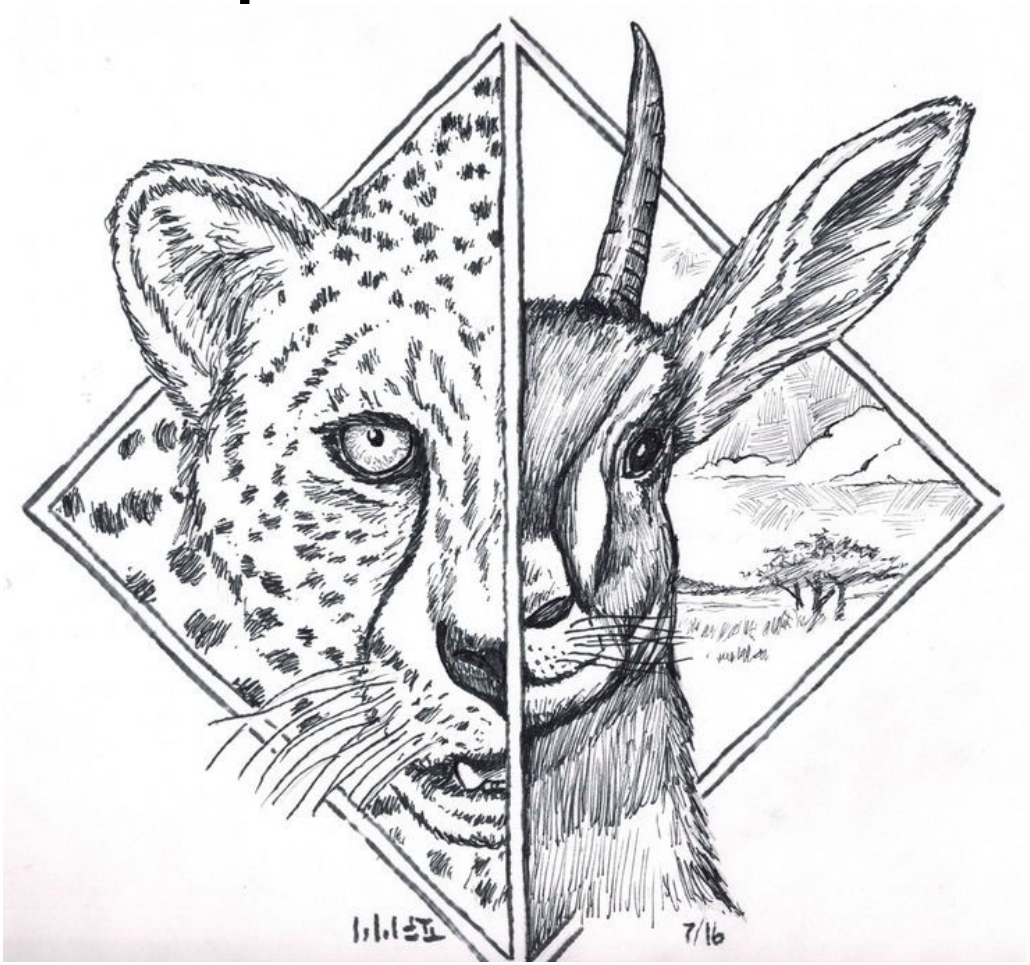
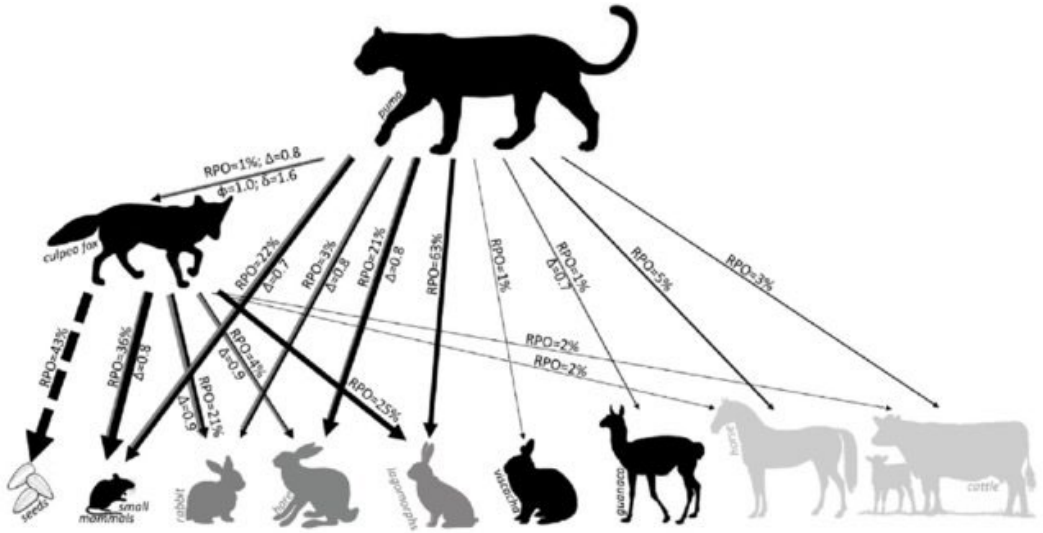
Computer modeling of physical phenomena



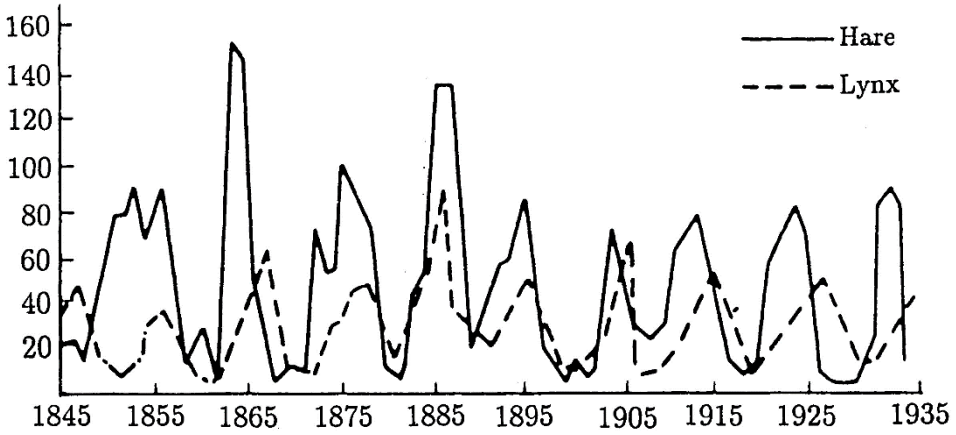
Predator-prey models

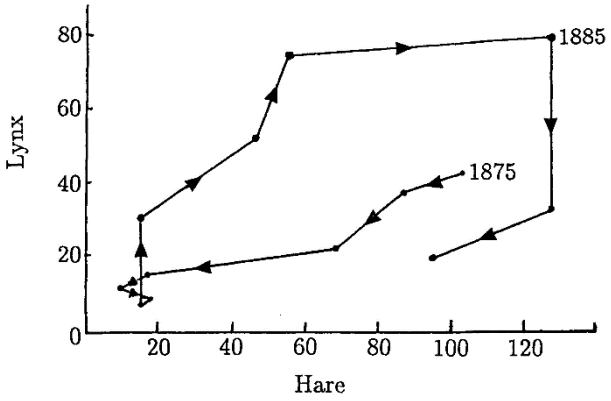
Predators and prey

- Predation is a one of the basic interspecies population interactions
- One species uses another as a food resource
- Predators play an important role in controlling prey population
- In simple systems, the predator-prey relationship results in coupled population oscillations



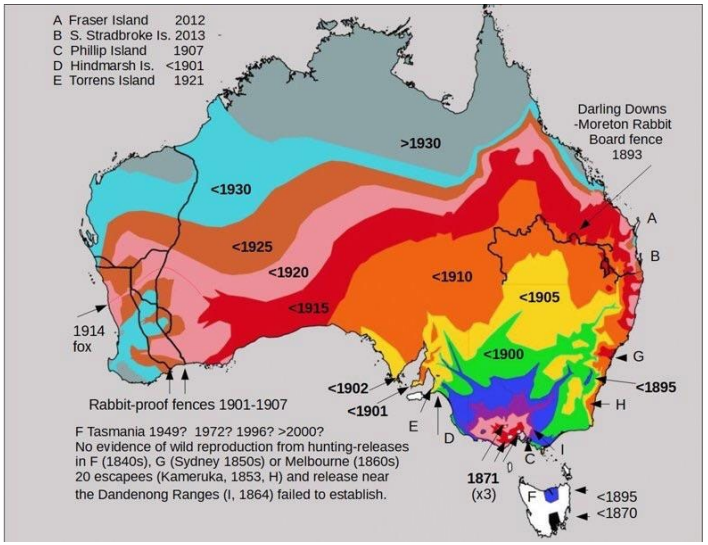
Hare vs lynx data





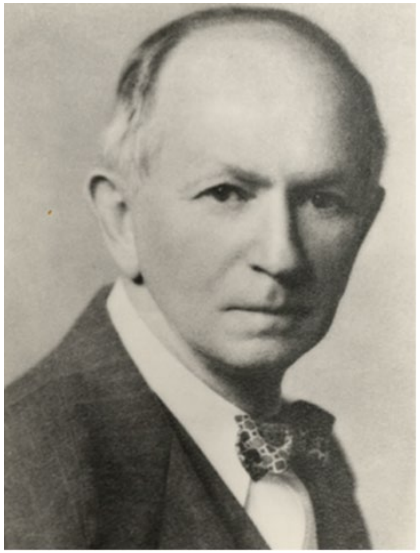
Data based on Hudson Bay Company's skin purchases between 1845 and 1930

No predators: rabbits in Australia…



Lotka-Volterra model

- Alfred Lotka

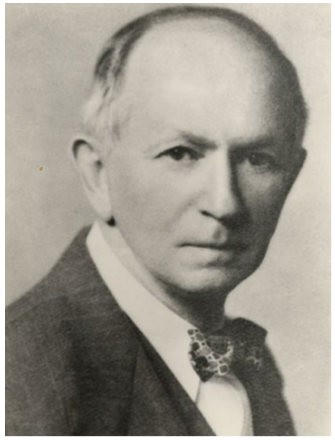


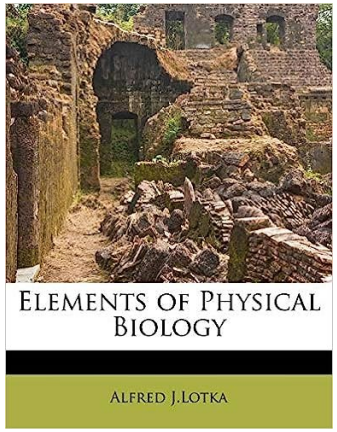
- American mathematician and physical chemist
- proposed the predator-prey model in 1920

- Vito Volterra

- Italian mathematician
- proposed the predator-prey model in 1926

Lotka: autocatalytic reactions





Alfred Lotka:

- Born in Lwów (1880) - the son of American expatriate missionaries
- From 1902 in USA
- Proposed a model for autocatalytic, periodic reactions:

  Contribution to the Theory of Periodic Reactions, J. Phys. Chem, 14, 271 (1910)

- Used these equations to analyze the predator-prey dynamics in his book *Elements of Physical Biology*

Lotka is also known for his work on evolution, demography, publick health, and scientific productivity (Lotka's law: *number of authors writing n papers is $1/n^2$ of the number of authors writing one paper*)

Volterra: Family bussiness

Luisa D'Ancona, Silvia D'Ancona and Umberto D'Ancona

Lotka-Volterra model

prey

$$\frac{dx}{dt} = \alpha x - \beta xy$$

births

deaths

predators

$$\frac{dy}{dt} = \delta xy - \gamma y$$

births (food-dependent)

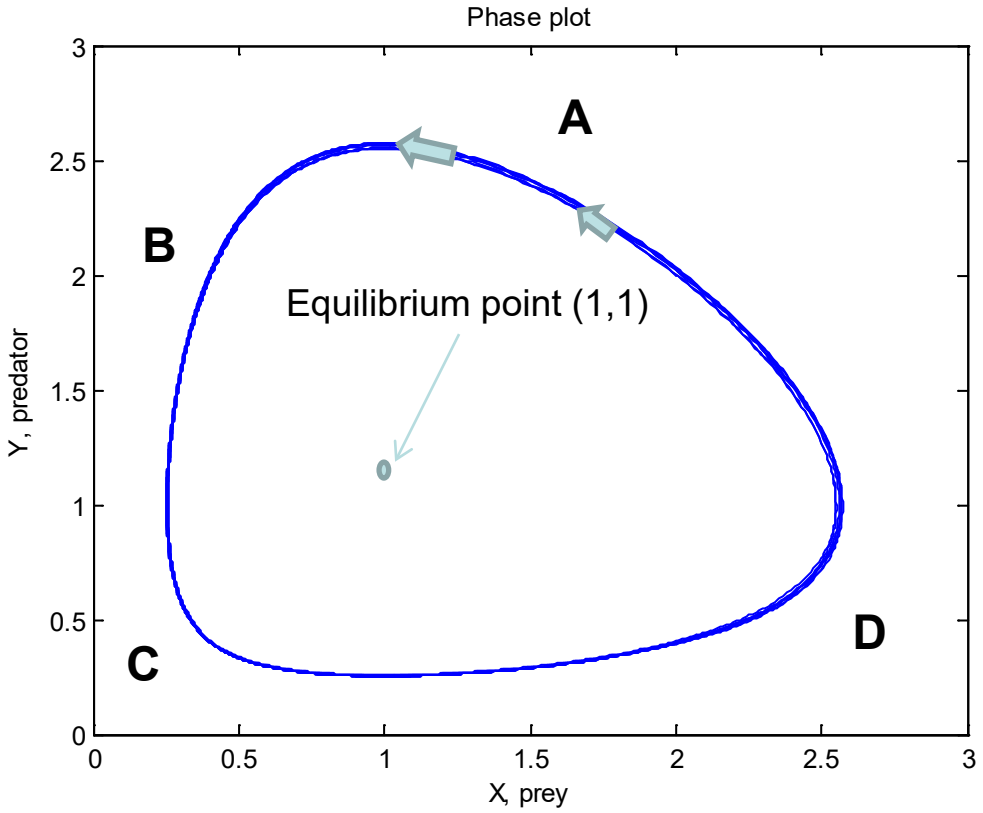
deaths

Assumptions

- The prey is sufficiently fed
- Predator preys exclusively on the prey,
- rate of change of population is proportional to its size
- environment does not change in favor of any species
- genetic adaptation is sufficiently slow
- predator have limitless appetite

Phase trajectories

- Equilibrium Point:
  x=(c/d), y= (a/b)
- Counter-clockwise motion



A = Too many predators.
B = Too few prey.
C = Few predator and prey; prey grows
D= Few predators, ample prey.

x(t=0)=y(t=0)=2

a=b=c=d=1

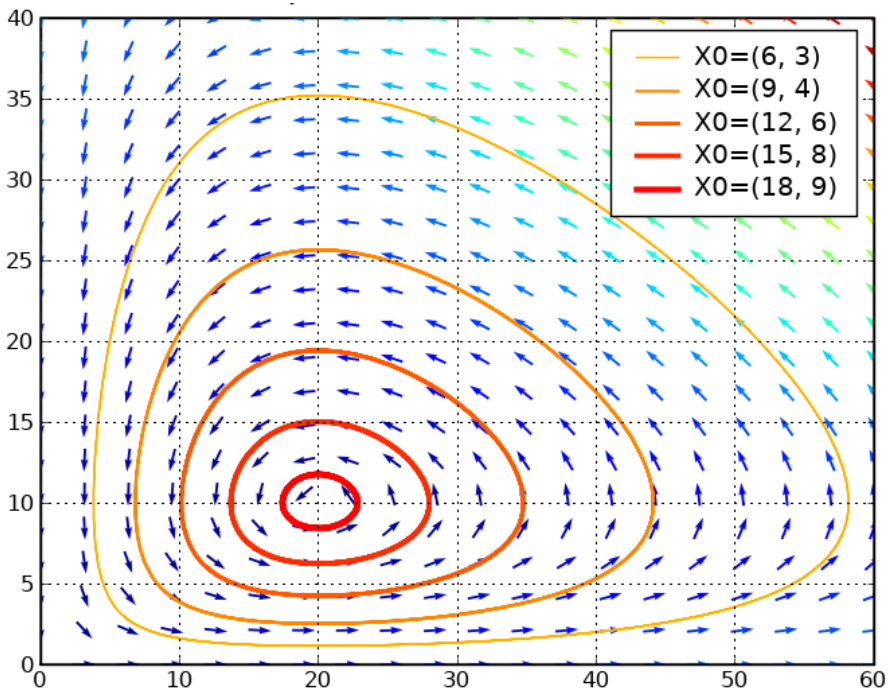
Phase trajectories

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \delta xy - \gamma y$$

phase trajectory

$$\frac{dy}{dx} = -\frac{y}{x}\frac{\delta x - \gamma}{\beta y - \alpha}$$

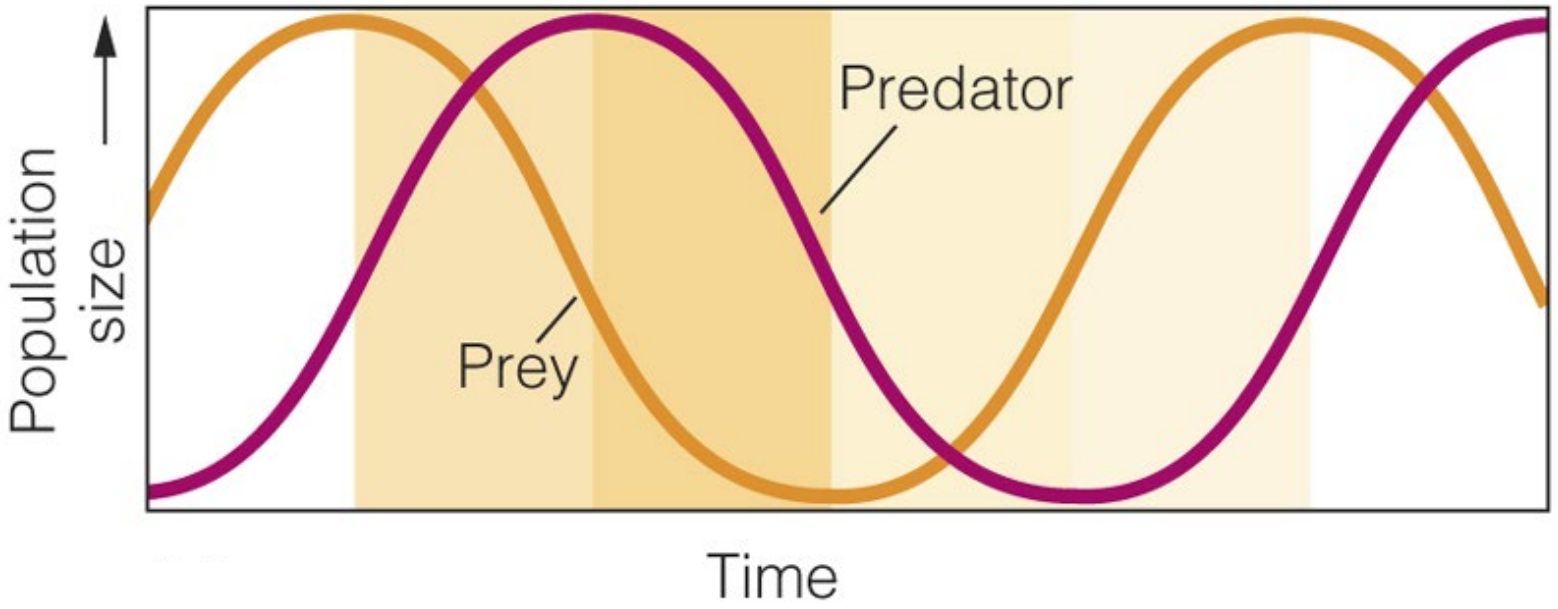


conserved quantity $V = \delta x - \gamma \log x + \beta y - \alpha \log y$

x(t=0)=y(t=0)=2

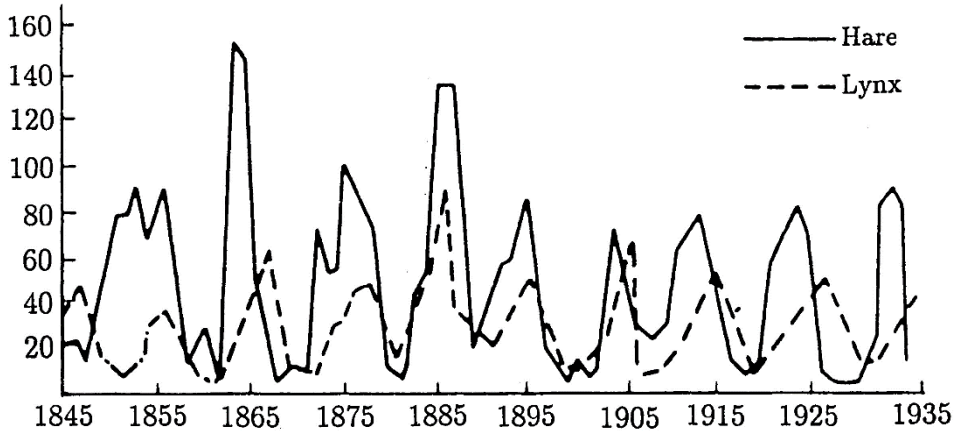
a=b=c=d=1

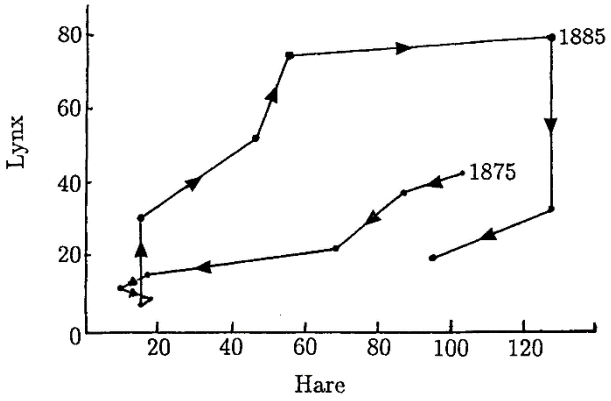
Population size evolution



Characteristic phase shift between predators and prey

Hare vs lynx again



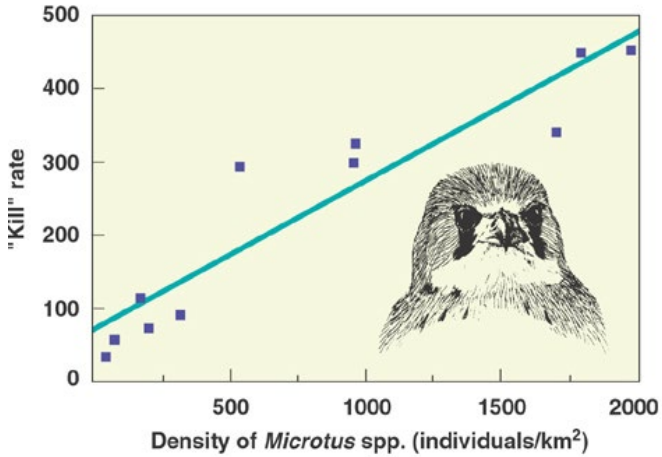


Data based on Hudson Bay Company's skin purchases between 1845 and 1930

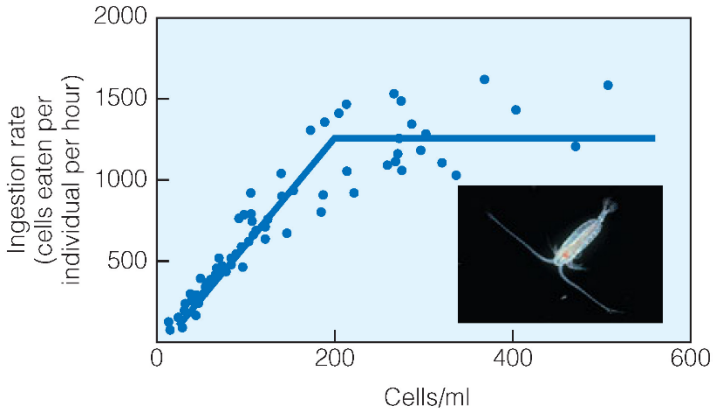
Problems of Lotka-Volterra

- No prey self limitation
- No predator self limitation
- No limit on prey consumption per predator
- solution of the Lotka-Volterra equations depends on the initial conditions

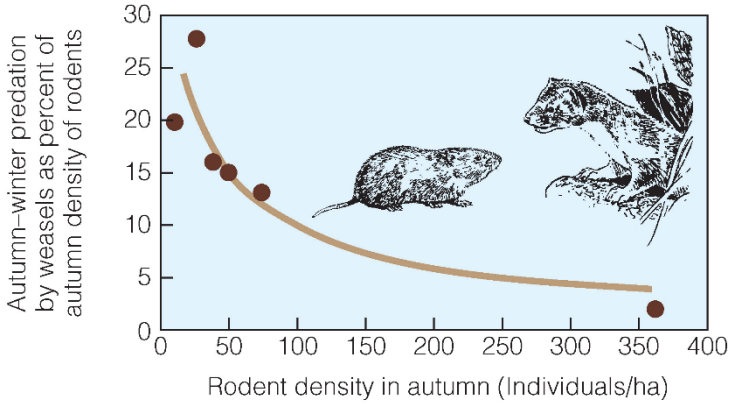
Predator-prey relationships



shrikes feeding on microtus

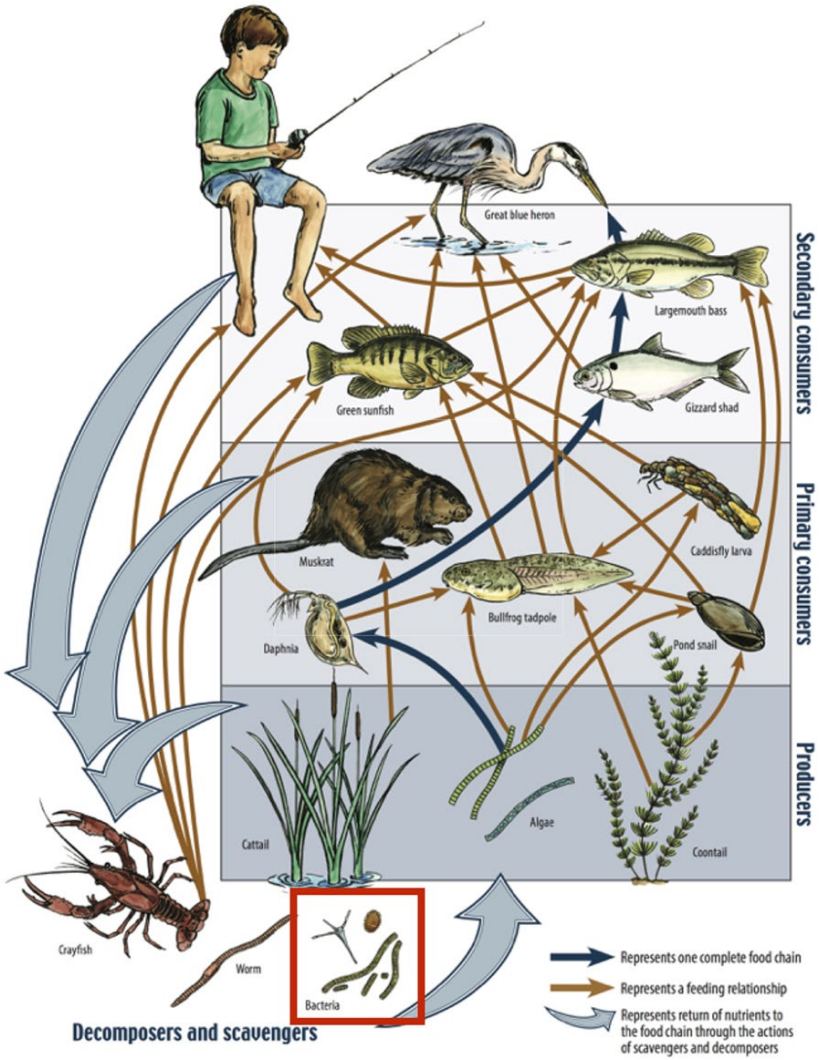


copepod feeding on plankton



weasels preying on rodents (Bialowieza)

Food webs


Great blue heron
Largemouth bass
Green sunfish
Gizzard shad
Muskrat
Caddisfly larva
Bullfrog tadpole
Daphnia
Pond snail
Cattail
Algae
Coontail
Crayfish
Worm
Bacteria
Decomposers and scavengers
Secondary consumers
Primary consumers
Producers
Represents one complete food chain
Represents a feeding relationship
Represents return of nutrients to the food chain through the actions of scavengers and decomposers