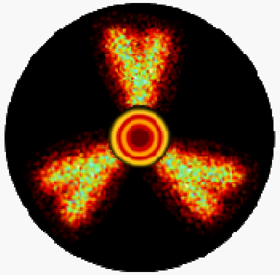
Particle radioactivity

Lecture 2



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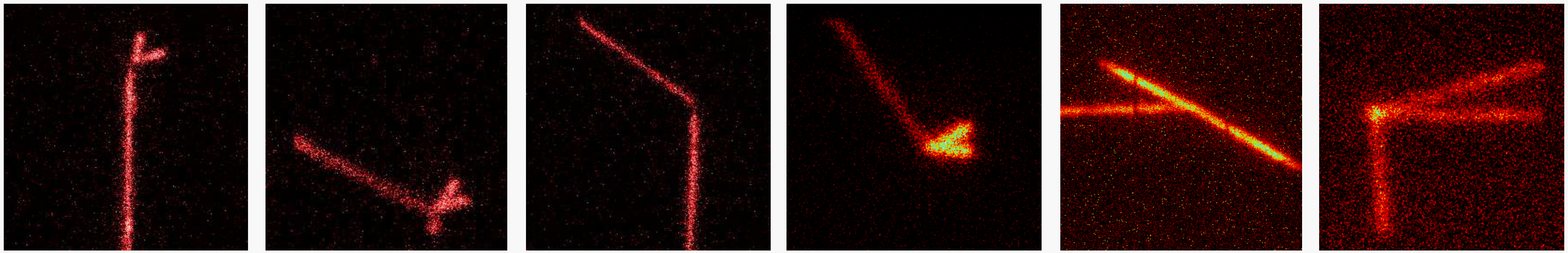
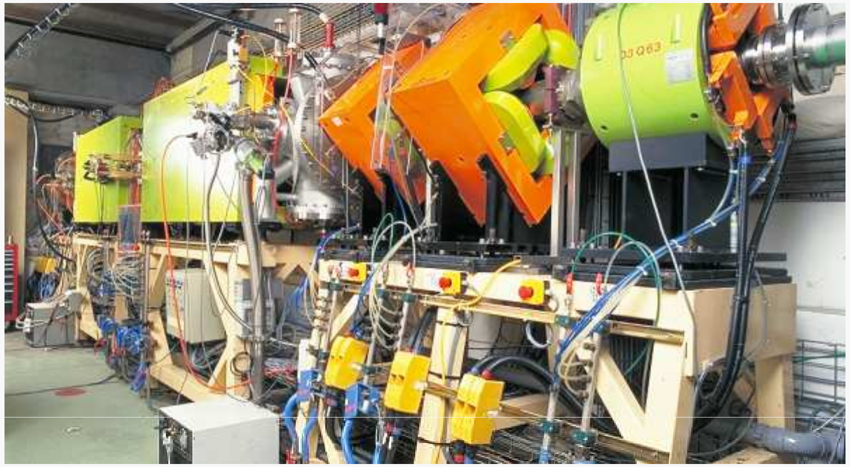
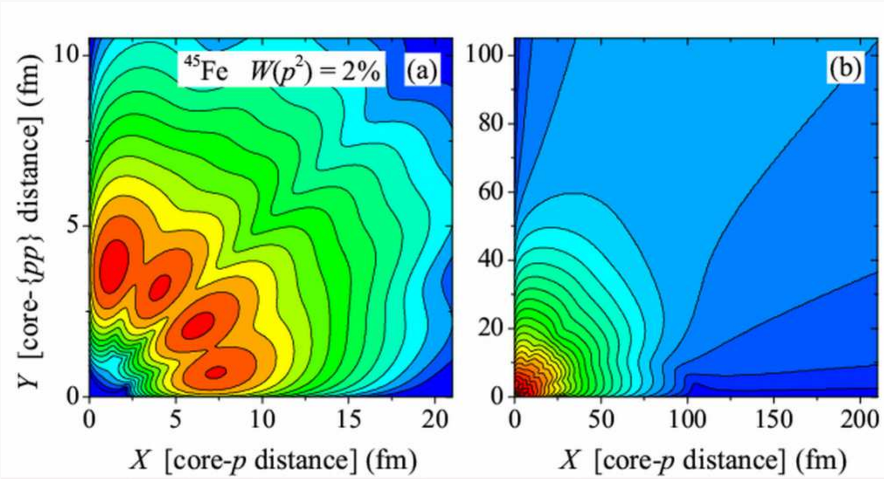


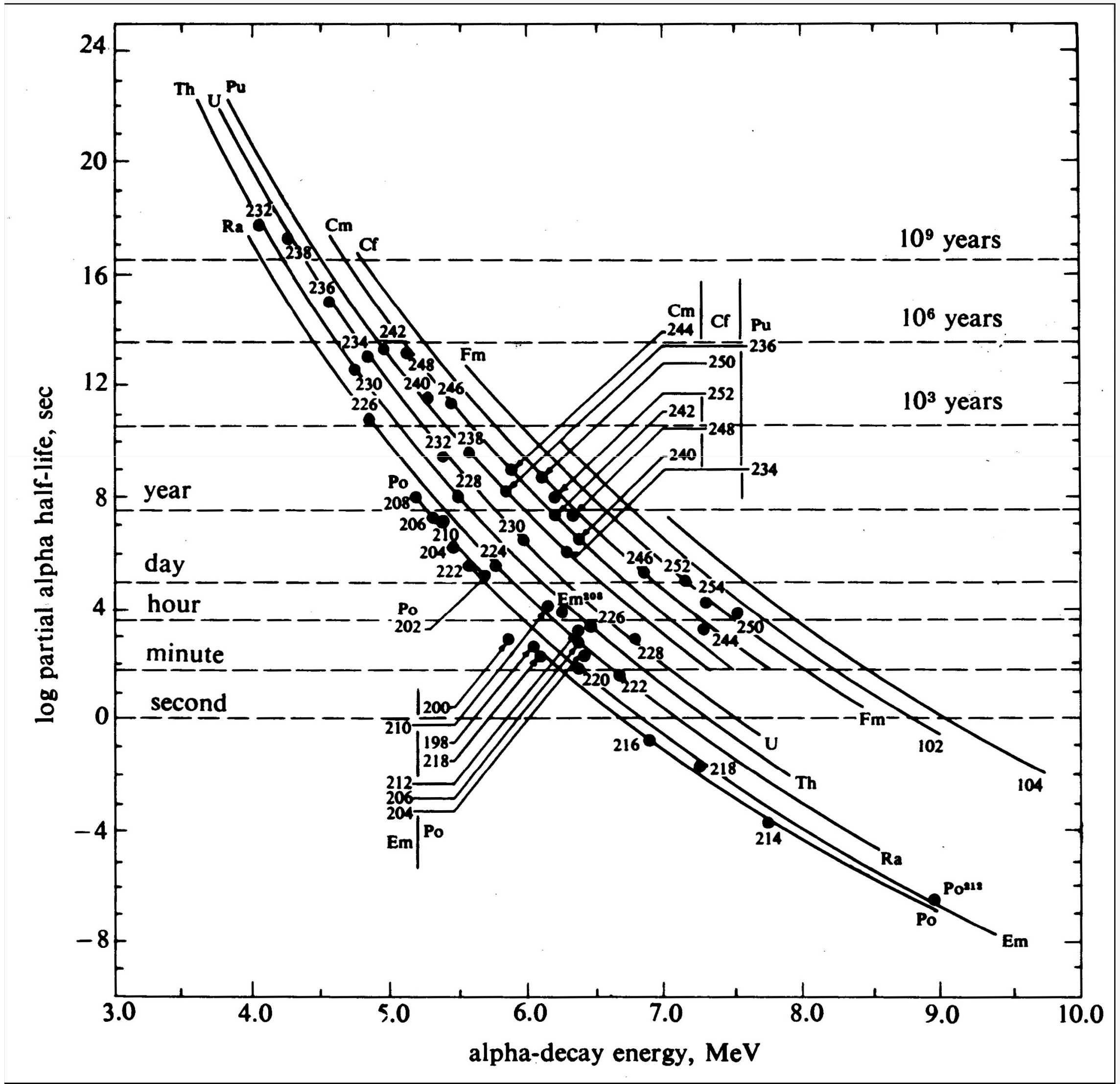
Outline





- Basic introduction
- Experimental techniques
  - reactions
  - separators
  - detection
- Theoretical models
  - Gamow idea
  - p, and 2p emission
  - 3-body model

Geiger-Nutall law



^{238}U : E_a = 4.2 MeV, $T_{1/2} = 4.5 \cdot 10^9$ y

^{222}Rn : E_a = 5.5 MeV, $T_{1/2}$ = 3.8 d

^{214}Po : E_a = 7.7 MeV, $T_{1/2}$ = 164 μs

Geiger-Nutall (1912)

$$\log T = a + \frac{b}{\sqrt{E_\alpha}}$$

Gamow picture

- Early puzzle of α decay: half-life is extremely dependent of on energy

  Solution by Gamow (1928) was a triumph of quantum mechanics!

→ Radioactive process is so slow (decaying state so narrow) that it can be approximated by a **stationary** picture:

$$\hat{H}\,\Phi(\vec{r},t) = i\hbar\frac{\partial\Phi(\vec{r},t)}{\partial t}\ ,\ \Phi(\vec{r},t) = \Psi(\vec{r})\,\mathrm{e}^{-iEt/\hbar}$$

but with the complex energy!

$$\hat{H}\,\Psi(\vec{r}) = E\,\Psi(\vec{r}) \qquad E = E_0 - i\,\Gamma/2$$

$$|\Phi(\vec{r},t)|^2 = |\Psi(\vec{r})|^2\,\mathrm{e}^{-i\Gamma t/\hbar}$$

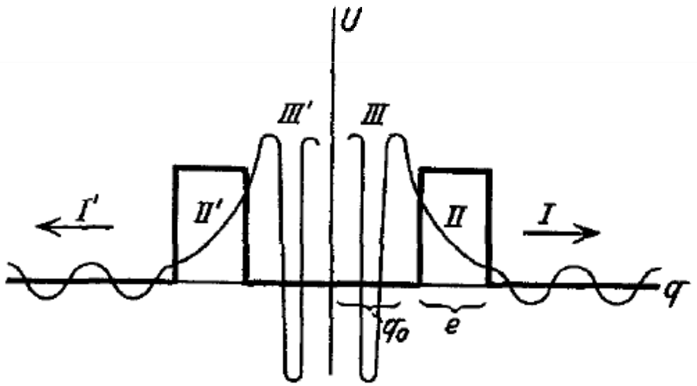


Fig. 3.

Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und E komplex setzen:

$$E = E_0 + i\,\frac{h\,\lambda}{4\,\pi},$$

Gamow, Z. Phys. 51 (1928) 204

Beyond the proton drip-line

Competition between two decay modes

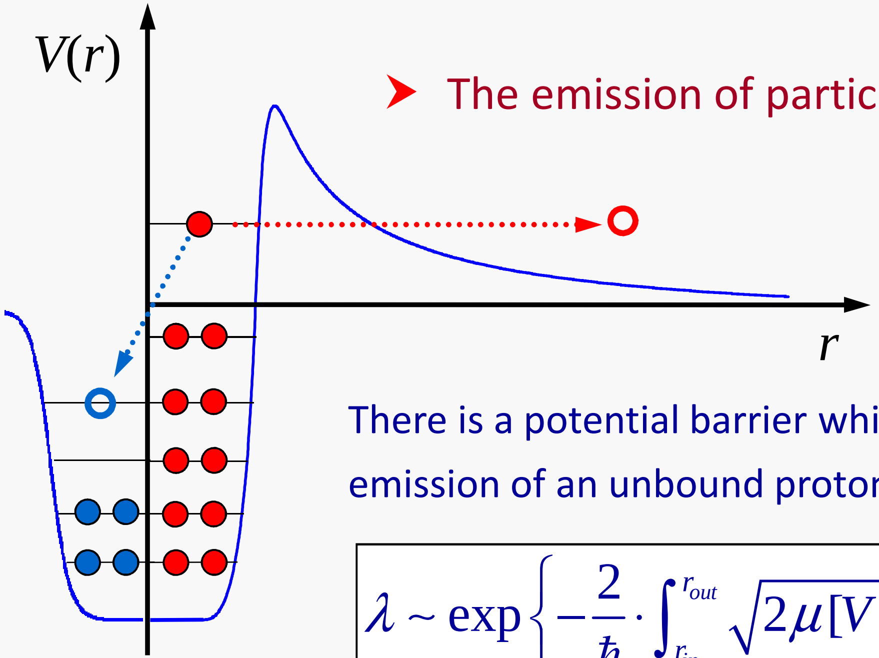
- The β^+ decay

Probability of transition:

$$\lambda \sim Q^5$$

Decay energy may be large,
but the weak interaction
is really weak

$$\rightarrow \; T_{1/2} > 1 \text{ ms}$$



- The emission of particles

There is a potential barrier which hampers emission of an unbound proton (α, 2p, ^{14}C,..)

$$\lambda \sim \exp\left\{ -\frac{2}{\hbar} \cdot \int_{r_{in}}^{r_{out}} \sqrt{2\mu [V(r) - Q_p]} \cdot dr \right\}$$

The realistic potential

Radial part of the potential (for protons!)

$$V_{\text{tot}} = V_N + V_C + V_L + V_{LS}$$

$$V_N(r) = -V_0\left(1+\kappa\frac{N-Z}{A}\right) f(r,R,a)$$

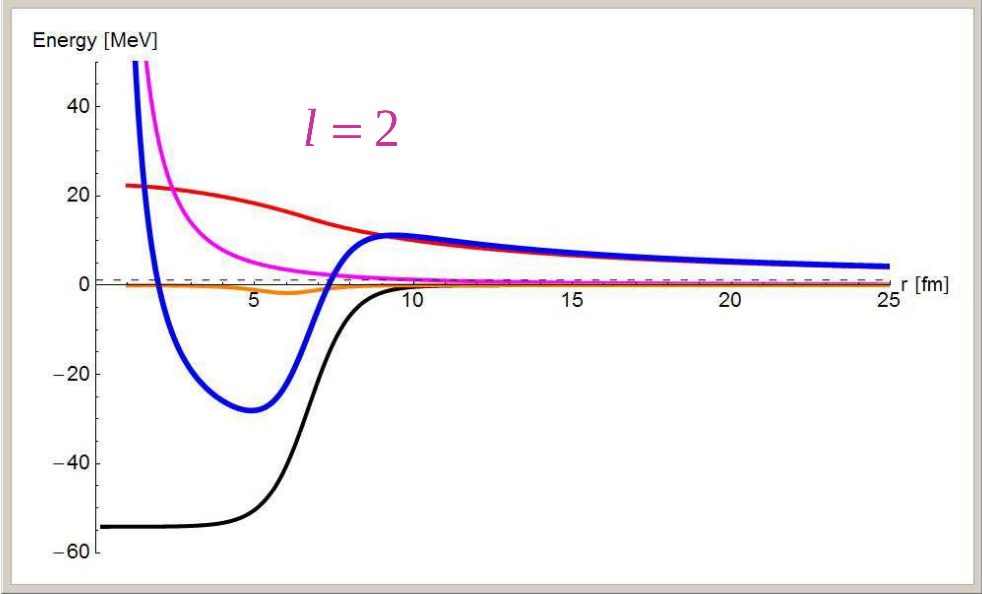
$$f = \frac{1}{1+\exp\left(\frac{r-R}{a}\right)}$$

$$V_C(r) = \begin{cases} (Z-1)e^2/r & r \geq R_C \\ (Z-1)e^2\left(3R_C^2 - r^2\right)/\left(2R_C^3\right) & r < R_C \end{cases}$$

$$V_L(r) = \frac{\hbar^2}{2\mu r^2} l(l+1) \qquad \mu = \frac{m_p M_{A-1}}{m_p + M_{A-1}}$$

$$V_{LS}(r) = \frac{\lambda V_0}{2\mu^2 r}\frac{df}{dr}\cdot\begin{cases} l & j = l+1/2 \\ -(l+1) & j = l-1/2 \end{cases}$$

N. Schwierz et al., http://arxiv.org/abs/0709.3525v1



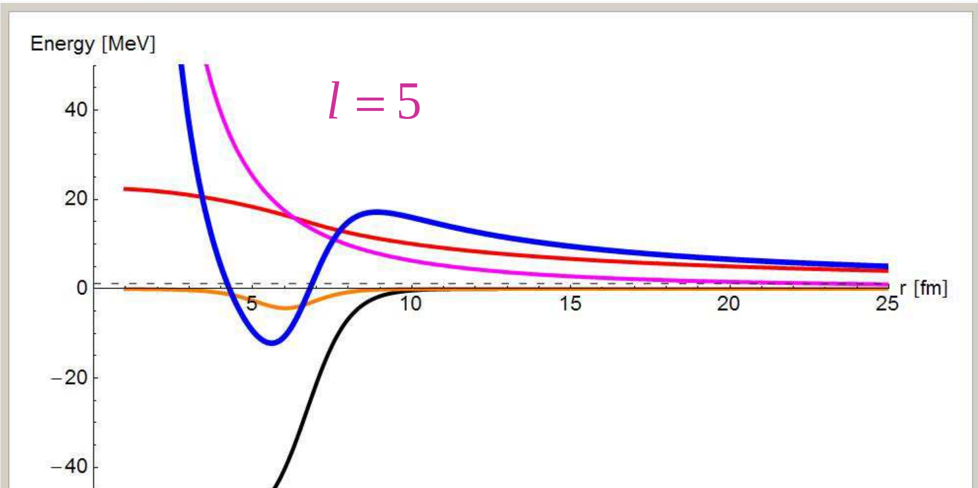
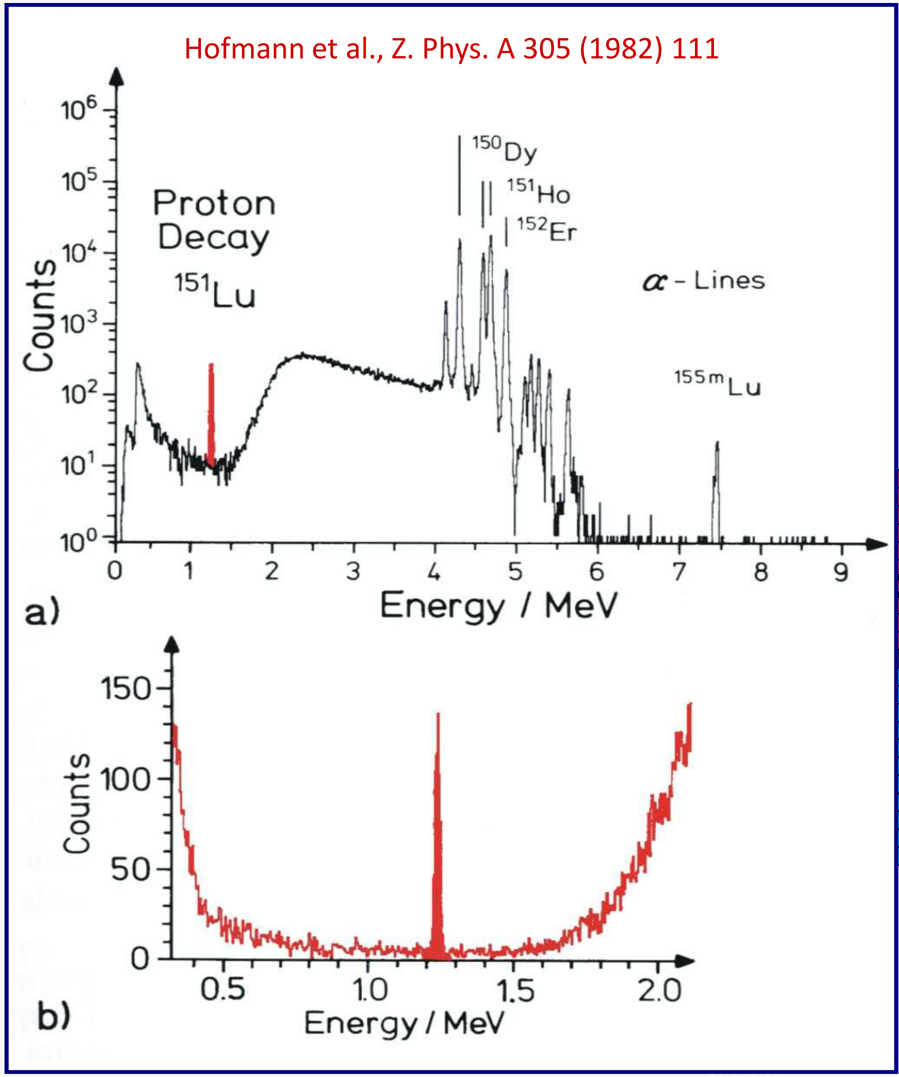
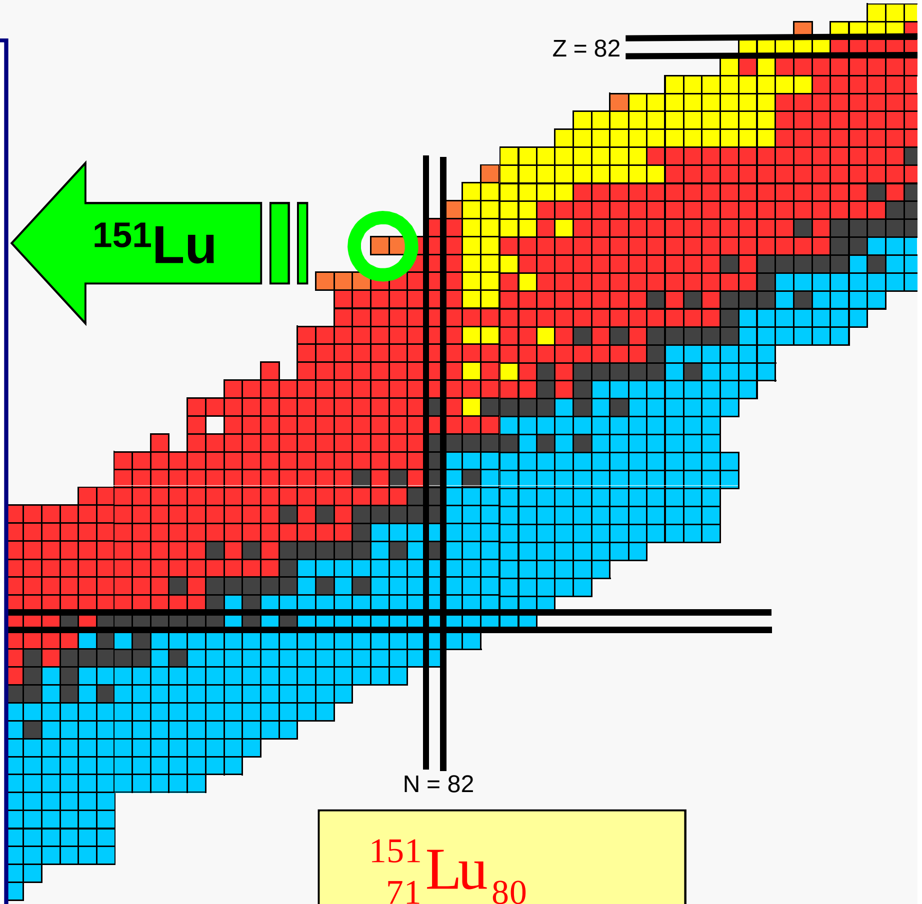


Illustration ⇨ *Mathematica_Adds-on*

The first g.s.proton emitter





$^{151}_{71}Lu_{80}$

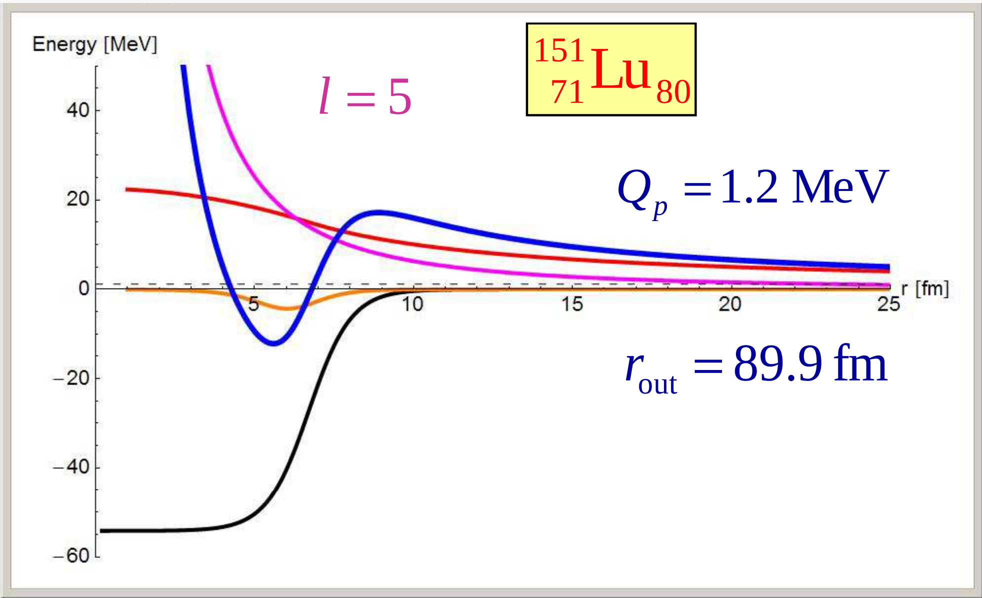
$Q_p = 1.2$ MeV

$T_{1/2} = 85$ ms

Simple model

- Decay constant for proton emission (WKB model):

$$\lambda_p \equiv \frac{\Gamma_p}{\hbar} \equiv \frac{\ln 2}{T_{1/2}^{p}} = S \cdot \nu \cdot T$$



S – spectroscopic factor. Assuming the proton emitted from the *i*-th single particle state and the BCS model of pairing:

$$S_i = (u_i)^2$$ – probability that the *i*-th state is empty in the daughter nucleus

ν – frequency factor:

$$\nu = \frac{\sqrt{2}\,\pi^2 (\hbar c)^2 c}{(\mu c^2)^{3/2} R_C^3 \sqrt{V_C(R_C) - Q_p}}$$

T – transmission (tunneling) probability trough the barrier

$$T = \exp\left\{ -\frac{2}{\hbar} \cdot \int_{r_{in}}^{r_{out}} \sqrt{2\mu[V(r) - Q_p]} \cdot dr \right\}$$

Calculation for ^{151}Lu

❶ Assume for an estimate: $S \cong 1$

❷ $V_C(R_C) = \frac{(Z-1)e^2}{R_C}$ $\left.\begin{array}{l} e^2 \cong 1.44 \text{ MeV} \cdot \text{fm} \\ R_C \cong 1.26 \cdot A^{1/3} \text{ fm} \end{array}\right\}$ $V_C(R_C) \cong 15 \text{ MeV}$

→ $\nu \cong 5 \cdot 10^{21} \text{ s}^{-1}$

❸ $$T = \exp\left\{ -\frac{2}{\hbar} \cdot \int_{r_{in}}^{r_{out}} \sqrt{2\mu[V(r) - Q_p]} \cdot dr \right\}$$

Numerical integration in *Mathematica*

↓

l	T	$T_{1/2}^p = \ln 2 / \nu T$
2	$2.04 \cdot 10^{-18}$	68 μs
4	$2.21 \cdot 10^{-20}$	6.3 ms
5	$9.58 \cdot 10^{-22}$	145 ms
6	$2.34 \cdot 10^{-23}$	5.92 s

- Experimental half-life is 85 ms.

  Which value of l fits? 4 or 5?

Is such comparison correct?

No !!!

Half-life prediction for ^{151}Lu

- The measured half-life reflects all possible decay channels!
  And the proton emitter can decay by the β^+ transition as well!

Decay probabilities have to be summed: $\lambda^{tot} = \lambda_p + \lambda_\beta$

$$\Rightarrow \quad \frac{1}{T_{1/2}} = \frac{1}{T_{1/2}^{p}} + \frac{1}{T_{1/2}^{\beta}}$$

The estimate of the $T_{1/2}^{\beta}$ for ^{151}Lu is 220 ms.

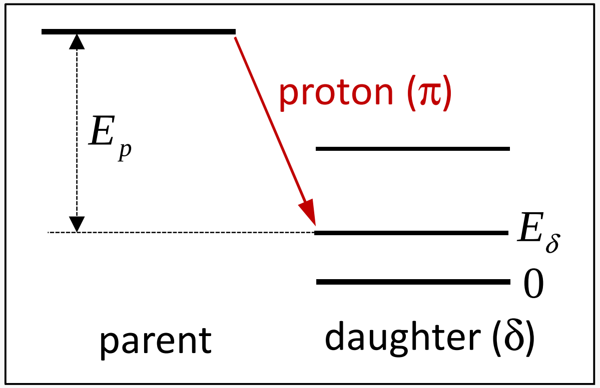
Combining this with previous calculations we obtain:

l	T	$T_{1/2}^{p} = \ln 2 / \nu T$	$T_{1/2}$
2	$2.04 \cdot 10^{-18}$	68 μs	68 μs
4	$2.21 \cdot 10^{-20}$	6.3 ms	6.1 ms
5	$9.58 \cdot 10^{-22}$	145 ms	87 ms
6	$2.34 \cdot 10^{-23}$	5.92 s	212 ms

$\Leftrightarrow$ $T_{1/2}^{exp} = 85$ ms — Bingo!

Conclusion:
proton is emitted from the spherical $h_{11/2}$ orbital!

More general model



- Cluster approximation (proton moves in the (deformed) potential of the daughter
- Parent wave function:

$$\Psi_{JM}(\vec{r}) = \frac{1}{r} \sum_{\alpha=\{\pi,\delta\}} u_\alpha(r) \left[\mathcal{Y}_\pi \otimes \Phi_\delta \right]_{JM}$$

radial part of relative motion — angular part

→ Schrödinger equation, integration over angles:

$$\left[\frac{d^2}{dr^2} - \frac{l_p(l_p+1)}{r^2} + \frac{2\mu}{\hbar^2} E_p \right] u_\alpha(r) = \frac{2\mu}{\hbar^2} \sum_{\alpha'} \left(\hat{V}_{\alpha,\alpha'} \right) u_{\alpha'}(r) \quad \textbf{(coupled channels)}$$

- Solutions must fulfill conditions:

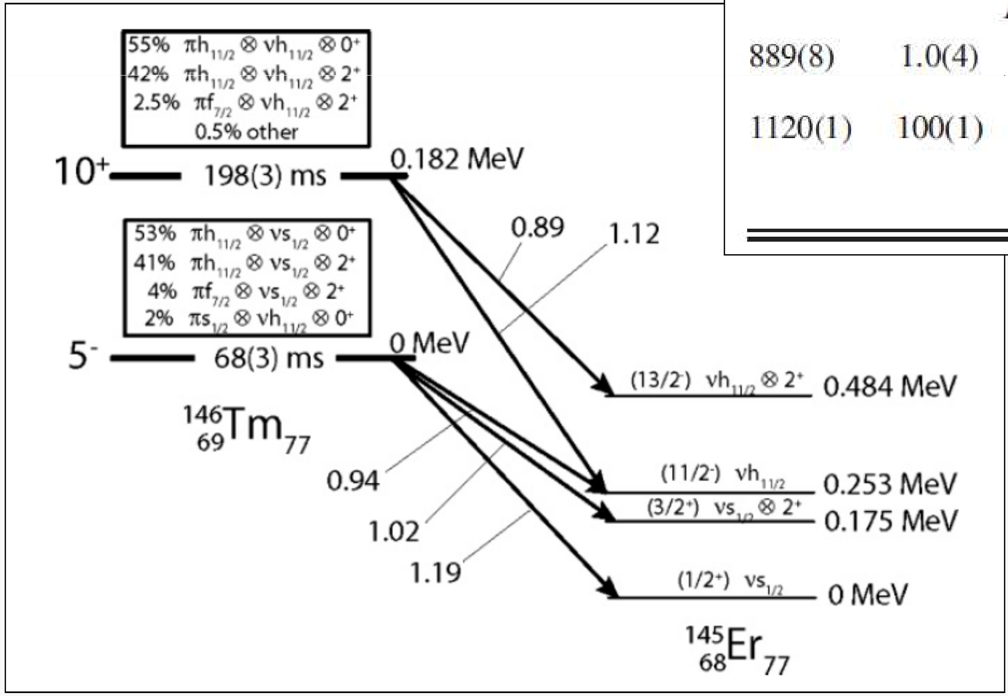
$$u_\alpha(r) \underset{r\to 0}{\longrightarrow} 0 \,, \qquad u_\alpha(r) \underset{r\to\infty}{\longrightarrow} \sim G_{l_p}(\eta,kr) + i\,F_{l_p}(\eta,kr) \quad \text{(outgoing Coulomb wave)}$$

→ From radial functions the width: $u_\alpha(r) \Rightarrow \Gamma_\alpha \,, \;\; \Gamma = \sum_\alpha \Gamma_\alpha \,, \;\; \Gamma_\delta = \sum_{\{\pi\}} \Gamma_{\{\pi,\delta\}}$

Odd-odd example: ^{146}Tm

- The richest emitter known: 5 proton lines observed!
- → In the Z – odd, N – odd nucleus the proton radioactivity provides data on neutron levels!

E_p	I_p^{exp} (%)	Wave function composition	I_p^{cal}	Δl	E_f
		Ground state $I^\pi = 5^-$, $T_{1/2} = 68(5)$ ms			
938(4)	13.8(9)	2% $\pi s_{1/2} \otimes \nu h_{11/2} \otimes 0^+$	(15)[a]	0	253
1016(4)	18.3(11)	4% $\pi f_{7/2} \otimes \nu s_{1/2} \otimes 2^+$	15	3	175
		41% $\pi h_{11/2} \otimes \nu s_{1/2} \otimes 2^+$	0.003	5	175
1191(1)	68.1(19)	53% $\pi h_{11/2} \otimes \nu s_{1/2} \otimes 0^+$	70	5	0
		Isomeric state $I^\pi = 10^+$, $T_{1/2} = 198(3)$ ms			
889(8)	1.0(4)	2.5% $\pi f_{7/2} \otimes \nu h_{11/2} \otimes 2^+$	1.2	3	484
		42% $\pi h_{11/2} \otimes \nu h_{11/2} \otimes 2^+$	0.04	5	484
1120(1)	100(1)	55% $\pi h_{11/2} \otimes \nu h_{11/2} \otimes 0^+$	98.6	5	253
		0.1% $\pi h_{9/2} \otimes \nu h_{11/2} \otimes 0^+$	0.2	5	253
		0.4% $\pi(l > 5) \otimes \nu h_{11/2}$			

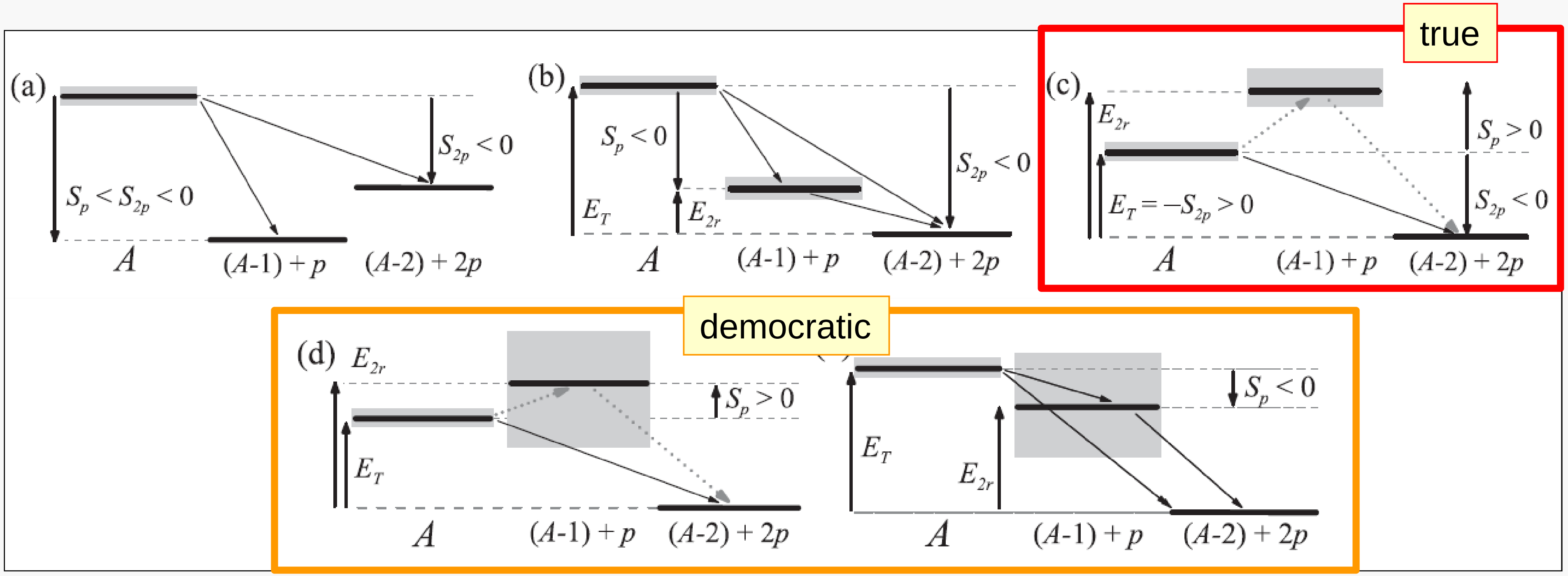


- Good agreement with the model assuming coupling of a particle to core vibrations (the c-c scheme)

Tantawy *et al.*, Phys. Rev. C 73 (2006) 024316
Hagino, Phys. Rev. C64 (2001) 041304(R)

Two-proton emission

Energy conditions for different modes of the 2p emission



a) $^{18}Ne^*$

b) $^{14}O^*$, $^{17}Ne^*$

b) ^{19}Mg , ^{45}Fe, ^{48}Ni, ^{54}Zn,...

d,e) ^{6}Be , ^{12}O(?)

Predicting masses

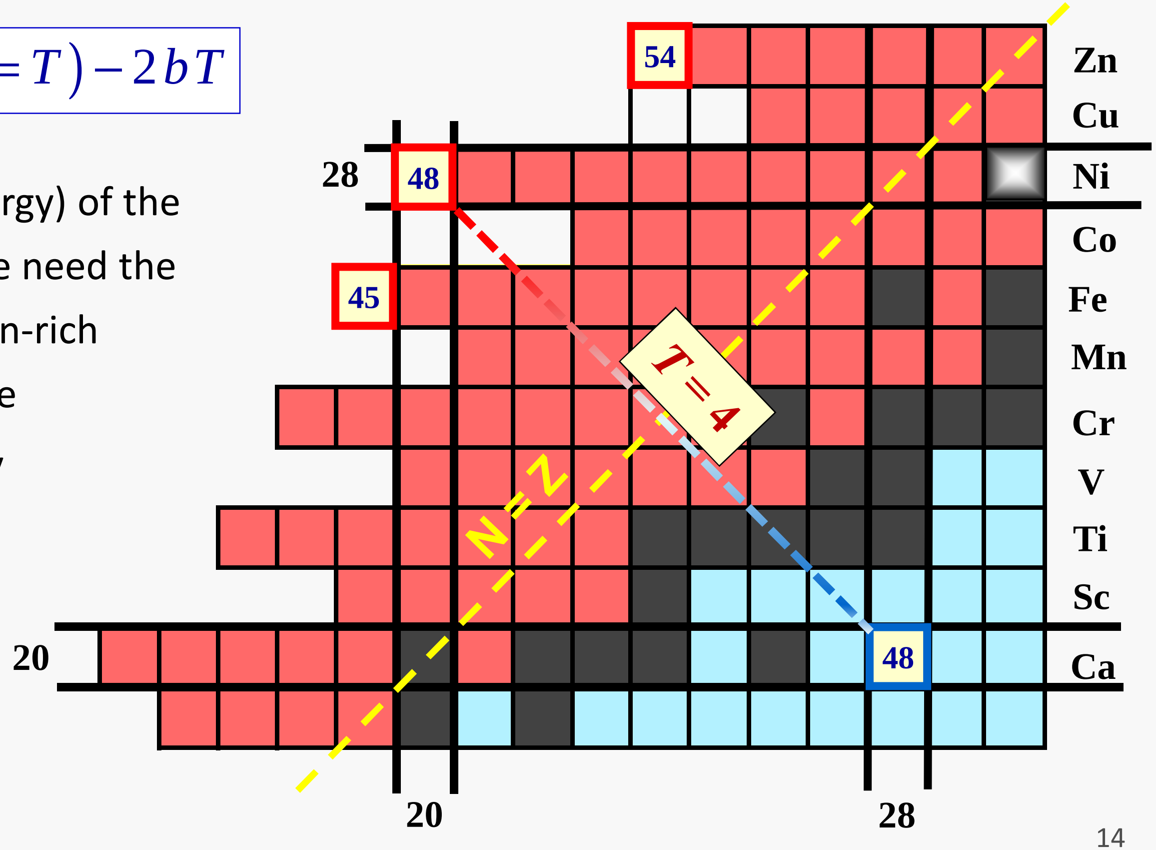
- Global mass models are not precise enough to determine the decay mode. However, there is a trick based on the Isobaric Multiplet Mass Equation (IMME):

$$BE(A,T,T_z) = a(A,T) + b(A,T)T_z + c(A,T)T_z^2$$

$$T_z = (N-Z)/2$$

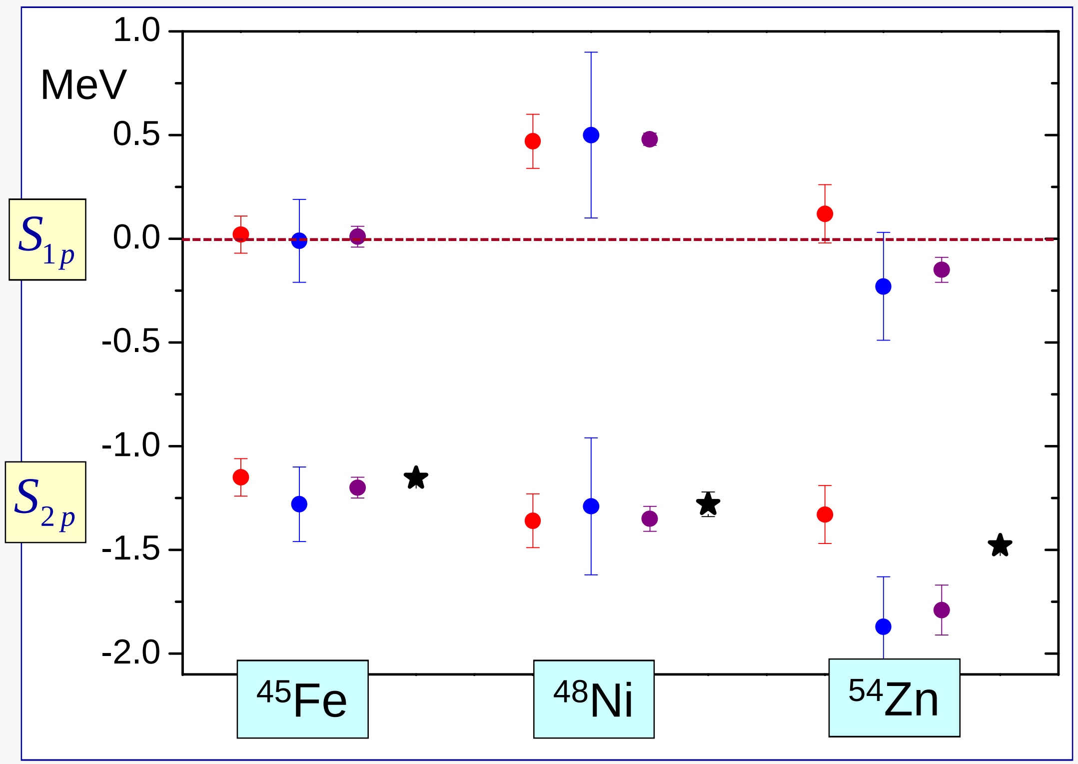
$$BE(T_z=-T) = BE(T_z=T) - 2bT$$

- To get the mass (binding energy) of the neutron-deficient nuclide, we need the **measured mass** of its neutron-rich analogue and the value of the *coefficient b* from the theory (shell-model, systematics…)

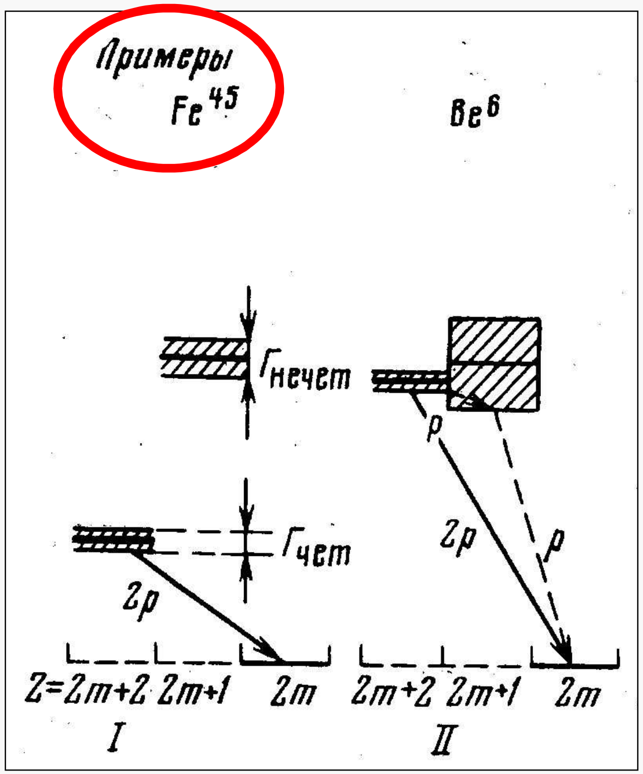


2*p* candidates

Predicted 1p and 2p separation energies



Pattern predicted by Goldansky



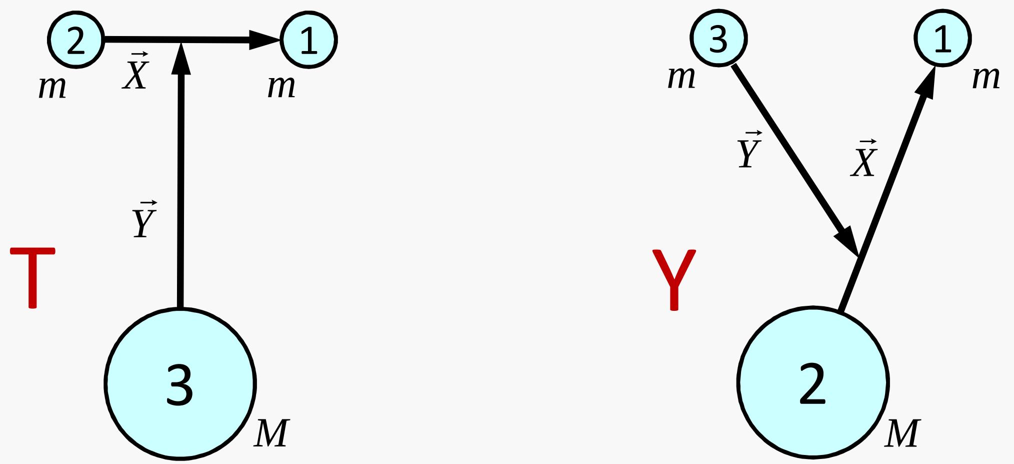
V.I. Goldanskii, Nucl. Phys. 19 (60) 482

- ● B.A. Brown, PRC 43 (91) R1513
- ● W.E. Ormand, PRC 55 (97) 2407
- ● B.J. Cole, PRC 54 (96) 1240

★ exp

Jacobi coordinates, positions

- Three-body kinematics is simpler in Jacobi coordinates



- In place of the radius and solid angle of one particle, the three particles are described by the *hyperradius* and *hyper solid angle*:

$$r,\ \Omega\ \rightarrow\ \rho,\ \Omega_5$$

$$\Omega_5 = \left\{\theta_\rho, \Omega_X, \Omega_Y\right\} \qquad \rho^2 = \frac{A_1 A_2 A_3}{A_1 + A_2 + A_3}\left(\frac{\vec{r}_{12}^{\,2}}{A_3} + \frac{\vec{r}_{23}^{\,2}}{A_1} + \frac{\vec{r}_{31}^{\,2}}{A_2}\right)$$

$$\tan\left(\theta_\rho\right) = \sqrt{M_x / M_y}\ X/Y \qquad M_x = \frac{A_1 A_2}{A_1 + A_2} u\ ,\ \ M_y = \frac{\left(A_1 + A_2\right) A_3}{A_1 + A_2 + A_3} u$$

Jacobi coordinates, momenta

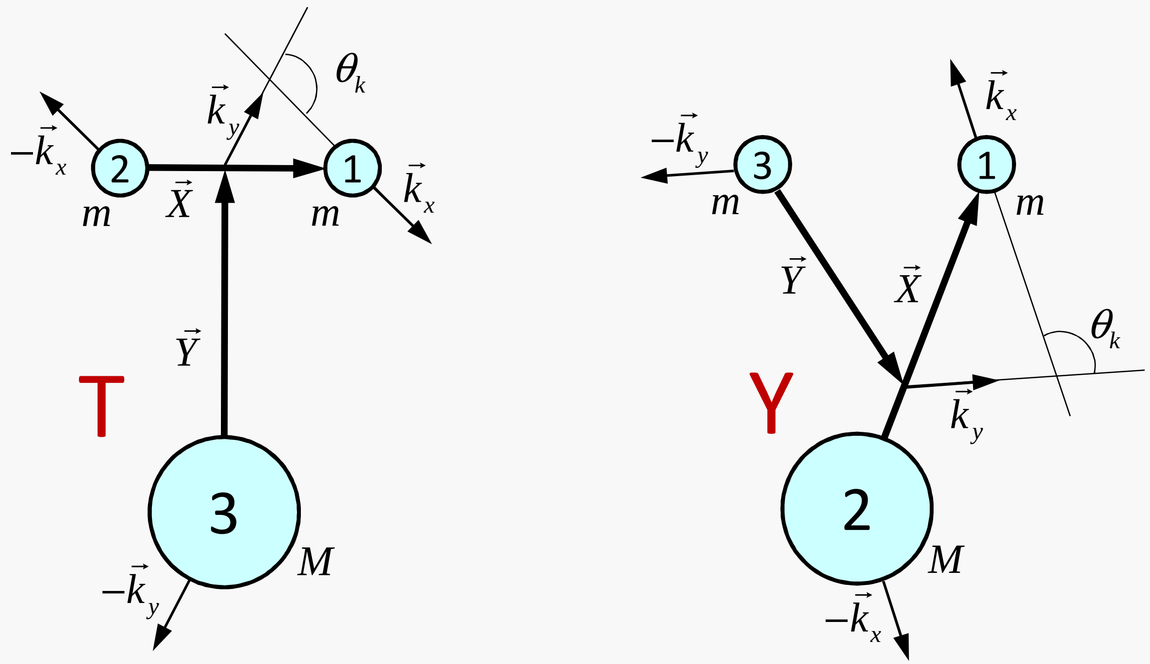


Illustration
⇨ *Mathematica_Adds-on*

- Complete correlation picture is given by two parameters: $\varepsilon = E_x / E_T \,, \quad \cos(\theta_k) = \dfrac{\vec{k}_x \cdot \vec{k}_y}{k_x k_y}$

E_T is the total decay energy: $E_T = E_x + E_y = \dfrac{k_x^2}{2M_x} + \dfrac{k_y^2}{2M_y}$

$$M_x = \frac{A_1 A_2}{A_1 + A_2} u$$

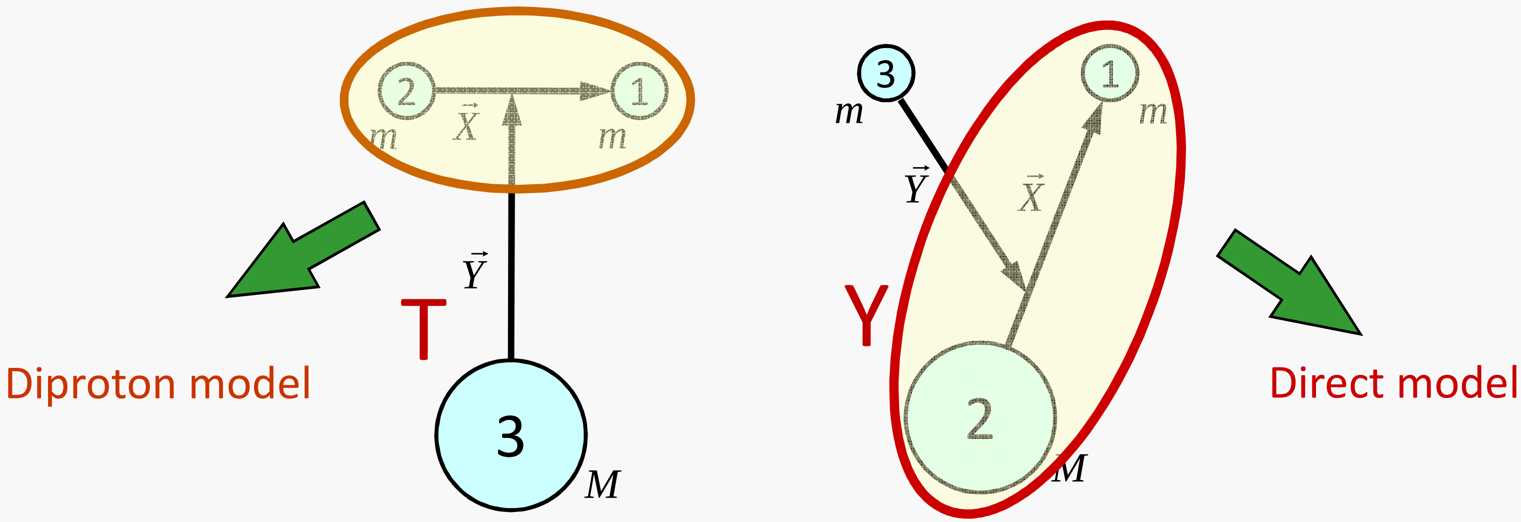
If the decay occurs at rest: $\vec{k}_x = \dfrac{A_2 \vec{k}_1 - A_1 \vec{k}_2}{A_1 + A_2} \,, \quad \vec{k}_y = -\vec{k}_3$

$$M_y = \frac{(A_1 + A_2) A_3}{A_1 + A_2 + A_3} u$$

Simplified models

- By simplifying interactions describing the *core+p+p* system, the three-body decay can be reduced to the combination of two-body processes. With the simplified Hamiltonian, the problem can be solved exactly.

→ Two types of approximations are considered:



- The simplified models are very useful to estimate decay rates and to veryfiy numerical procedures used in the full three-body model.

Direct model

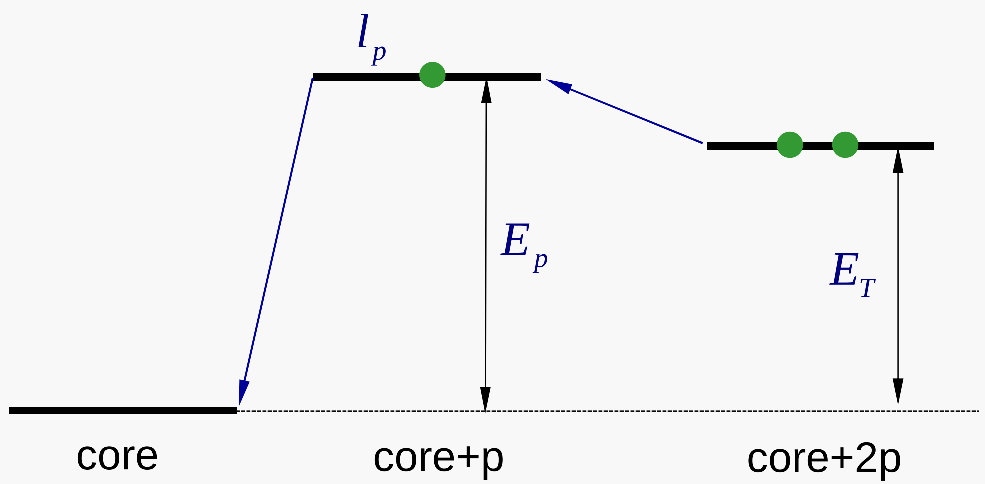
- In the Y Jacobi system: $\vec{k}_x = \vec{k}_1 + \frac{\mu}{M}\vec{k}_3, \quad \vec{k}_y = -\vec{k}_3, \quad E_x = \frac{k_x^2}{2\mu} \qquad \mu = \frac{mM}{m+M}$

❶ Assume: $M >> m \ \rightarrow \mu \cong m \ \rightarrow \vec{k}_x \cong \vec{k}_1, \ E_x = \frac{k_1^2}{2m}$

→ Then ε is the fraction of the decay energy taken by one proton and

θ_k is the angle between momenta of both protons + π

❷ Assume: both protons occupy the same orbital with angular momentum l_p



Direct model

- The *2p* decay width in the direct model is given by:

$$\Gamma_{dir} = \frac{E_T}{2\pi}\left(E_T - 2E_p\right)^2 \int_0^1 d\varepsilon \frac{\Gamma_x\left(\varepsilon E_T\right)}{\left(\varepsilon E_T - E_p\right)^2 + \Gamma_x\left(\varepsilon E_T\right)^2/4} \times \frac{\Gamma_y\left(\left(1-\varepsilon\right)E_T\right)}{\left(\left(1-\varepsilon\right)E_T - E_p\right)^2 + \Gamma_y\left(\left(1-\varepsilon\right)E_T\right)^2/4}$$

where Γ_i is the width of the two-body subsystem: $\Gamma_i\left(E\right) = 2\gamma_i^2 P_{l_p}\left(E, R, Z_i\right)$

reduced width: $\gamma_i^2 = \frac{\hbar^2}{2\mu_i R^2}\theta_i^2$

penetrability: $P_{l_p}\left(E, R, Z_i\right) = \frac{kR}{F_{l_p}^2\left(\eta, kR\right) + G_{l_p}^2\left(\eta, kR\right)}$

$Z_x = Z_{\text{core}}$

$Z_y = Z_{\text{core}} + 1$

radius: $R = 1.4\left(A_{\text{core}} + 1\right)^{1/3}$ fm

Sommerfeld parameter: $\eta = \mu Z e^2/\hbar^2 k$

wave number: $k = \sqrt{2\mu E}/\hbar$

Direct model

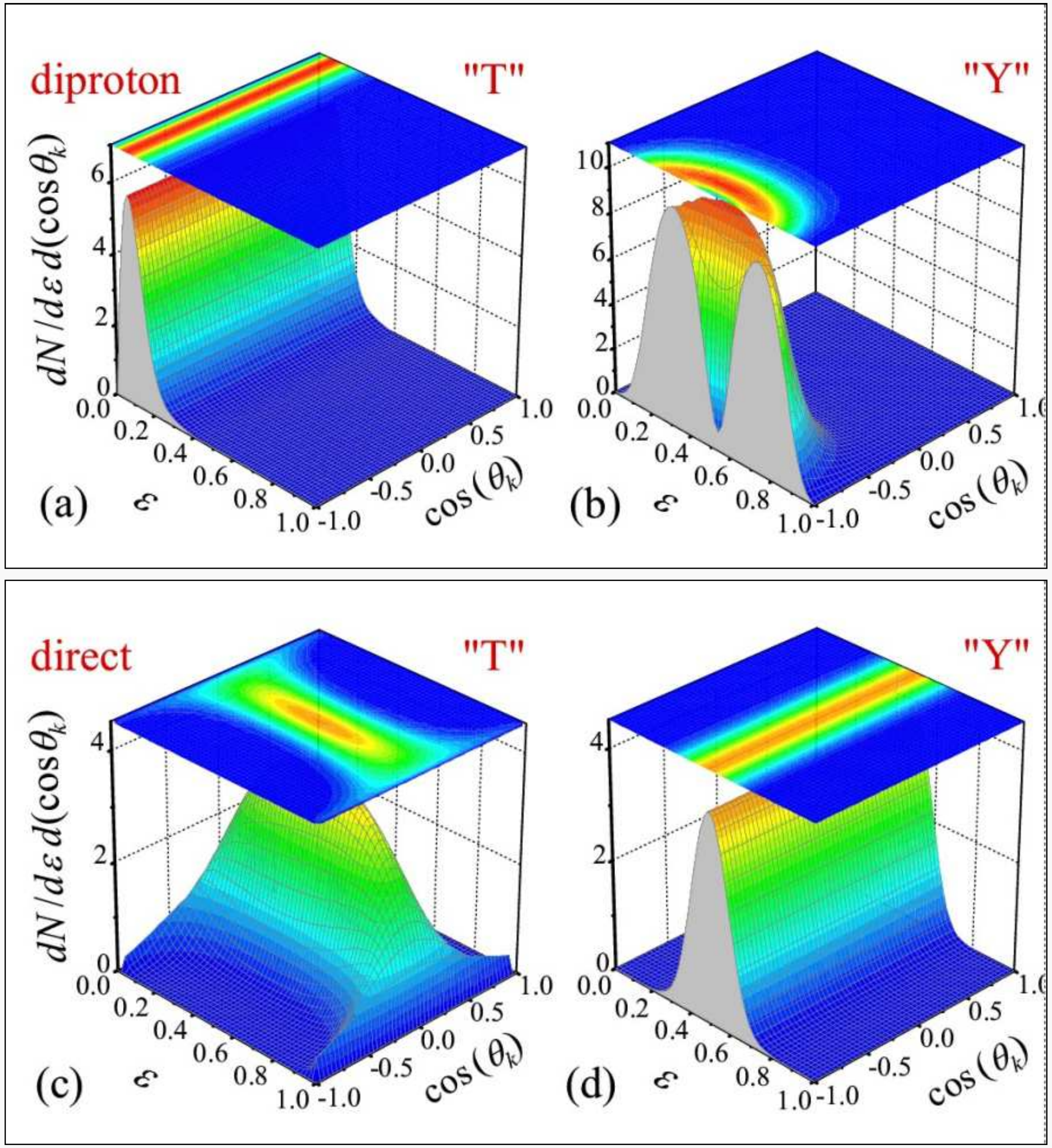
- The 2p decay half-lives vs. the direct model $T_{1/2} = \frac{\ln 2\hbar}{\Gamma}$

^{A}Z	$T_{1/2}$ *exp*	E_T [MeV]	E_p [MeV]	l_p	$T_{1/2}$ *dir*
^{19}Mg	4.0(15) ps	0.75(5)	1.3	0	0.18 ps
^{45}Fe	3.7(4) ms	1.154(16)	1.178	1	0.37 ms
^{48}Ni	$3.0^{+2.2}_{-1.2}$ ms	1.28(6)	1.794	1	2.0 ms
^{54}Zn	$1.98^{+0.73}_{-0.41}$ ms	1.48(2)	1.6	1	0.25 ms

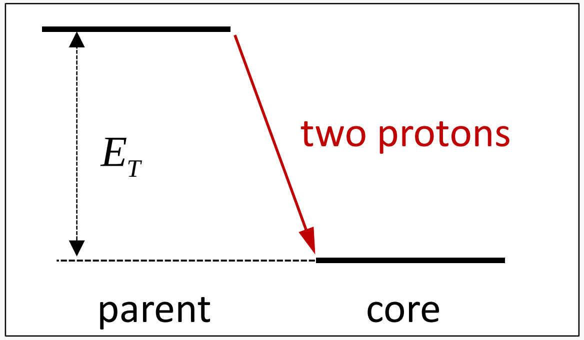
- The direct model allows to investigate the transition from the simultaneous to the sequential emission

Illustration ⇨ *Mathematica_Adds-on*

^{45}Fe: p-p correlations in simplified models



Three-body model



- Cluster approximation (two protons and the core)
- Parent wave function:

$$\Psi_{JM}(\rho,\Omega_5) = \frac{1}{\rho^{5/2}} \sum_{\alpha=\{K,\dots\}} \chi_\alpha(\rho)\, \mathcal{J}_\alpha^{JM}(\Omega_5)$$

radial functions — hyperspherical harmonics

→ Schrödinger equation, integration over angles:

$$\left[\frac{d^2}{d\rho^2} - \frac{\mathcal{L}_K(\mathcal{L}_K+1)}{\rho^2} + \frac{2\mu}{\hbar^2}E_T\right]\chi_\alpha(\rho) = \frac{2\mu}{\hbar^2}\sum_{\alpha'}\left(\hat{V}_{\alpha,\alpha'}\right)\chi_{\alpha'}(\rho)$$

(coupled channels)

- Main problem: the asymptotic form of radial functions is not known!

  Solution of Grigorenko and Zhukov:

$$\chi_\alpha(\rho) \underset{\rho\to\infty}{\to} \sim \sum_{\alpha'} \hat{A}_{\alpha,\alpha'}\left[G_{\mathcal{L}_0}(\eta_\alpha,\kappa\rho) + i\,F_{\mathcal{L}_0}(\eta_\alpha,\kappa\rho)\right]$$

$\hat{A}_{\alpha,\alpha'}$ is the matrix which diagonalizes the Coulomb part of $\hat{V}_{\alpha,\alpha'}$

→ From radial functions the width and correlations :

$$\chi_\alpha(\rho) \Rightarrow \Gamma\ ,\quad dj/d\varepsilon\, d(\cos\theta_k)$$

Grigorenko and Zhukov, PRC 68 (03) 054005
M.P. et al, RMP (2012) 567

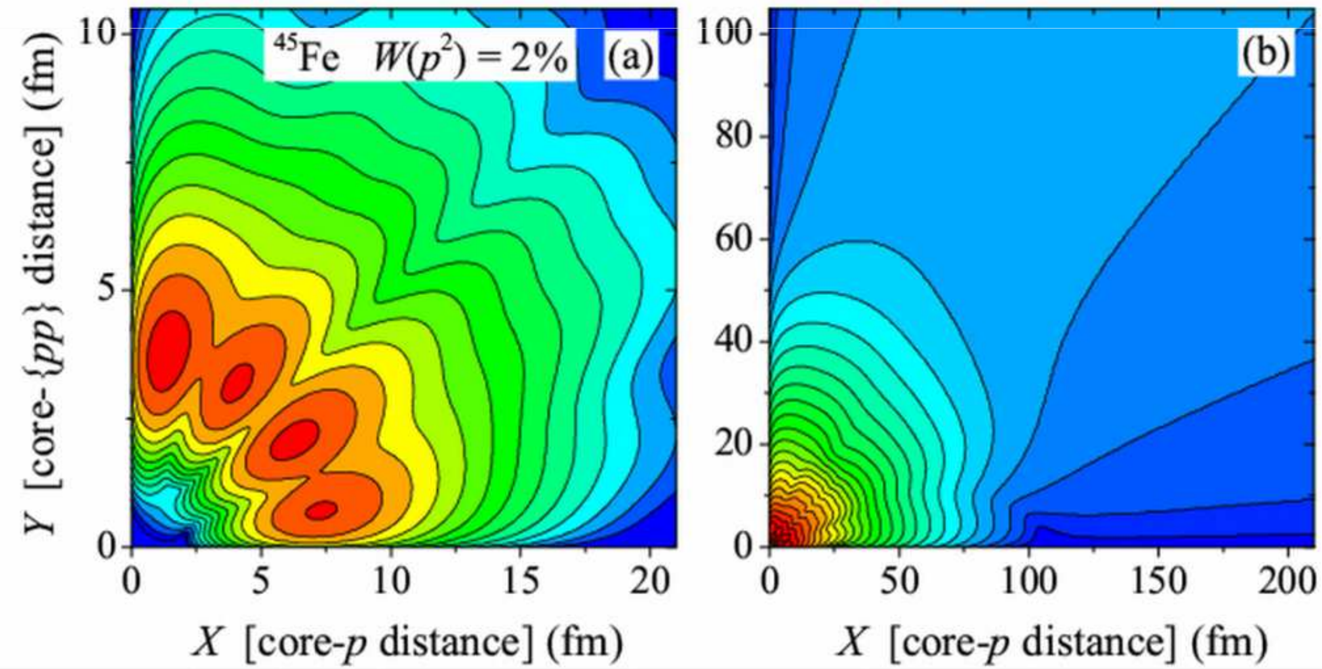
^{45}Fe: the wave function

- The 3-body wave function can be expressed as a sum of terms having defined l^2 configuration:

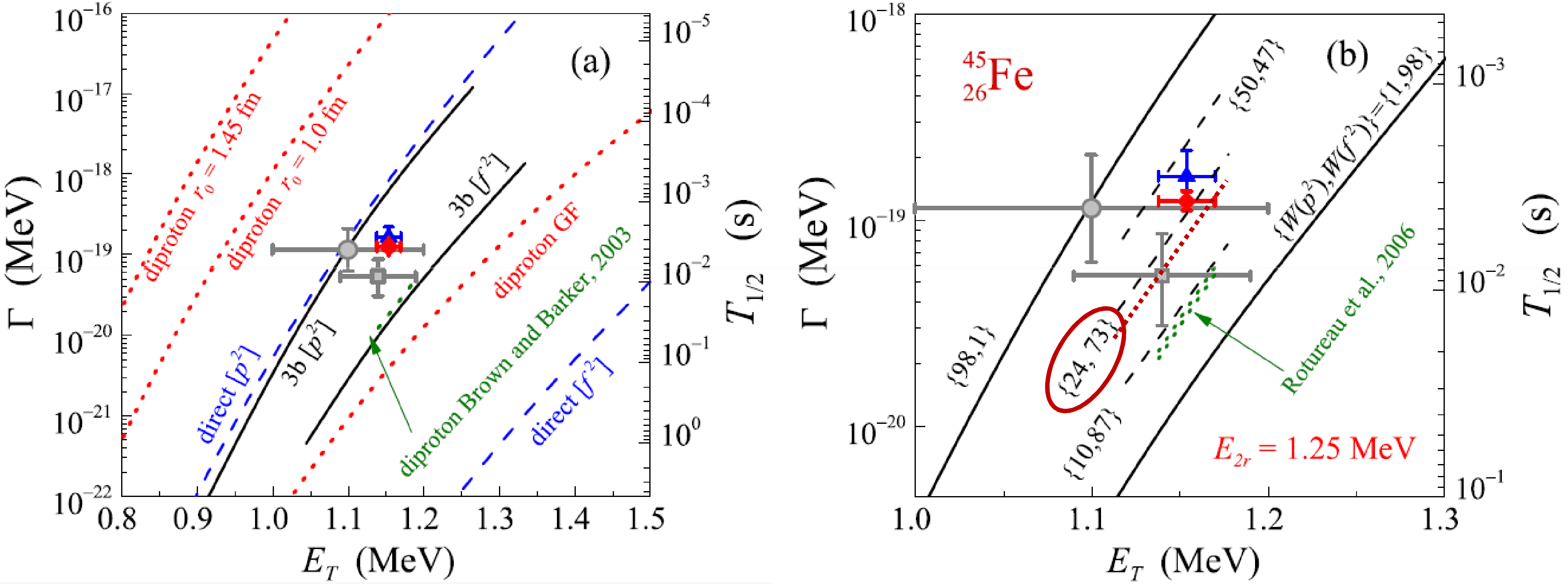
$$\Psi_{JM} = \sum_i W_i \left[l_i^2 \right]_0$$

By adjusting the potentials, the weights of different l^2 configurations can be modified

The ^{45}Fe wave function density $|\Psi|^2$ in the T system for $W(f^2) = 98\%$, $W(p^2) = 2\%$



^{45}Fe: decay energy and time

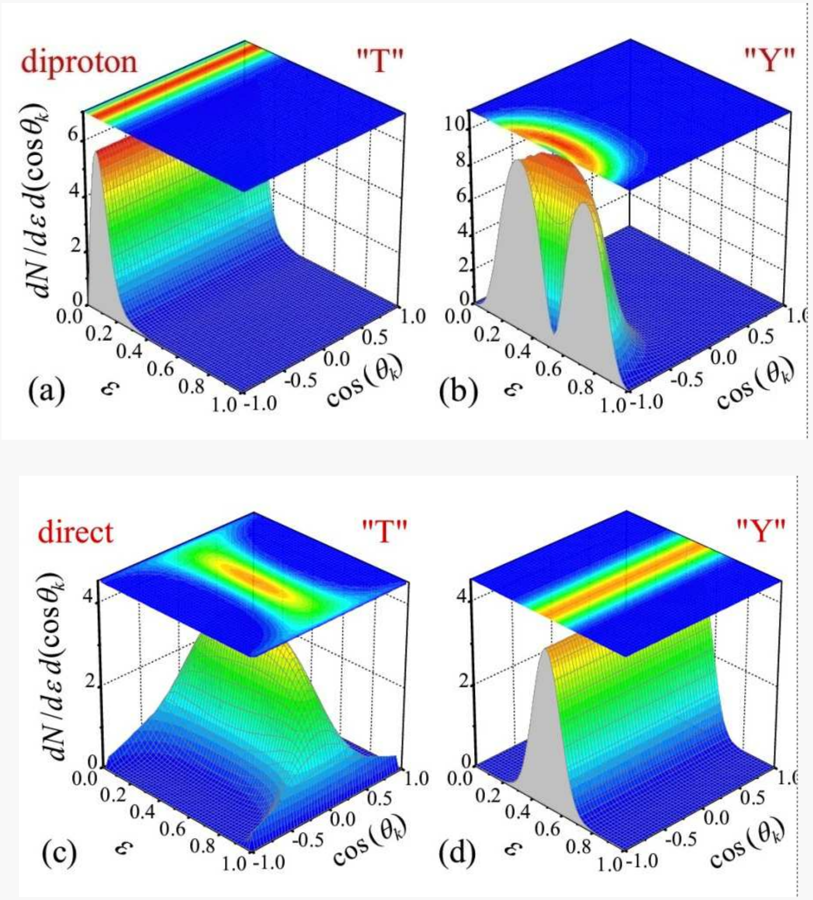


Grigorenko and Zhukov, Phys. Rev. C 68 (2003) 054005

Brown and Barker, PRC 67 (2003) 041304(R)

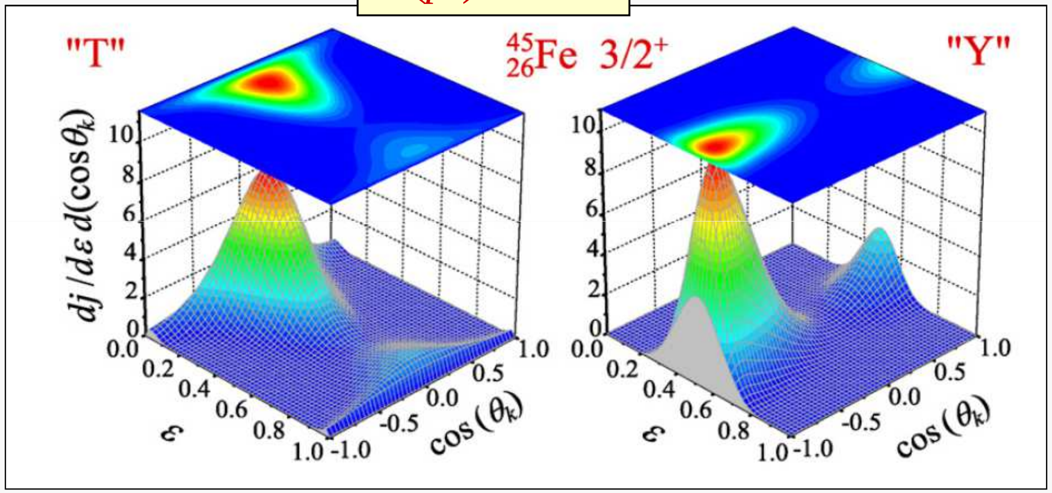
Rotureau, Okołowicz, and Płoszajczak, Nucl. Phys. A767 (2006) 13

^{45}Fe: p-p correlations



Three-body model

$W(p^2) = 24\%$



Grigorenko and Zhukov, Phys. Rev. C 68 (2003) 054005

This prediction was done before the experiment!
And how does the measured distribution look like ?

Thanks! See you tomorrow ☺

"Particles, particles, particles."