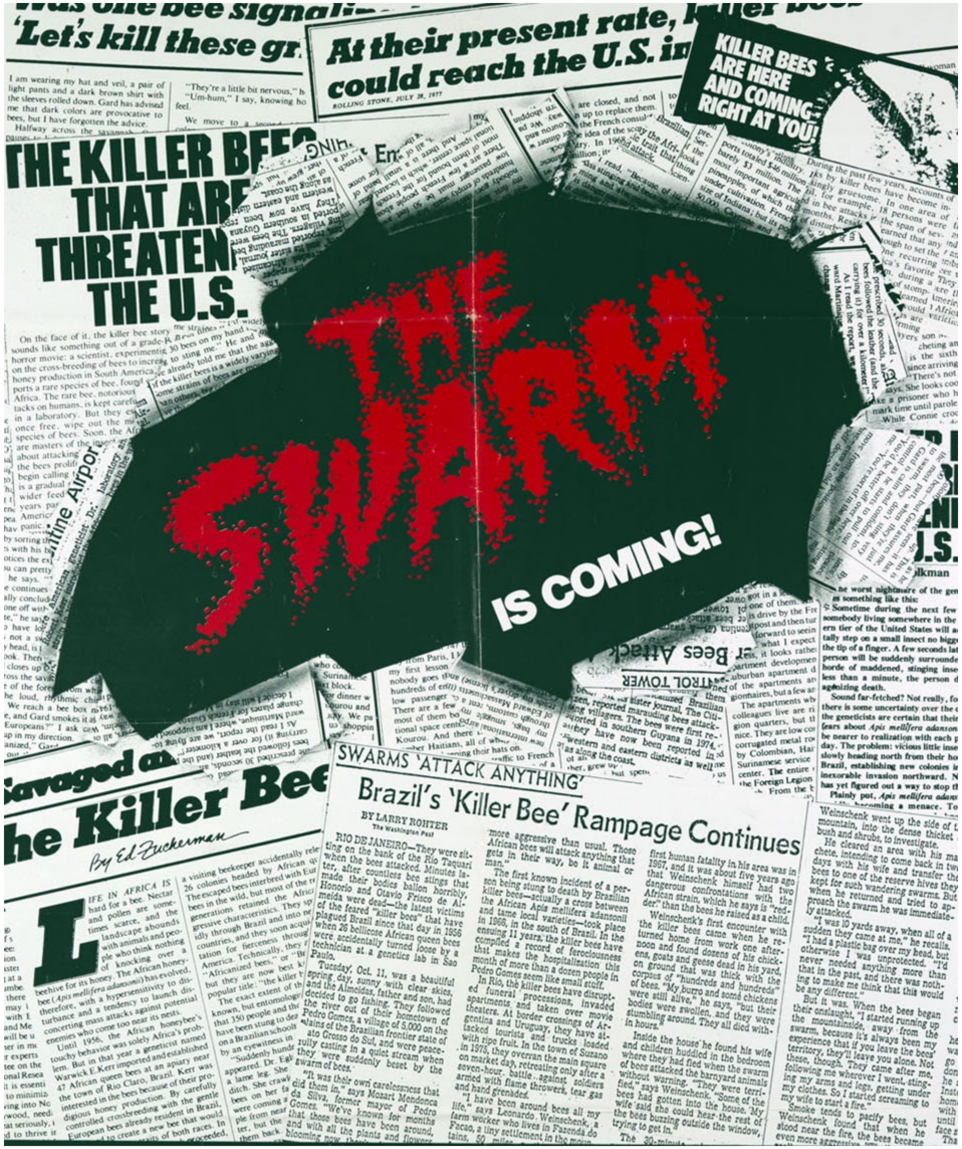
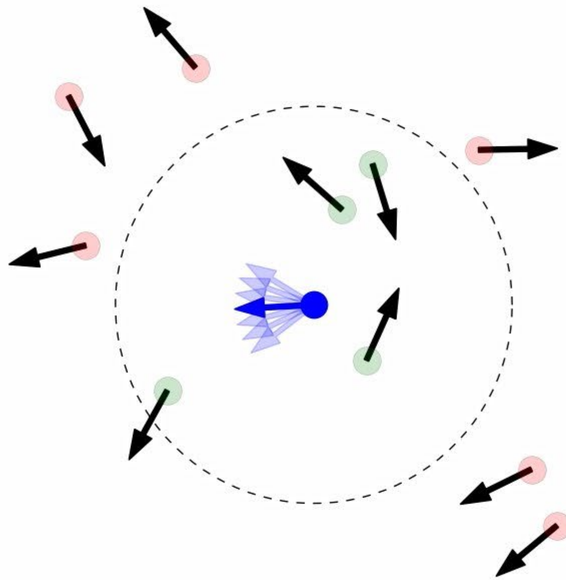
Computer simulations of complex systems



Lab PS II – The Swarm

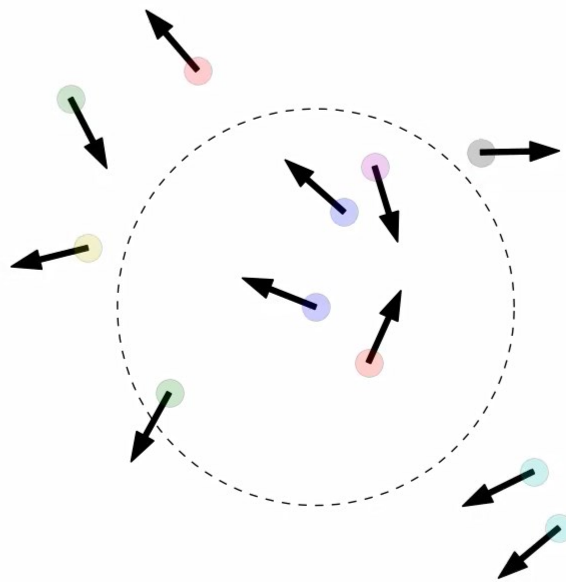
A simple swarming model



Vicsek, T., Czirók, A., Ben-Jacob, E., Cohen, I. and Shochet, O., 1995. Novel type of phase transition in a system of self-driven particles. *Physical review letters*, *75*(6), p.1226.

Step I

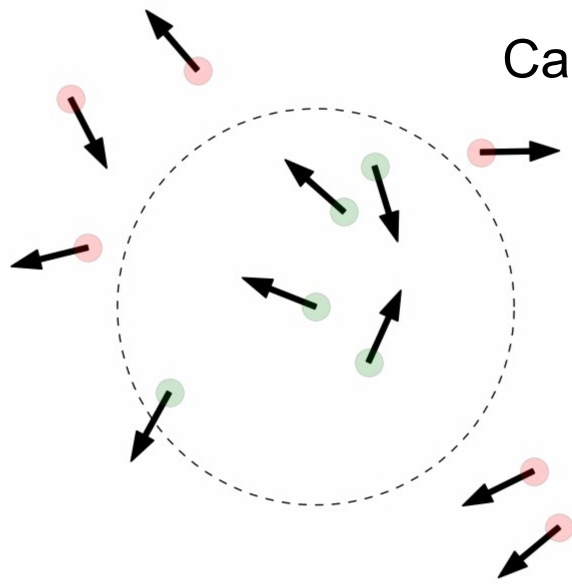
Find nearest neighbours



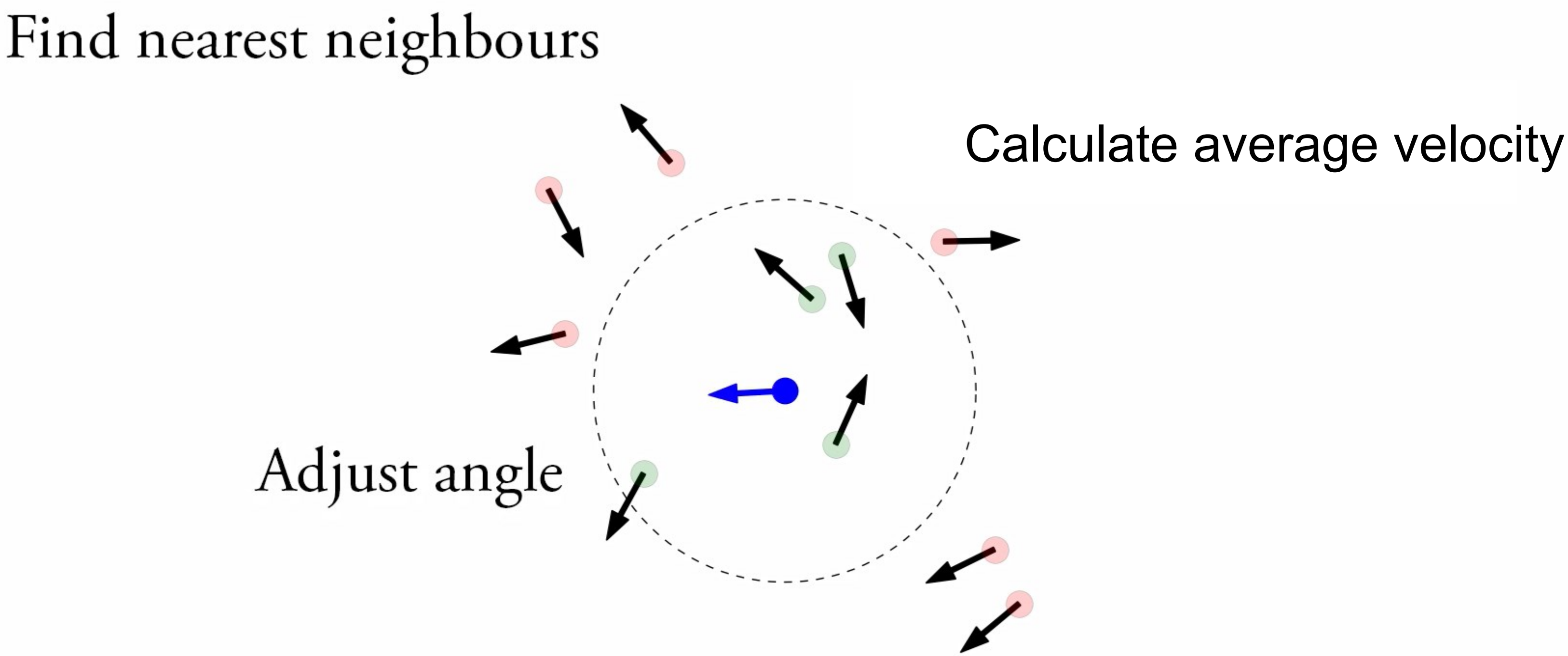
Step II

Find nearest neighbours

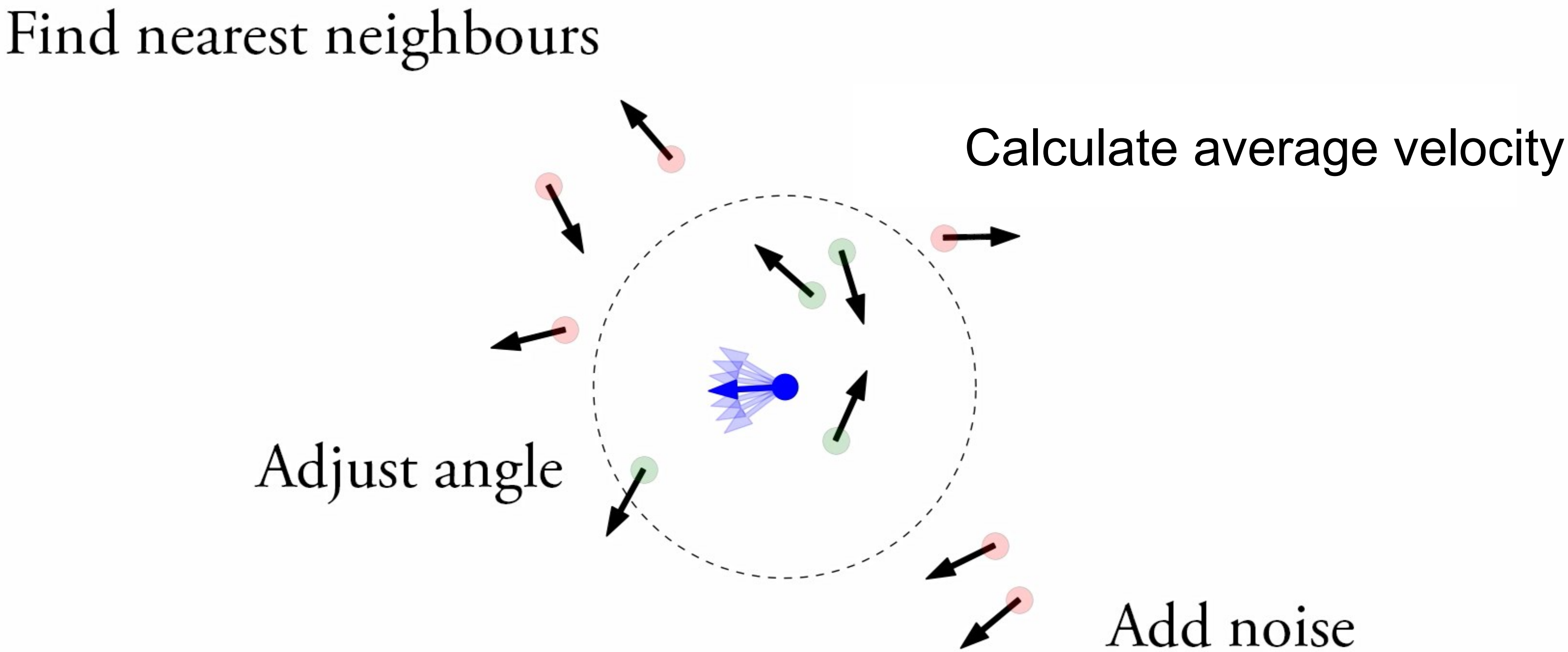
Calculate average velocity



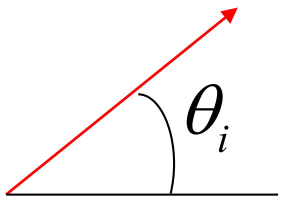
Step III


Find nearest neighbours
Calculate average velocity
Adjust angle

Step IV


Find nearest neighbours
Calculate average velocity
Adjust angle
Add noise

Summary



new direction of bird i

amplitude

$$\theta_i = \arg\left(\sum_k e^{i\theta_k}\right) + a\eta_i$$

direction of average velocity of the neighbours

random term

(uniform in the angle)

new position of bird i

$$r_i(t+dt) = r_i(t) + v_0 \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix} dt$$

Your task

- Simulate swarming model for 5000 birds (0.5p)
- Add a bird of prey (0.5p)

- follows the closest bird
- all birds in a radius r_b run away from it

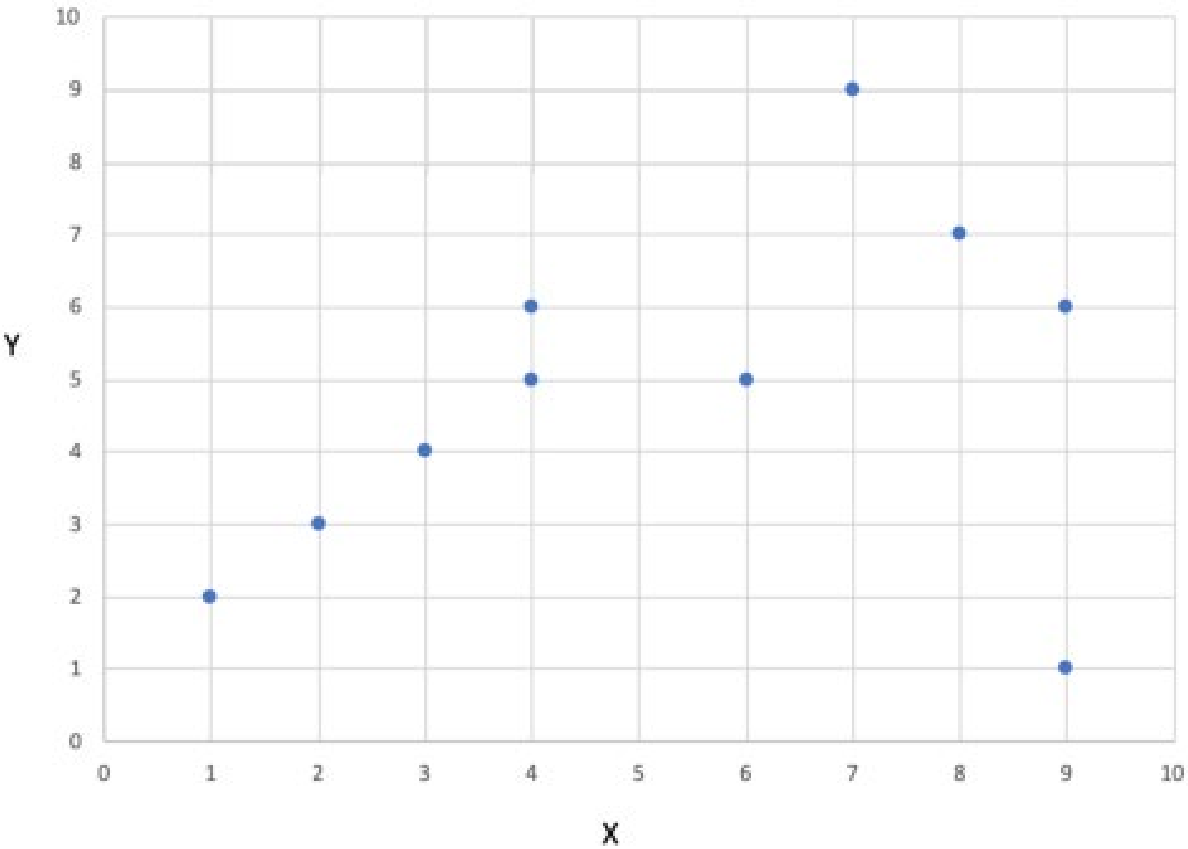
The (devil in the) details

- you need a fast way of getting the neighbours of bird *i*
- try to explore tree algorithms:

  https://towardsdatascience.com/tree-algorithms-explained-ball-tree-algorithm-vs-kd-tree-vs-brute-force-9746debcd940

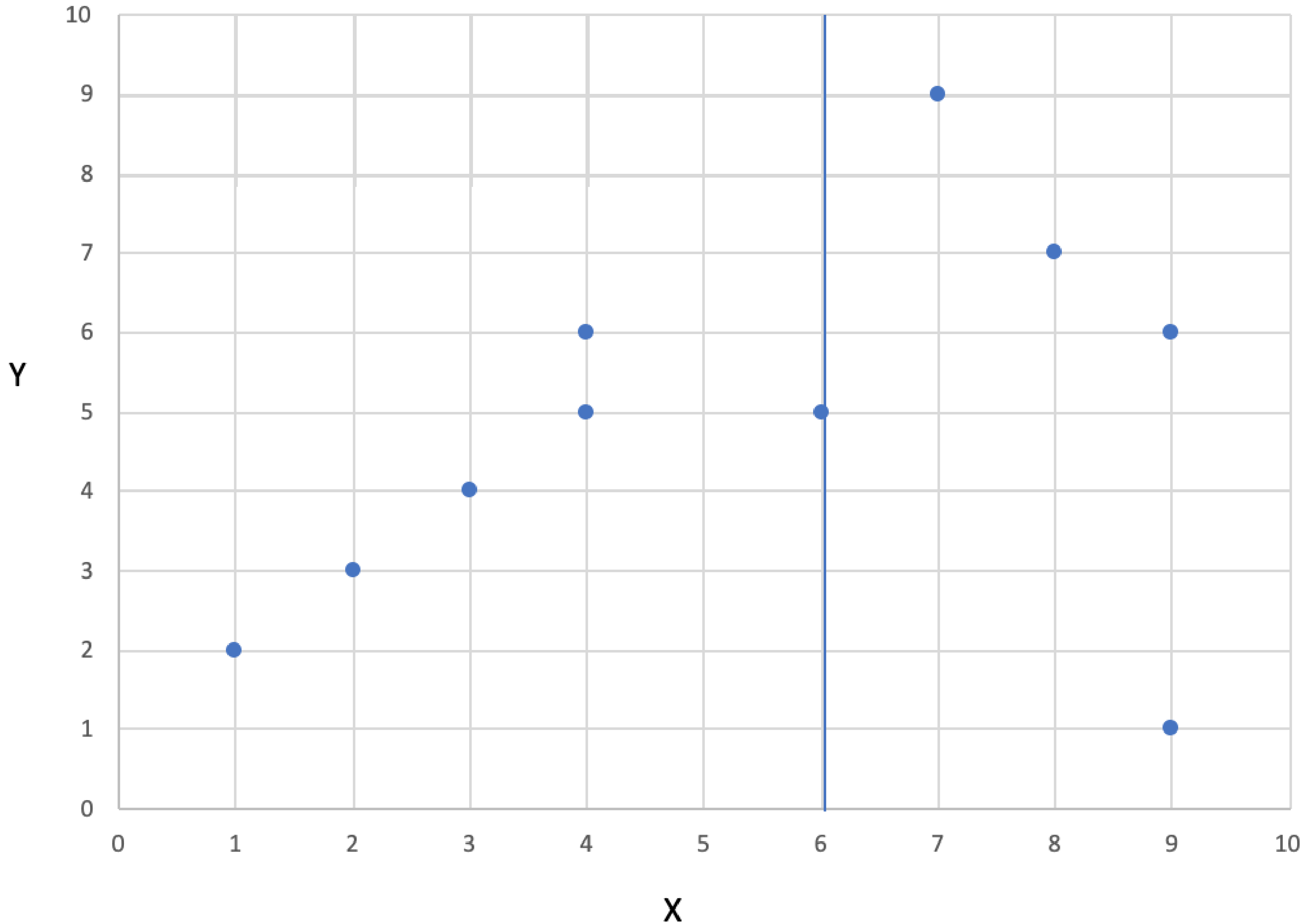
- one possibility is to use a kd-tree:

KDTree - example

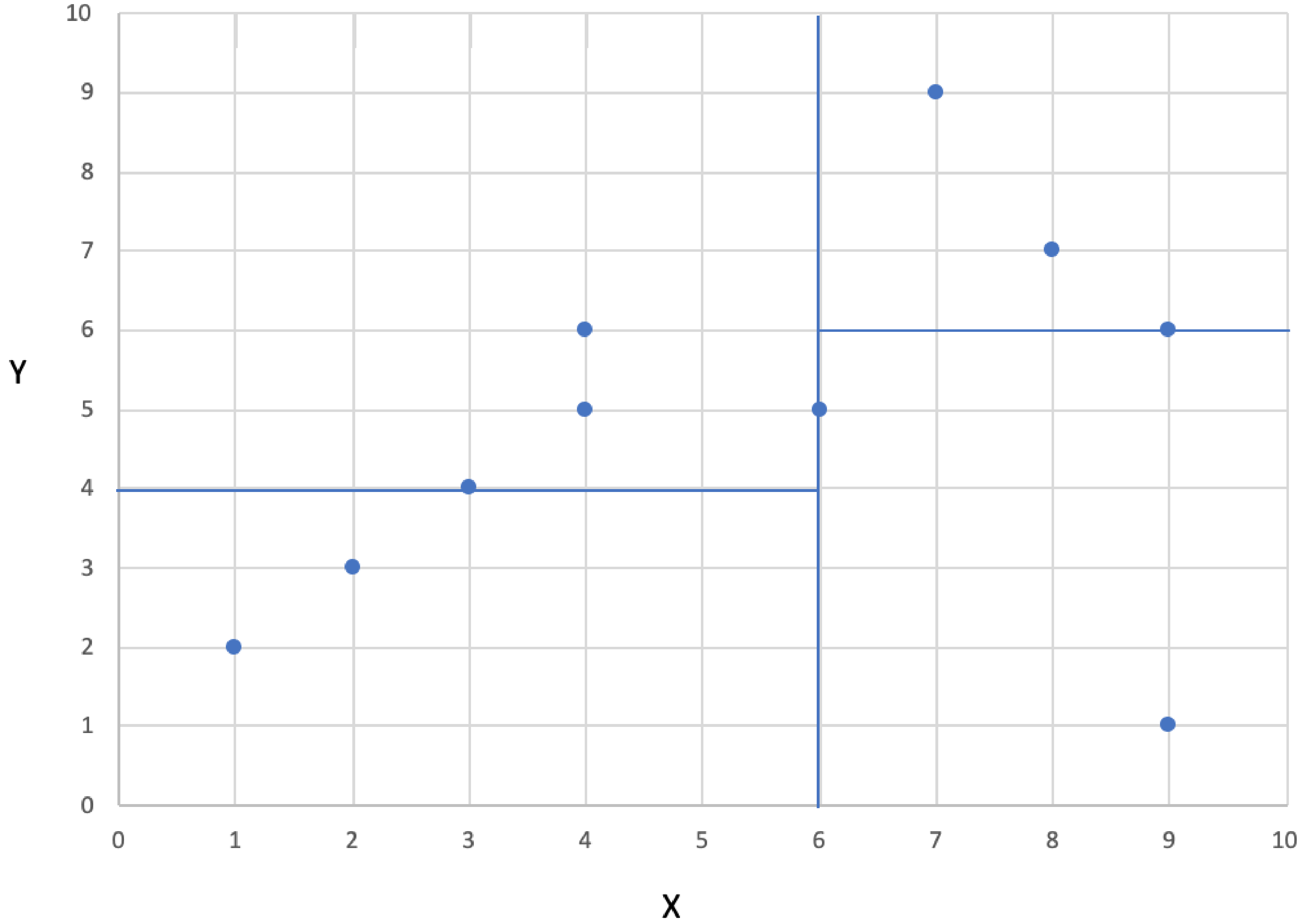


X	Y
9	1
3	4
4	6
6	5
2	3
8	7
7	9
9	6
1	2
4	5

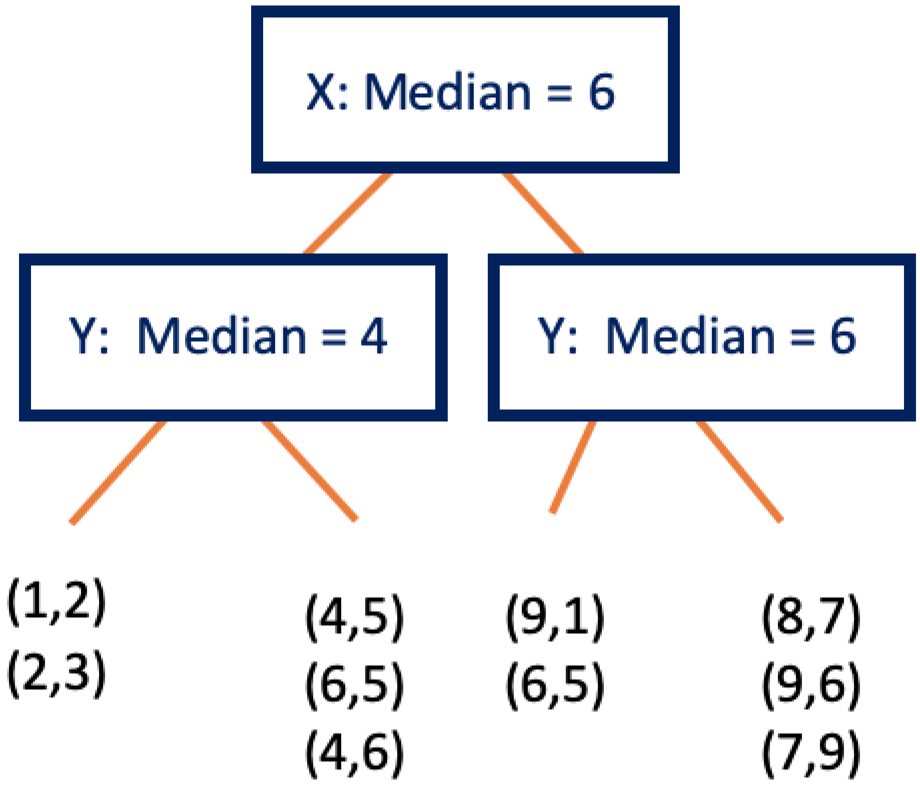
Division at the median along x



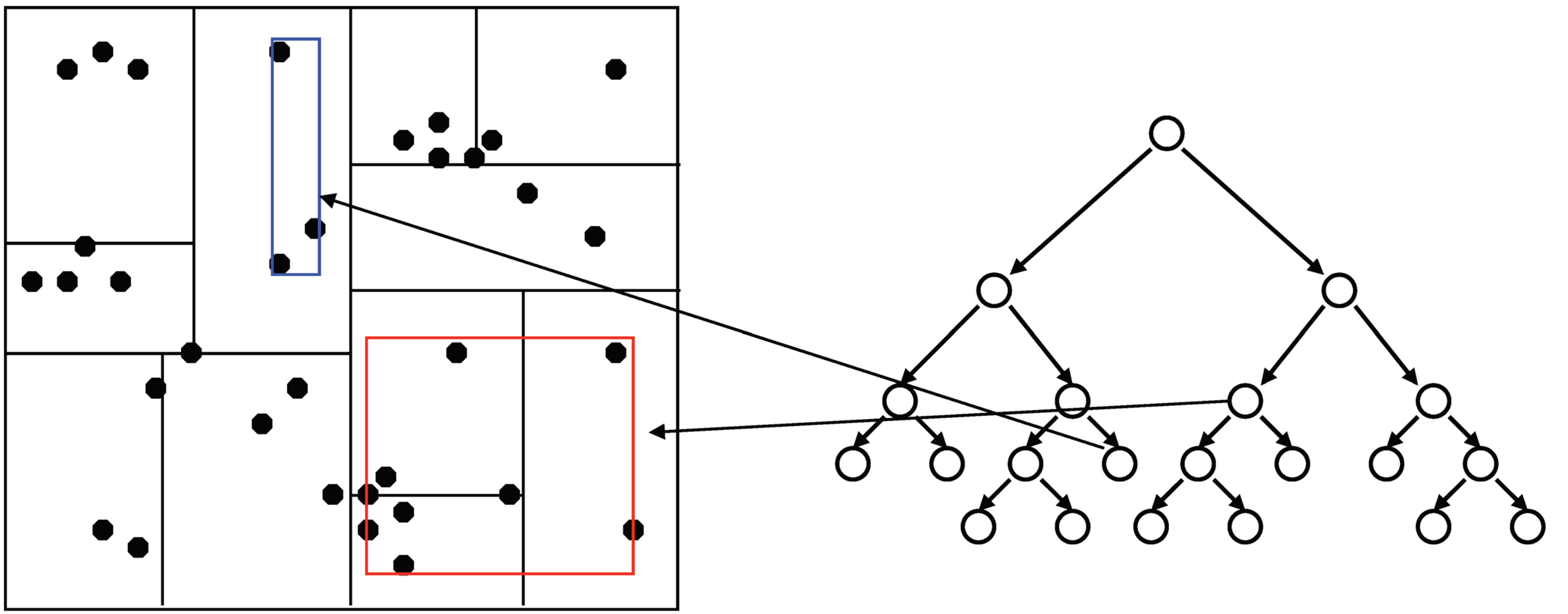
Division along y



Tree structure

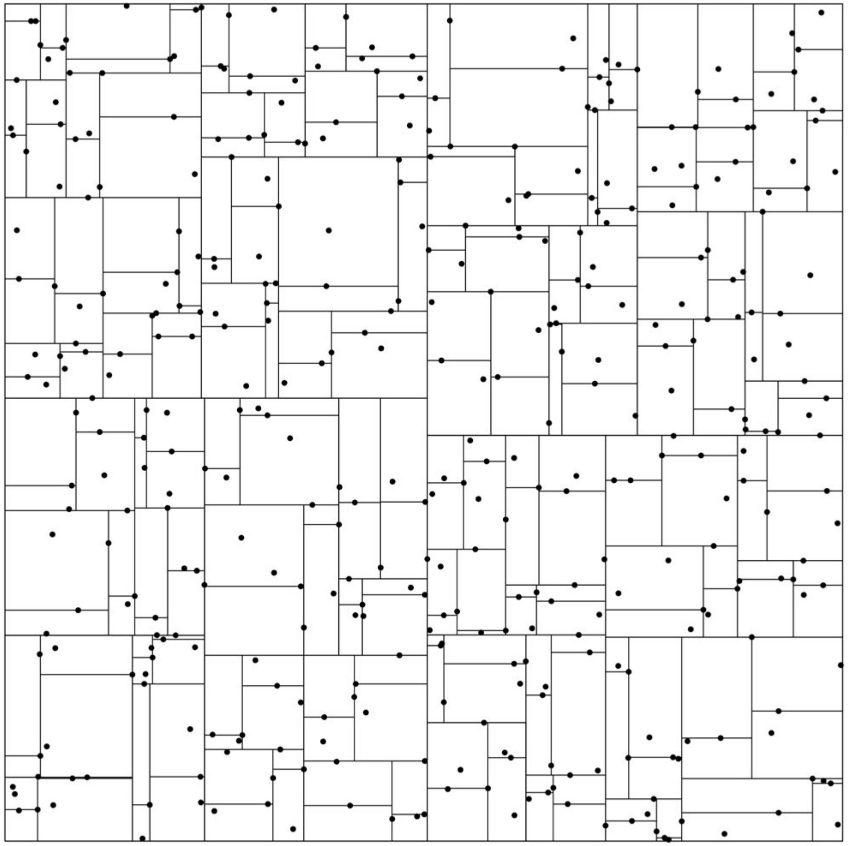


Tree structure



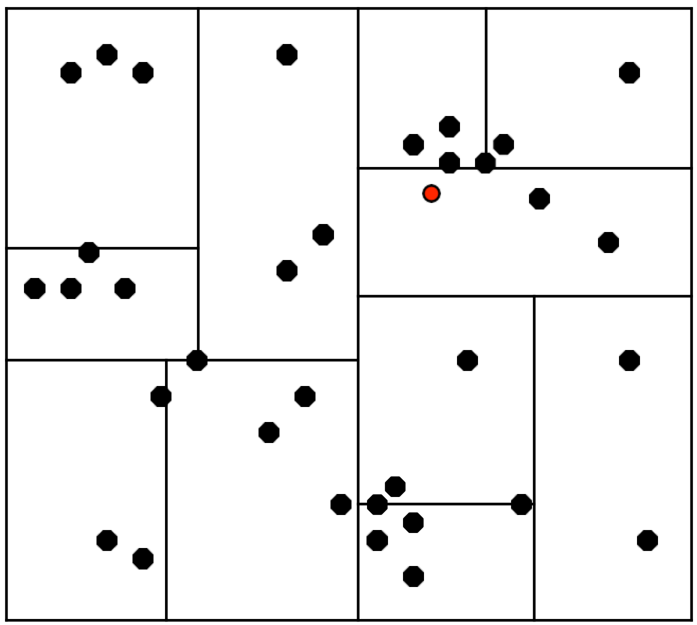
Larger tree

KD- Tree for 100 points



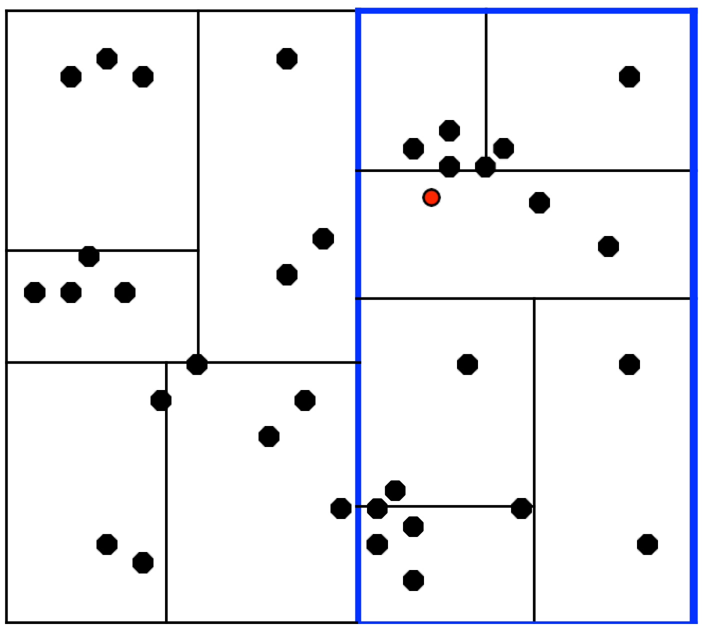
Construction - N log N
Nearest neighbor finding – log N

Finding nearest neighbor to a point



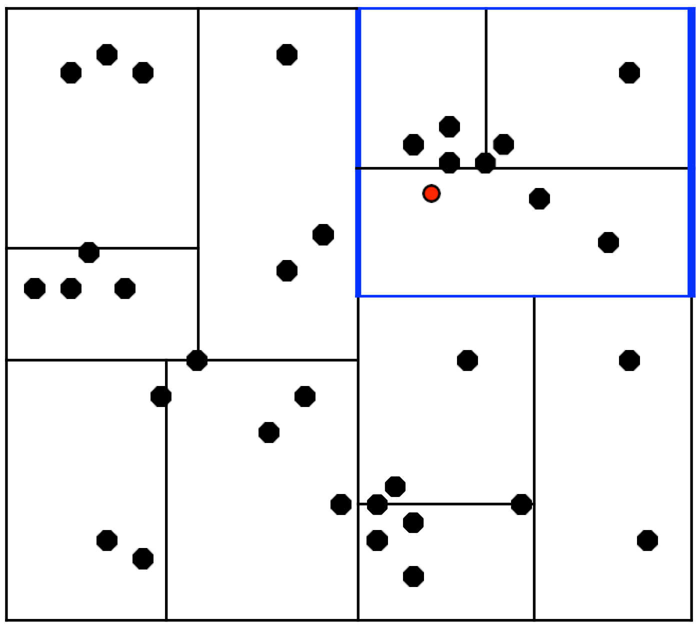
We traverse the tree looking for the nearest neighbor of the query point.

Finding nearest neighbor to a point



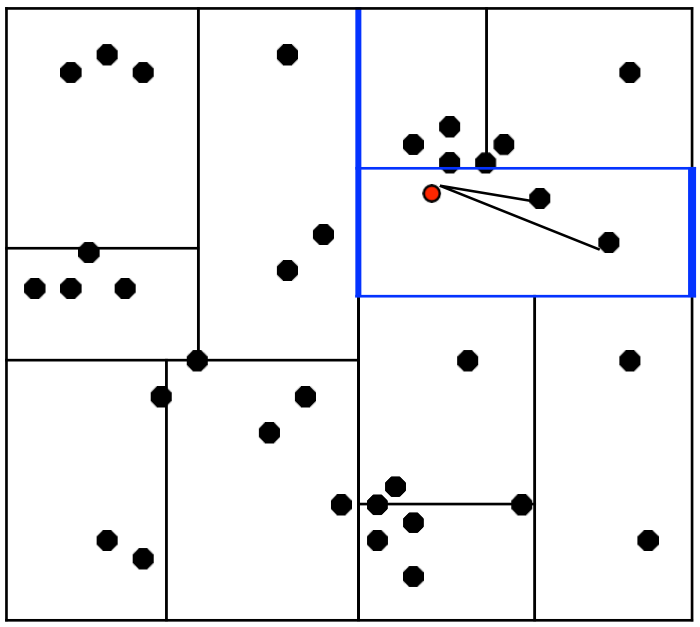
<u>Examine nearby points first</u>: Explore the branch of the tree that is closest to the query point first.

Finding nearest neighbor to a point



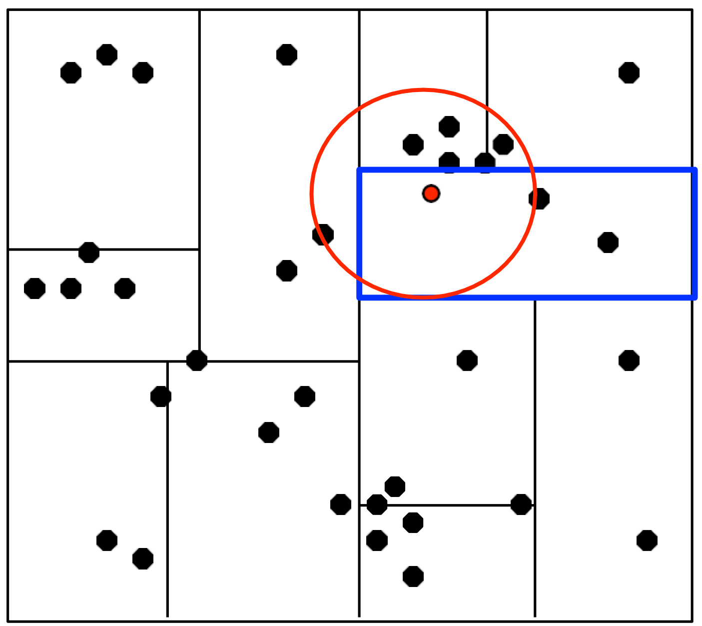
Examine nearby points first: Explore the branch of the tree that is closest to the query point first.

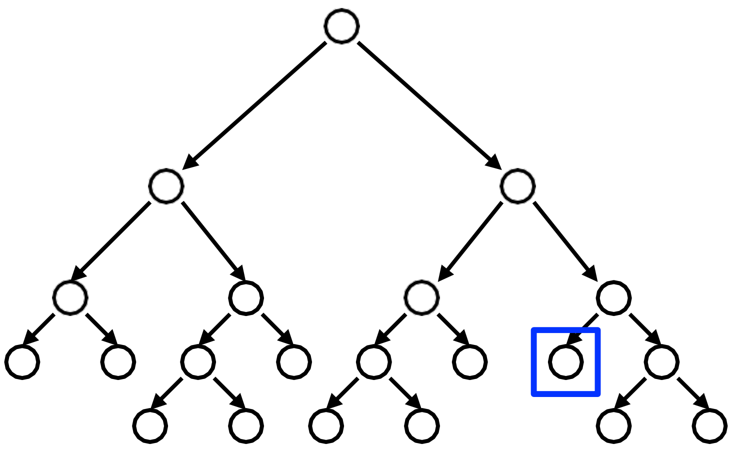
Finding nearest neighbor to a point



When we reach a leaf node: compute the distance to each point in the node.

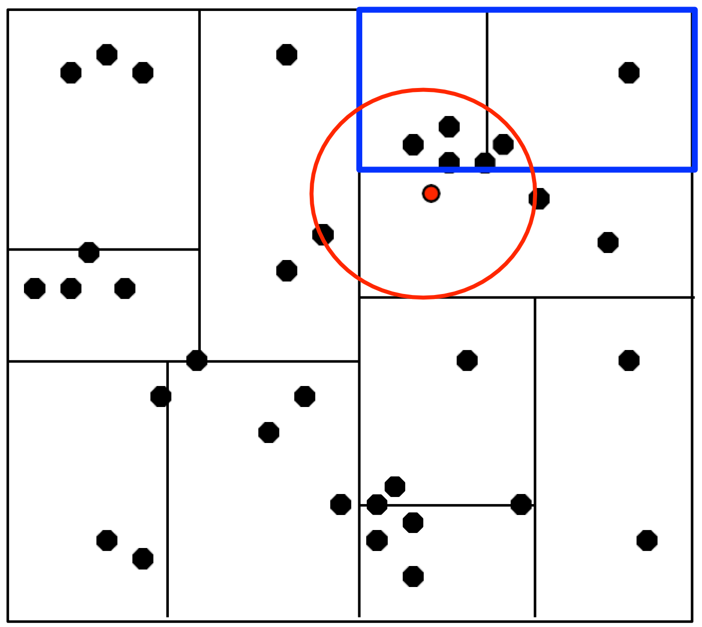
Finding nearest neighbor to a point

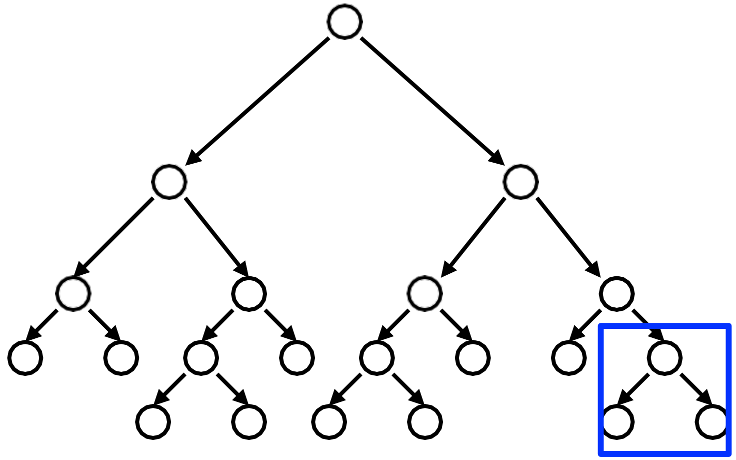




When we reach a leaf node: compute the distance to each point in the node.

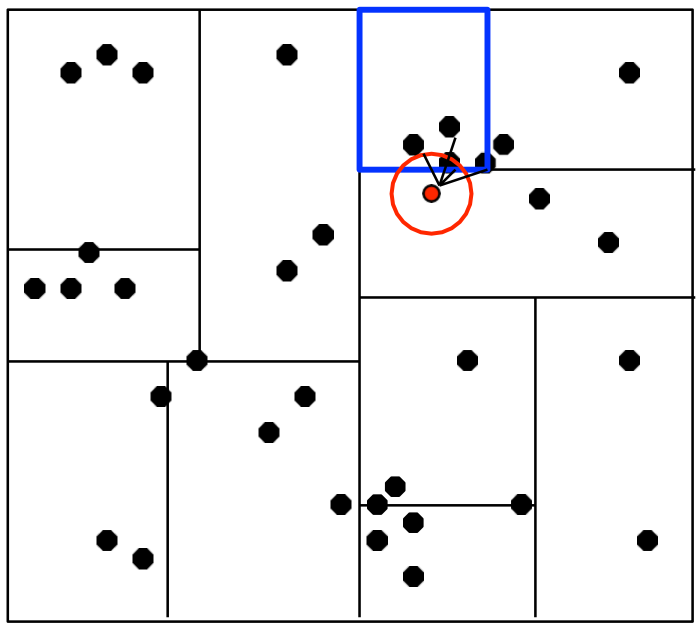
Finding nearest neighbor to a point

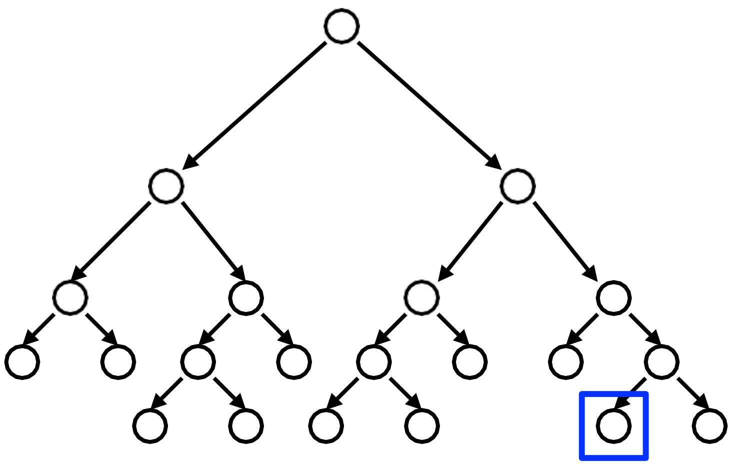




Then we can backtrack and try the other branch at each node visited.

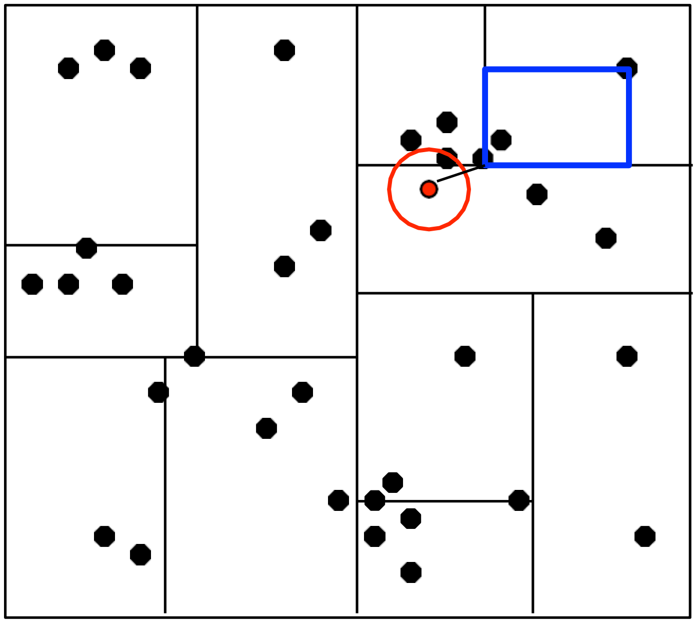
Finding nearest neighbor to a point

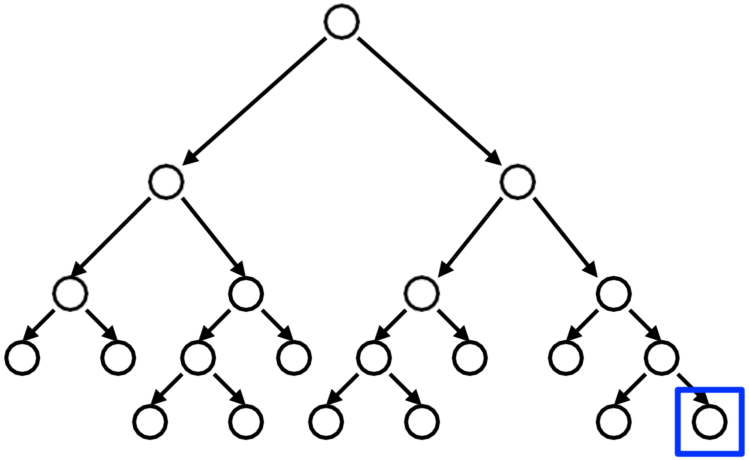




Each time a new closest node is found, we can update the distance bounds.

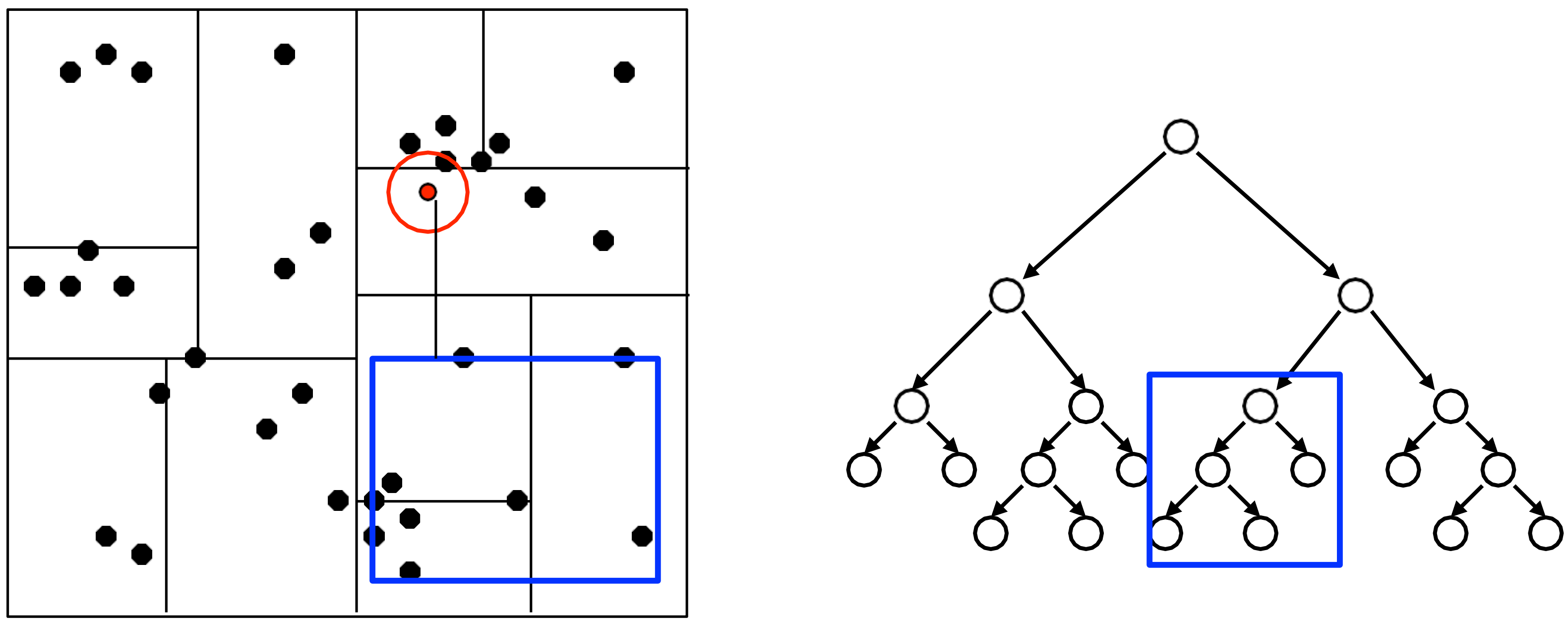
Finding nearest neighbor to a point





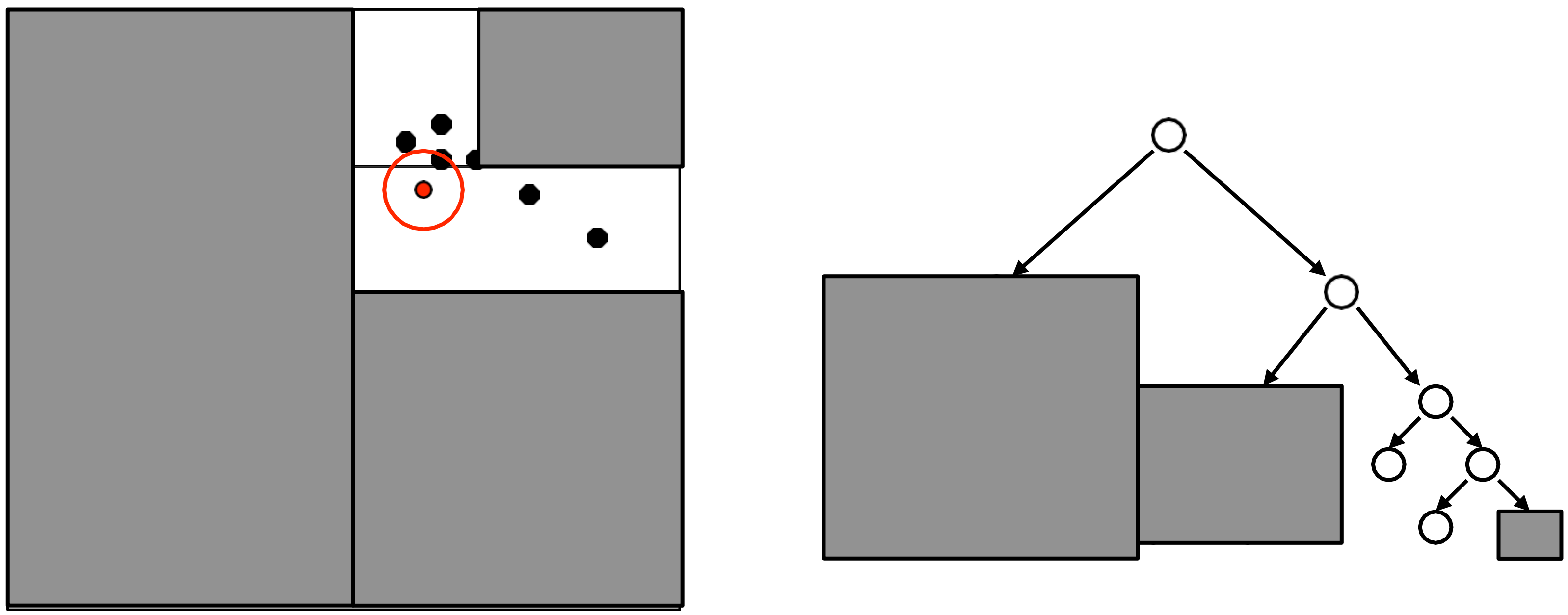
Using the distance bounds and the bounds of the data below each node, we can prune parts of the tree that could NOT include the nearest neighbor.

Finding nearest neighbor to a point



Using the distance bounds and the bounds of the data below each node, we can prune parts of the tree that could NOT include the nearest neighbor.

Finding nearest neighbor to a point



Using the distance bounds and the bounds of the data below each node, we can prune parts of the tree that could NOT include the nearest neighbor.

Birds tree

from scipy.spatial import cKDTree

periodic boundary conditions

birds_tree = cKDTree(positions,boxsize=[L,L])

dist = birds_tree.sparse_distance_matrix(birds_tree,max_distance=r,output_type='coo_matrix')

this produces a (sparse) matrix with distances between birds (if they are smaller than r)

- COO format stores data as a list of tuple with three elements; row, column, value
- the tuple is present only for non-zero elements.

Other methods in cKDTree

bird_tree.query(x, k)	find k nearest neighbours to point x
bird_tree,query_ball_point(x, r)	Find all points within distance r of point(s) x.

- do not use any loops (except over time) - use numpy.sum to sum over the columns/rows of a matrix

Distance matrix

dist = birds_tree.sparse_distance_matrix(birds_tree,max_distance=r,output_type='coo_matrix')

$$
\begin{array}{cc}
 & \begin{array}{cccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} &
\begin{bmatrix}
0.0 & & 2.3 & & & & & & & \\
 & 0.0 & & & 1.7 & & & & 1.85 & \\
2.3 & & 0.0 & & & & & & & \\
 & & & 0.0 & & & 0.66 & & & \\
 & 1.7 & & & 0.0 & & & & & \\
 & & & & & 0.0 & & & & \\
 & & & 0.66 & & & 0.0 & & & \\
 & & & & & & & 0.0 & & \\
 & 1.85 & & & & & & & 0.0 & \\
 & & & & & & & & & 0.0
\end{bmatrix}
\end{array}
$$

Nearest neighbours to bird no. 1 (including self)

Orientation matrix

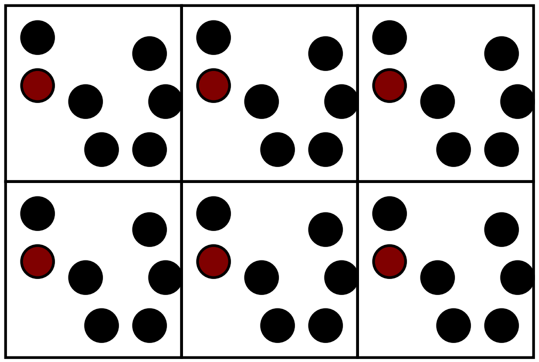
$$
\begin{bmatrix}
0.0 & & 2.3 & & & & & & & \\
& 0.0 & & & 1.7 & & & & 1.85 & \\
2.3 & & 0.0 & & & & & & & \\
& & & 0.0 & & & 0.66 & & & \\
& 1.7 & & & 0.0 & & & & & \\
& & & & & 0.0 & & & & \\
& & & 0.66 & & & 0.0 & & & \\
& & & & & & & 0.0 & & \\
& 1.85 & & & & & & & 0.0 & \\
& & & & & & & & & 0.0
\end{bmatrix}
$$

$$
\begin{bmatrix}
\exp(i\theta_0) & & \exp(\theta_2) & & & & & & & \\
& \exp(i\theta_1) & & & \exp(i\theta_4) & & & & \exp(i\theta_8) & \\
\exp(i\theta_0) & & \exp(i\theta_2) & & & & & & & \\
& & & \exp(i\theta_3) & & & \exp(i\theta_6) & & & \\
& \exp(i\theta_1) & & & \exp(i\theta_4) & & & & & \\
& & & & & \exp(i\theta_5) & & & & \\
& & & \exp(i\theta_3) & & & \exp(i\theta_6) & & & \\
& & & & & & & \exp(i\theta_7) & & \\
& \exp(i\theta_1) & & & & & & & \exp(i\theta_8) & \\
& & & & & & & & & \exp(i\theta_9)
\end{bmatrix}
$$

Exactly what we need to update orientation of bird no. 1

- A new sparse matrix with the same positions of nonzero elements, but containing orientations

Periodic boundary conditions

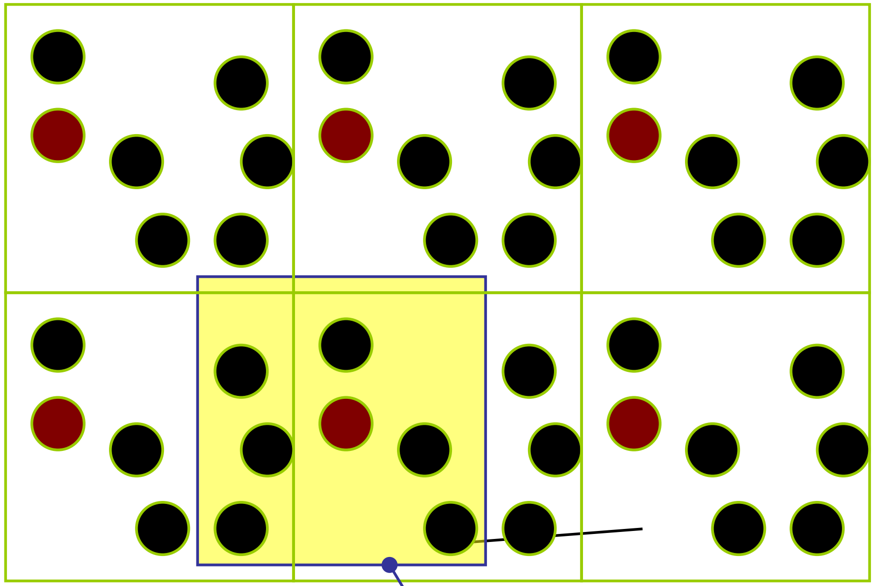


```
if newX > L:
    newX -= L
if newX < 0:
    newX += L

if newY >L:
    newY -= L
if newY < 0:
    newY += L
```

size of the box

Closest periodic image



The images of other particles closest to the red one

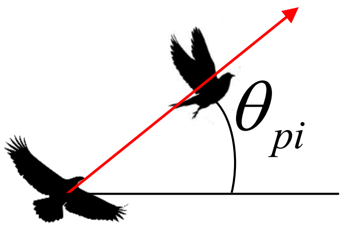
vector joining particle (i) and the closest periodic image of (j)

```
dr=np.remainder(ri - rj + L/2., L) - L/2
```

Birds

- all fly with the same velocity v_0
- their initial distribution is uniform (both in positions and in orientations)

Bird of prey



- moves with the same velocity v_0 as the birds

- follows the closest bird

$$\theta_p = \theta_{pi} + a\eta_p$$

- all the birds within a range of r_b fly away from it, ignoring other birds:

$$\theta_i = \theta_{pi} + a\eta_i$$

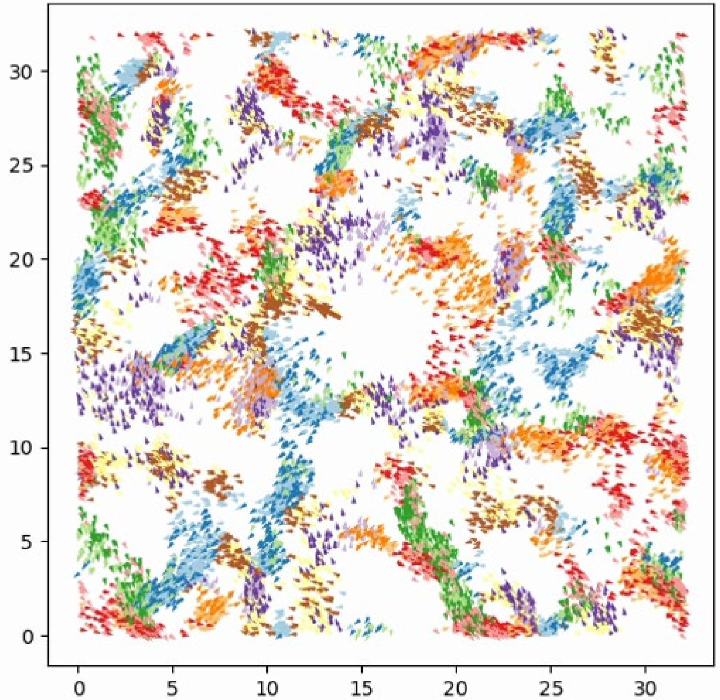
Predator interaction radius

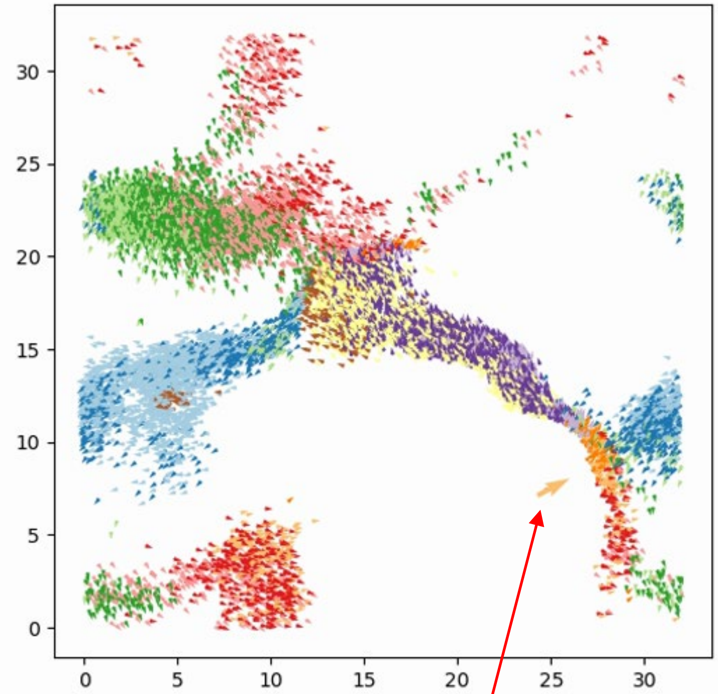
Visualization

matplotlib.pyplot.quiver([X, Y], U, V, [C])

- plots a 2D field of arrows
- X, Y define the arrow locations, U, V define the arrow directions, and C optionally sets the color

use θ here





(here colors are related to orientations which helps in identifying clusters)

bird of prey

The details

- take e.g.:

  $L=32$, $N=5000\text{-}10000$, $r=1$, $r_b=4$, $v_0=2$, $a=0.15$

- make a movie of several hundred frames
- for task one (w/o bird of prey) check if the system self-organizes after sufficiently long time (all birds flying in the same direction)
- how does the presence of the bird of prey impacts such a self-organization process?

Extra task

- look at the phase transition at the intensity of a noise is changed between 0 and 1

order parameter

$$\chi = \frac{1}{N v_0} \left| \sum_i \vec{v}_i \right|$$

- plot and analyze $\chi(a)$


It's more than speculation-
It's a prediction!
THE SWARM