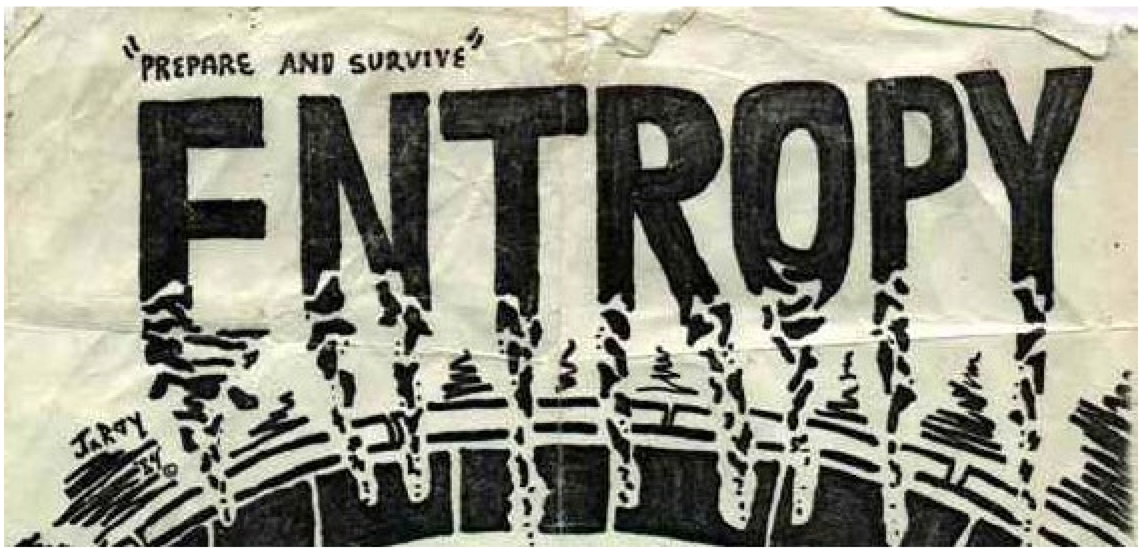
Computer modeling of physical phenomena



Lab I – Entropy of Cellular Automata

Task 1: plotting 1d CA

- write a general code to simulate the evolution of any 1d cellular automaton
- the code should read the rule number (0-255), based on this construct the transition rules and track the evolution
- for the initial conditions take either a random string of length L of 0s and 1s or a string of k 1s surrounded by L-k 0s
- assume periodic boundary conditions, i.e. cell L-1 is the neighbour of cell 0
- the code should work both for standard 1d rules as well as for the reversible ones
- play with the code and explore different rules and resulting patterns

Task 2: microscopic entropy

Microscopic (Gibbs/Shannon) Entropy:

$$S = -\sum_{i=1}^{2^L} p_i \log_2 p_i$$

← probability of occurence of a given state

↖ sum over the ensemble of all different initial conditions

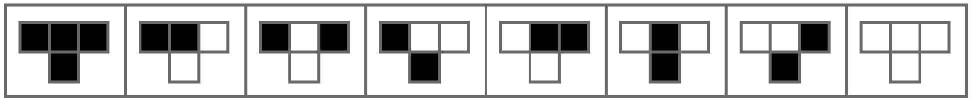
- calculate and plot S(t) for automaton 110 on 12 cells with all possible initial conditions
- calculate and plot S(t) for automaton 122R on 12 cells with all possible initial conditions
- compare the two graphs, comment on the differences

counting occurences in a list l:

```
from collections import Counter
Counter(l)
```

Some hints

- Use binary encoding!



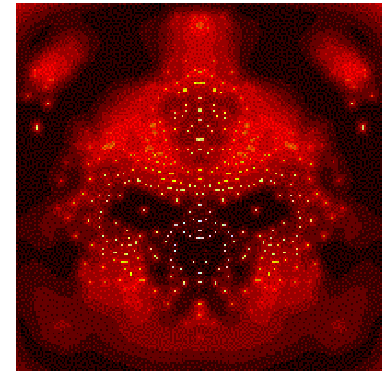
111 110 101 100 011 010 001 000

1 0 0 1 0 1 1 0 ← transform rule number into bits

- periodic boundary conditions – use modulo function (*j%width*)
- fast plotting:

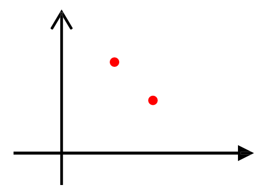
```
from PIL import Image, ImageDraw
img = Image.new("RGB",(width,height),(255,255,255))
draw = ImageDraw.Draw(img)
for y in range(height):
        for x in range(width):
                if data[y][x]: draw.point((x,y),(0,0,0))
img.save(fname,"PNG")
```

Gibbs vs Boltzmann



Gibbs entropy (microscopic, for demons only)

$$S_G = -\sum_i p_i \log p_i$$

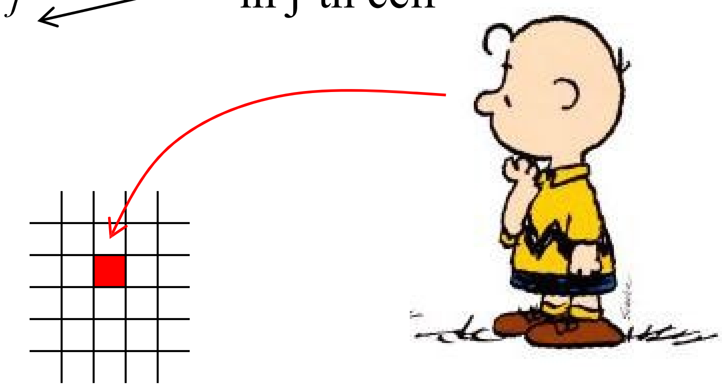


Boltzmann (macroscopic, for ordinary people)

$$S(t) = \sum_j \log N^j$$

no. of microstates consistent with the macrostate observed in j-th cell

sum over the finite-size compartments in the phase space



Task 3: Coarse-grained entropy

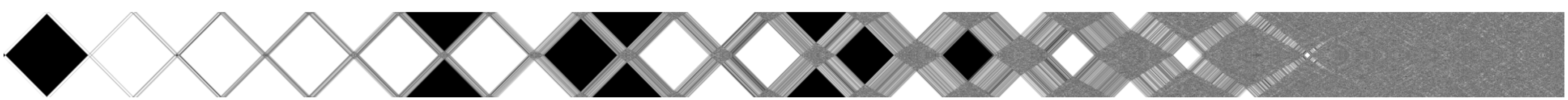
- find the evolution on a line of N=500 cells with a given initial condition (consisting of two lines)

  Example parameters:

  width=500, length=15000,

  Initial condition:

  empty (white) cells + n black cells in the center with e.g. n=41 (two identical rows)



Arrow of time? Entropy growth?

Coarse grained entropy

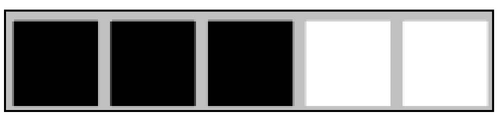
$$S(t) = \frac{1}{T}\sum_{i=0}^{T-1}\sum_{j}\log N_{m,k}^{j}(t+i)$$

average over time

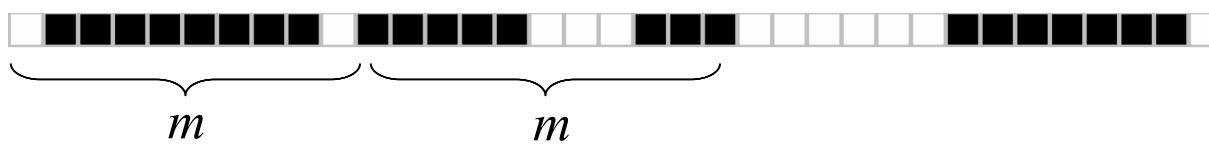
sum over groups of cells

number of microscopic configurations corresponding to the observation of k black cells in jth group of cells (each of length m)

$$N_{m,k} = \binom{m}{k} = \frac{m!}{k!(m-k)!}$$



e.g. m=5, k=3, $N_{5,3} = \binom{5}{3} = 10$



Find the evolution of S(t) for your trajectory. Take e.g. m=5, T=300