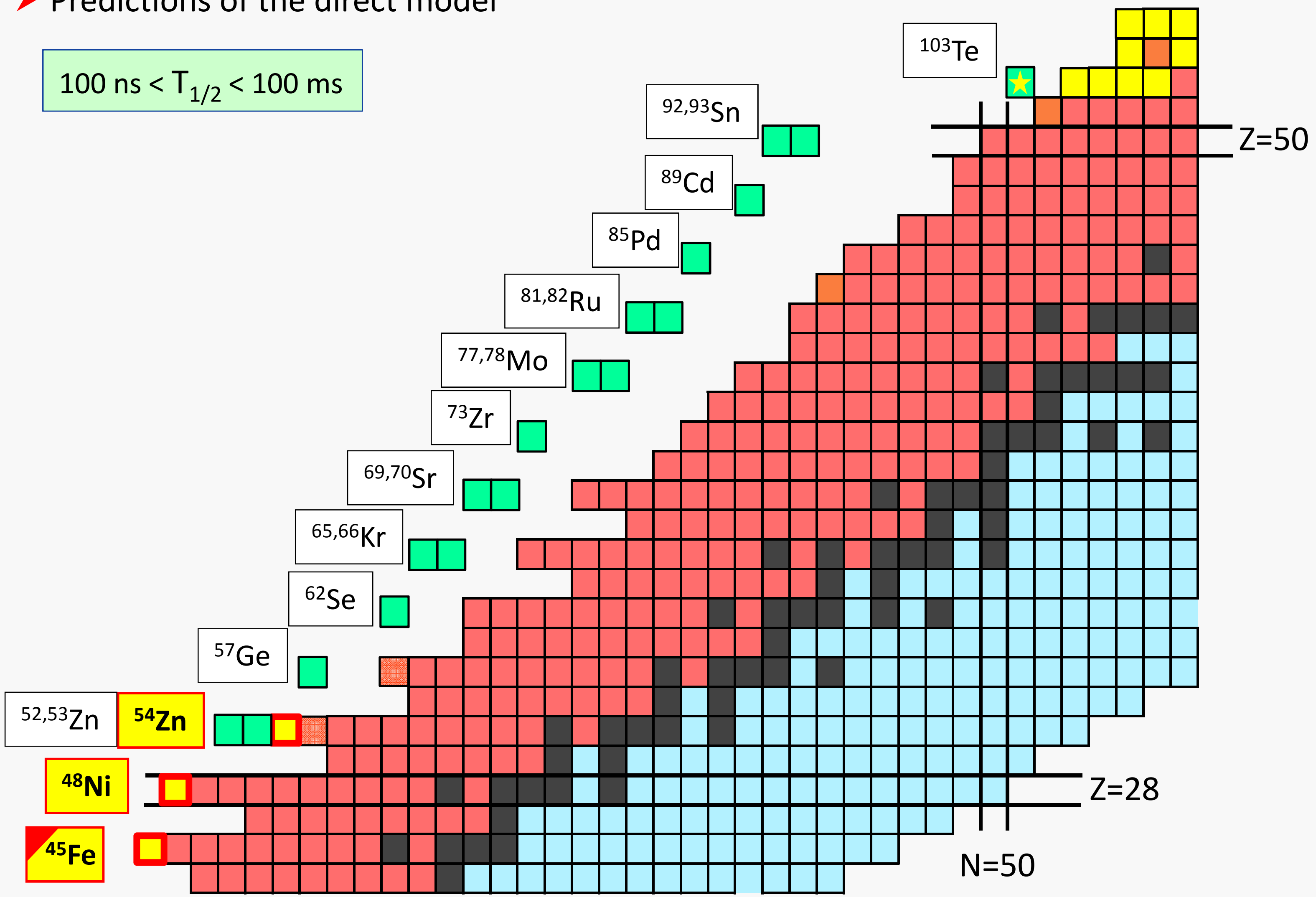


- When the energy condition for the true 2p decay is fulfilled, the predicted half-life is extremely long
- When the fast proton emission becomes possible, it proceeds as the sequential 2p decay

Heavy 2p landscape

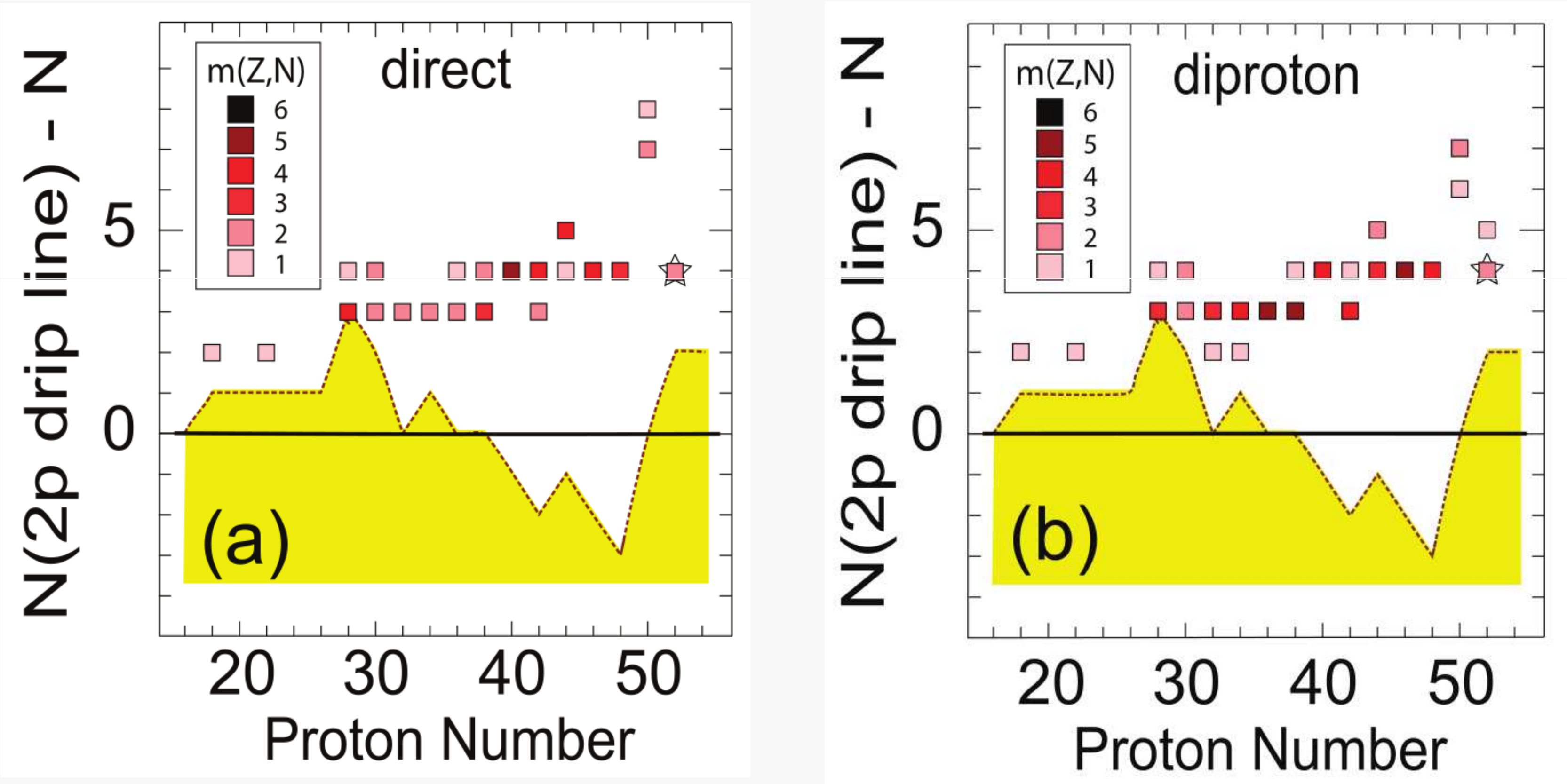
- Predictions of the direct model

100 ns < $T_{1/2}$ < 100 ms



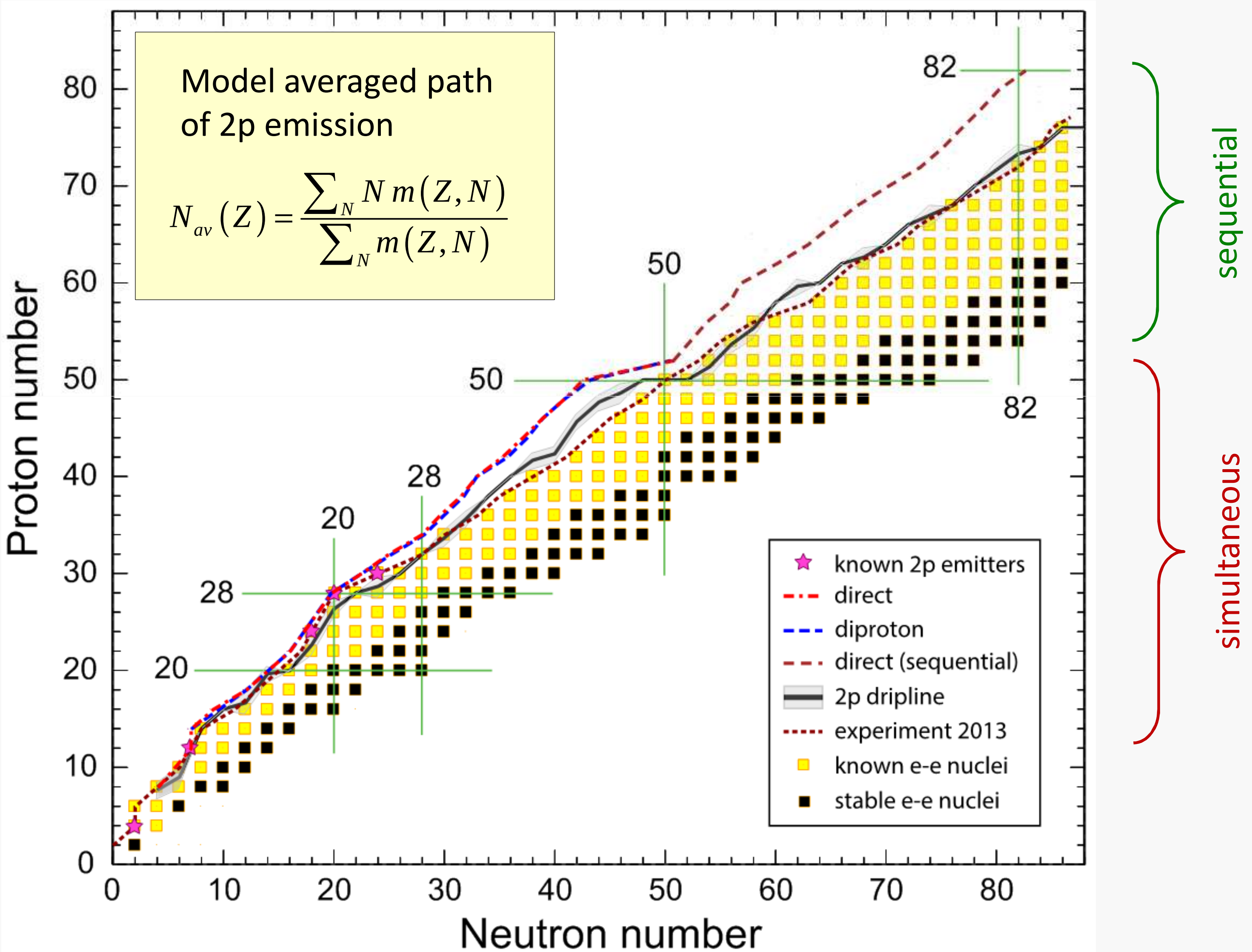
Heavy 2p landscape

- Selection criteria: $100 \text{ ns} < T_{2p} < 100 \text{ ms}$ $T_{2p} < 10 \cdot T_{\alpha}$



Olsen et al., PRL 110 (2013) 222501

Full 2p landscape



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2p landscape

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Thank you!