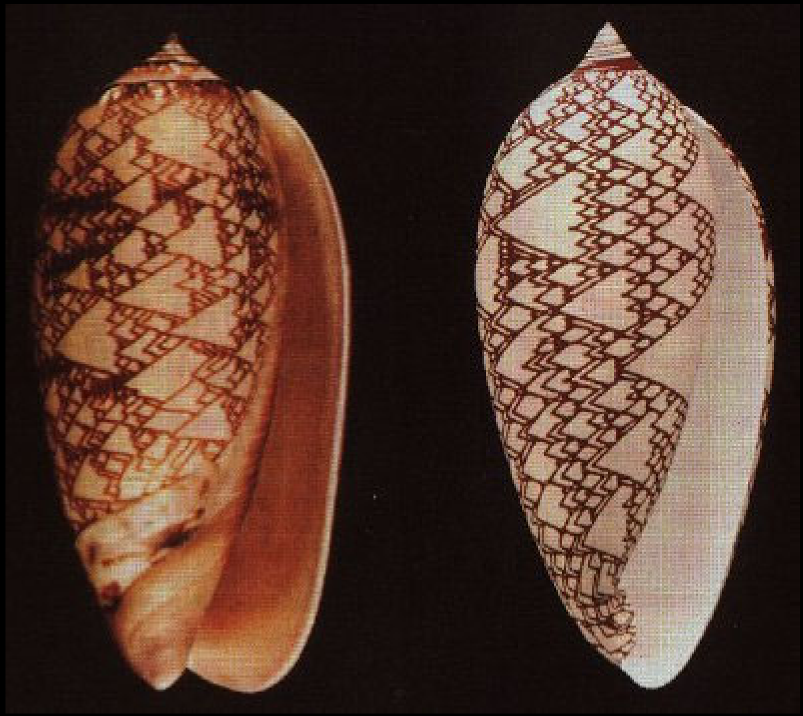
Computer modeling of physical phenomena



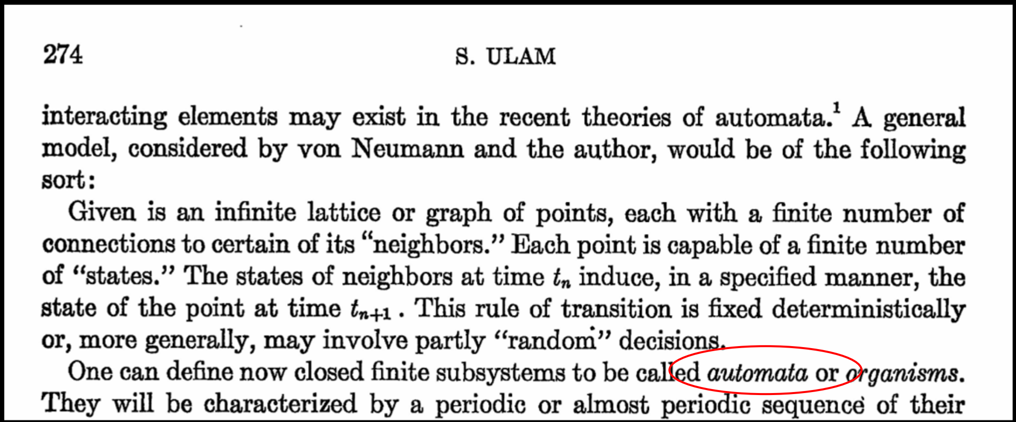
Lecture I CA

Cellular automata

History

'The general and logical theory of automata', 1948

(explored the idea of constructing self-replicating automata)



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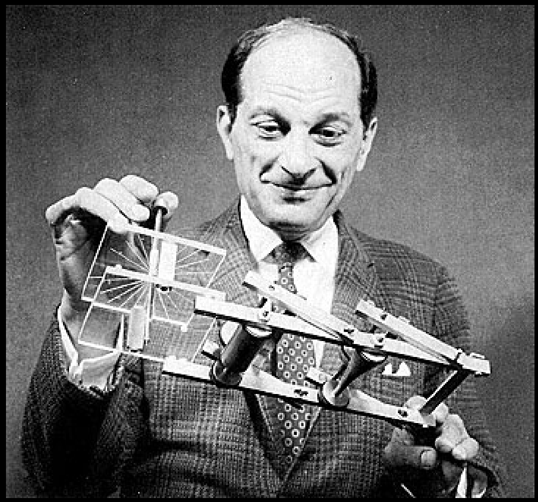
interacting elements may exist in the recent theories of automata.[1] A general model, considered by von Neumann and the author, would be of the following sort:

Given is an infinite lattice or graph of points, each with a finite number of connections to certain of its "neighbors." Each point is capable of a finite number of "states." The states of neighbors at time t_n induce, in a specified manner, the state of the point at time t_{n+1}. This rule of transition is fixed deterministically or, more generally, may involve partly "random" decisions.

One can define now closed finite subsystems to be called *automata* or *organisms*. They will be characterized by a periodic or almost periodic sequence of their

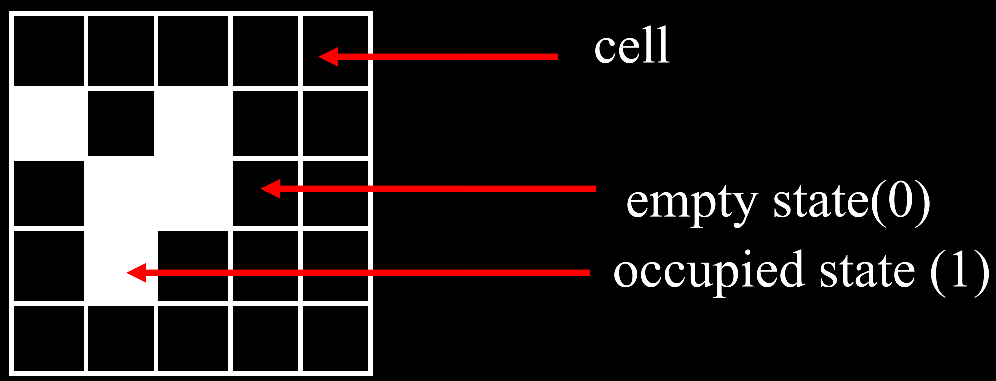


Stanisław Ulam, 1909-1984

At the end of 40ties in Los Alamos studied the lattice models of crystal growth

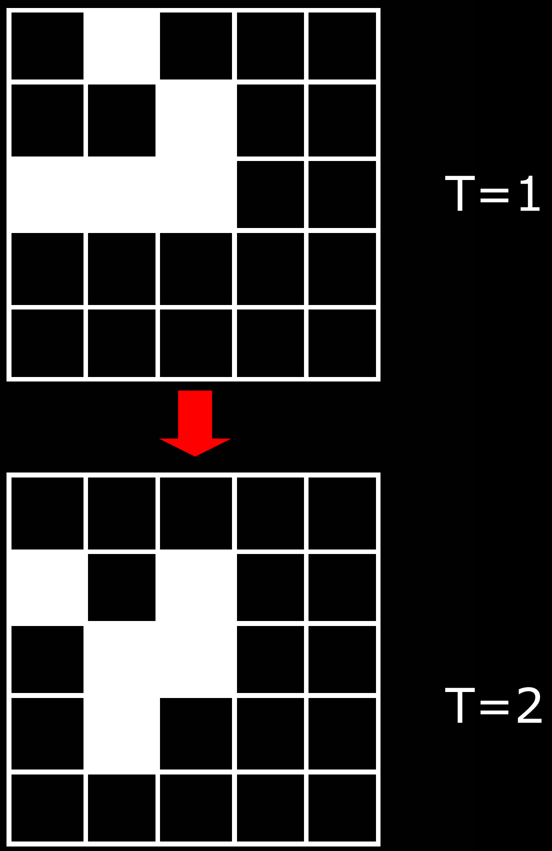
Ulam, S. M., Random processes and transformations, Proc. Int. Cong. Math. , Cambridge, MA , 1950, Vol, 2, 264-275

Cellular automaton



Automaton is defined on a grid of cells, each of which can attain a discrete number of states

Evolution

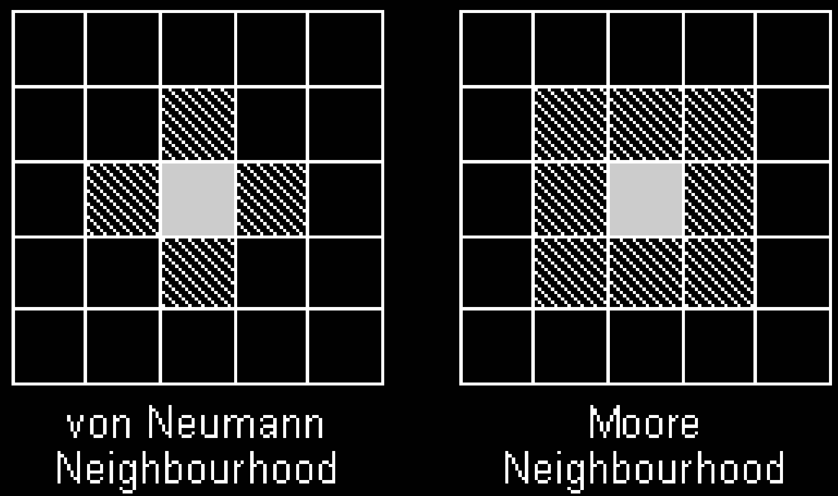


Cellular automaton evolves according to the *transition rules*, which determine the future state of the cell based on the present state of both the cell and its neighbourhood.

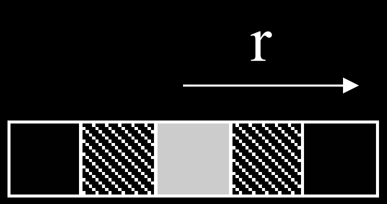
Neighbourhood

- determines the level of coupling between the cells on the grid

- two most popular neighbourhoods (1n 2d):



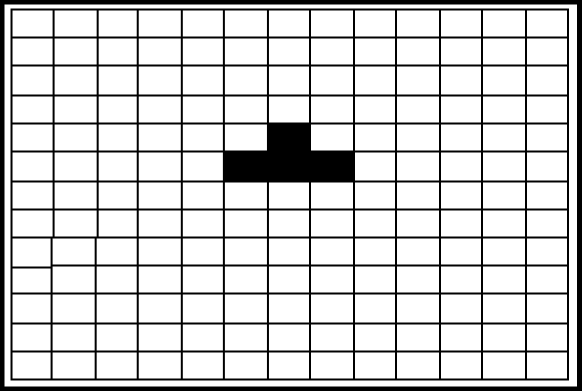
- in one dimension:



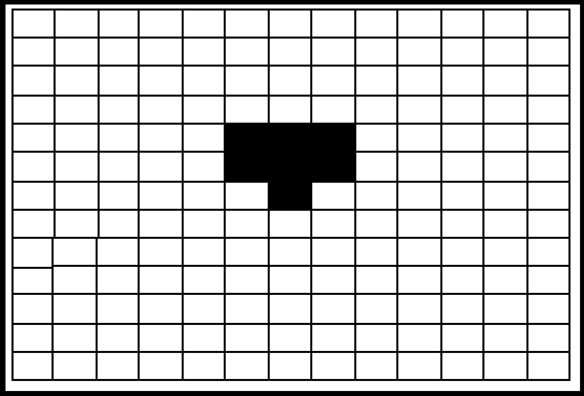
Example (Game of life)

Let us take the following transition rules :

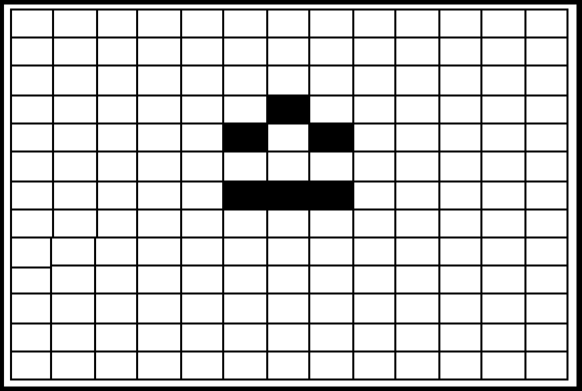
- the cell remains in state 1 (black), if it has 2 or 3 black neighbours (Moore neighbourhood)
- the cell gets transformed to black, if it has exactly 3 black neighbours (birth)
- in other cases the cell remains (or gets transformed to) white



initial state

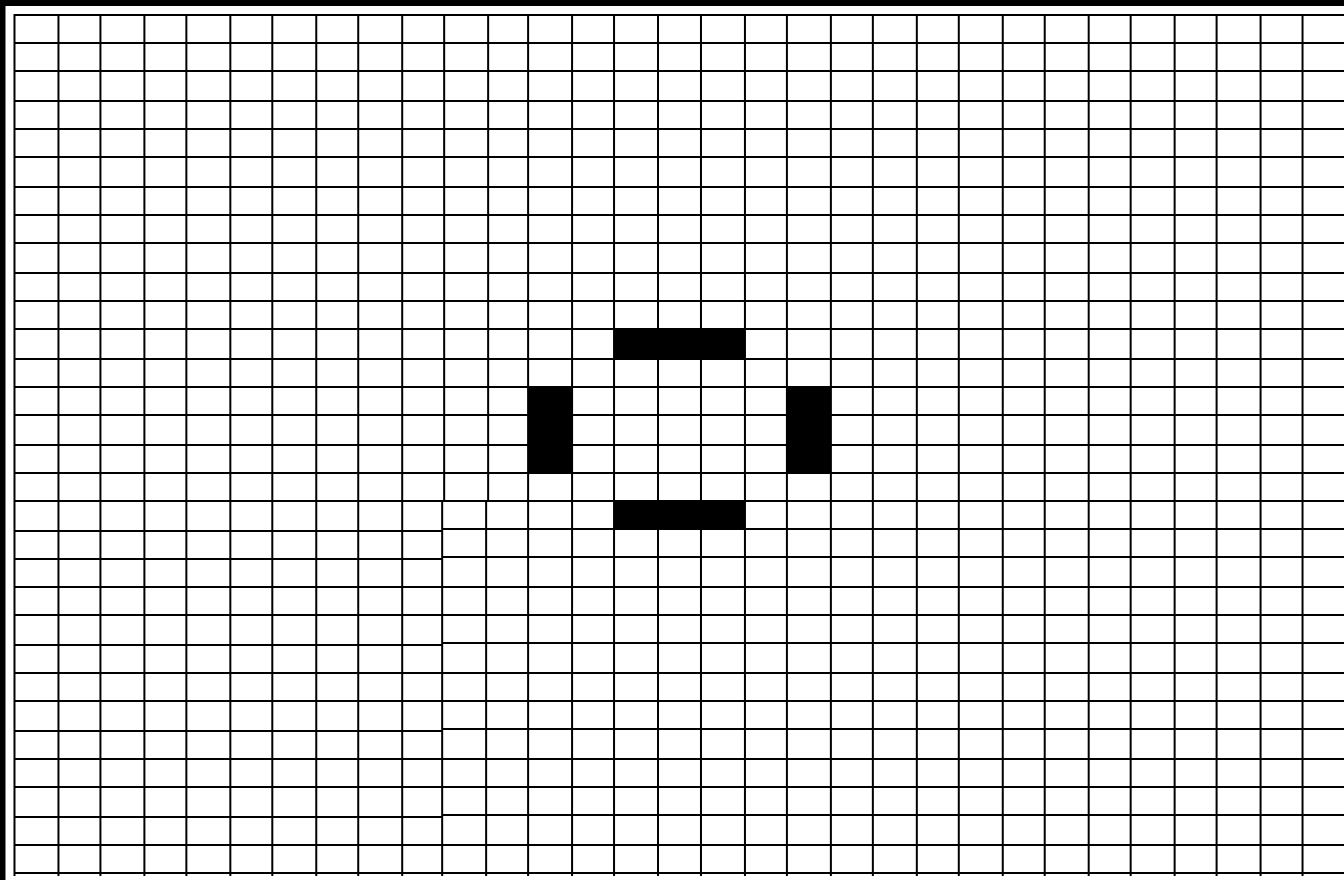


step 1

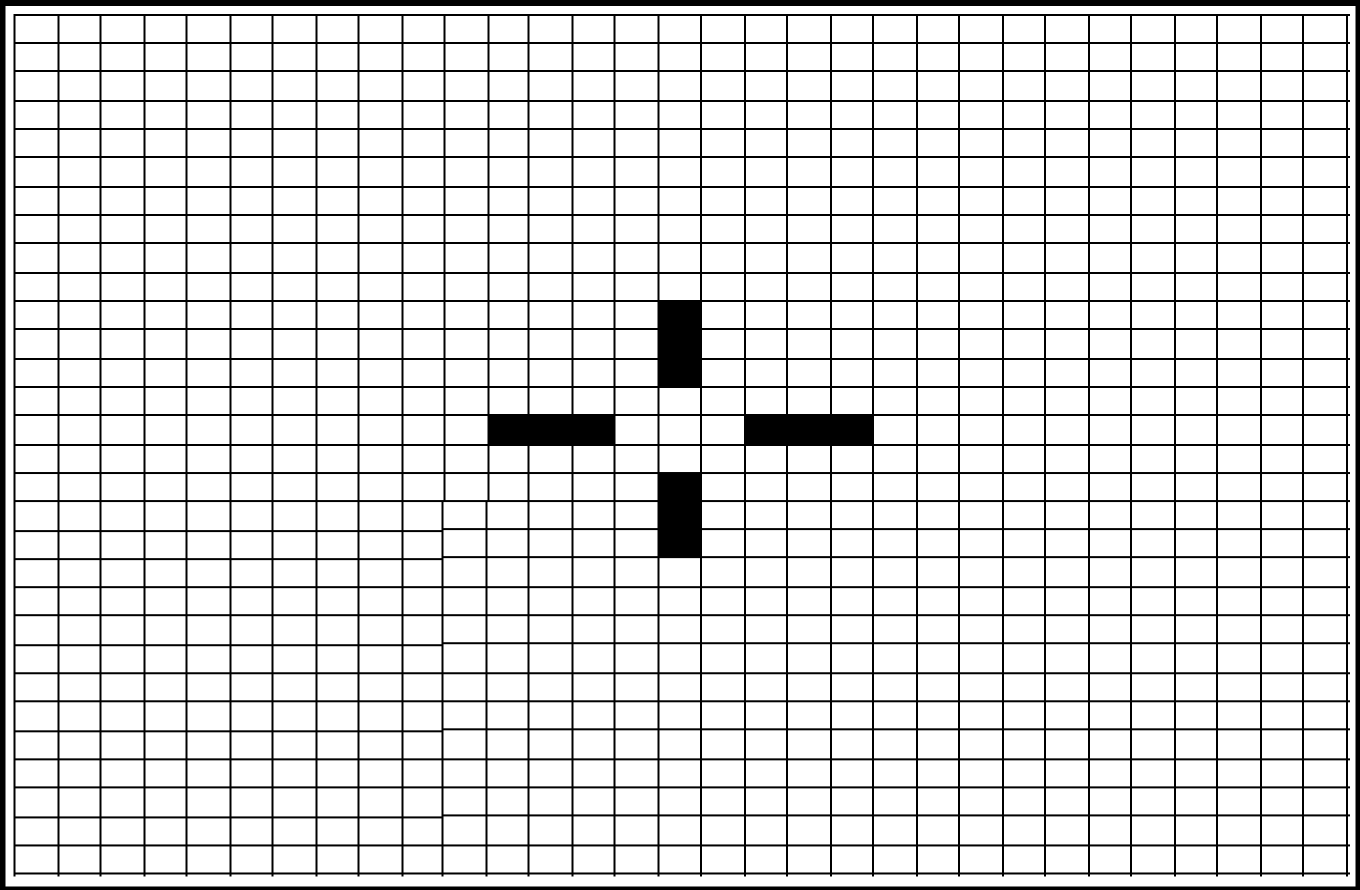


step 2

Evolution...



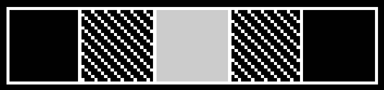
Limit cycle...



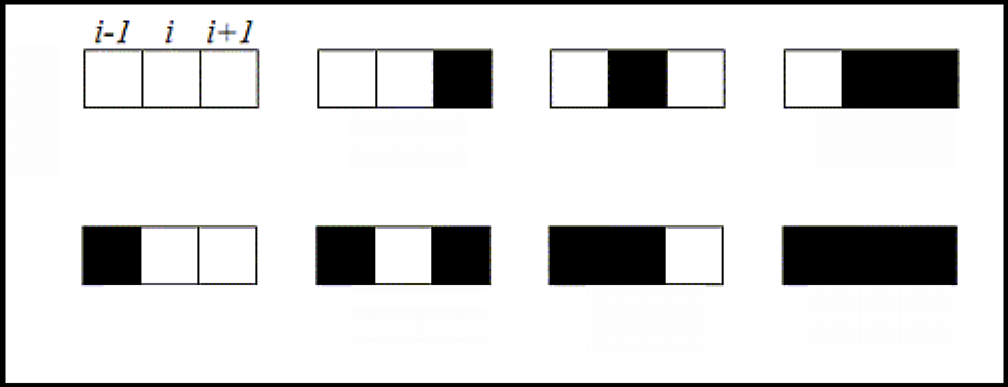
existence of limit cycles ⟹ irreversibility!

1d cellular automata

Let us consider 1d automata with neighbourhood of radius 1



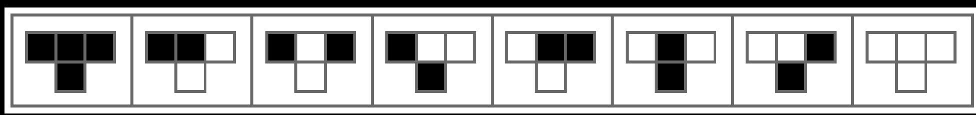
Such a neighbourhood can be in one of 8 states



Transitions rules determine what happens in each of these cases

$$c_i(t+1) = \varphi(c_{i-1}(t), c_i(t), c_{i+1}(t))$$

Example

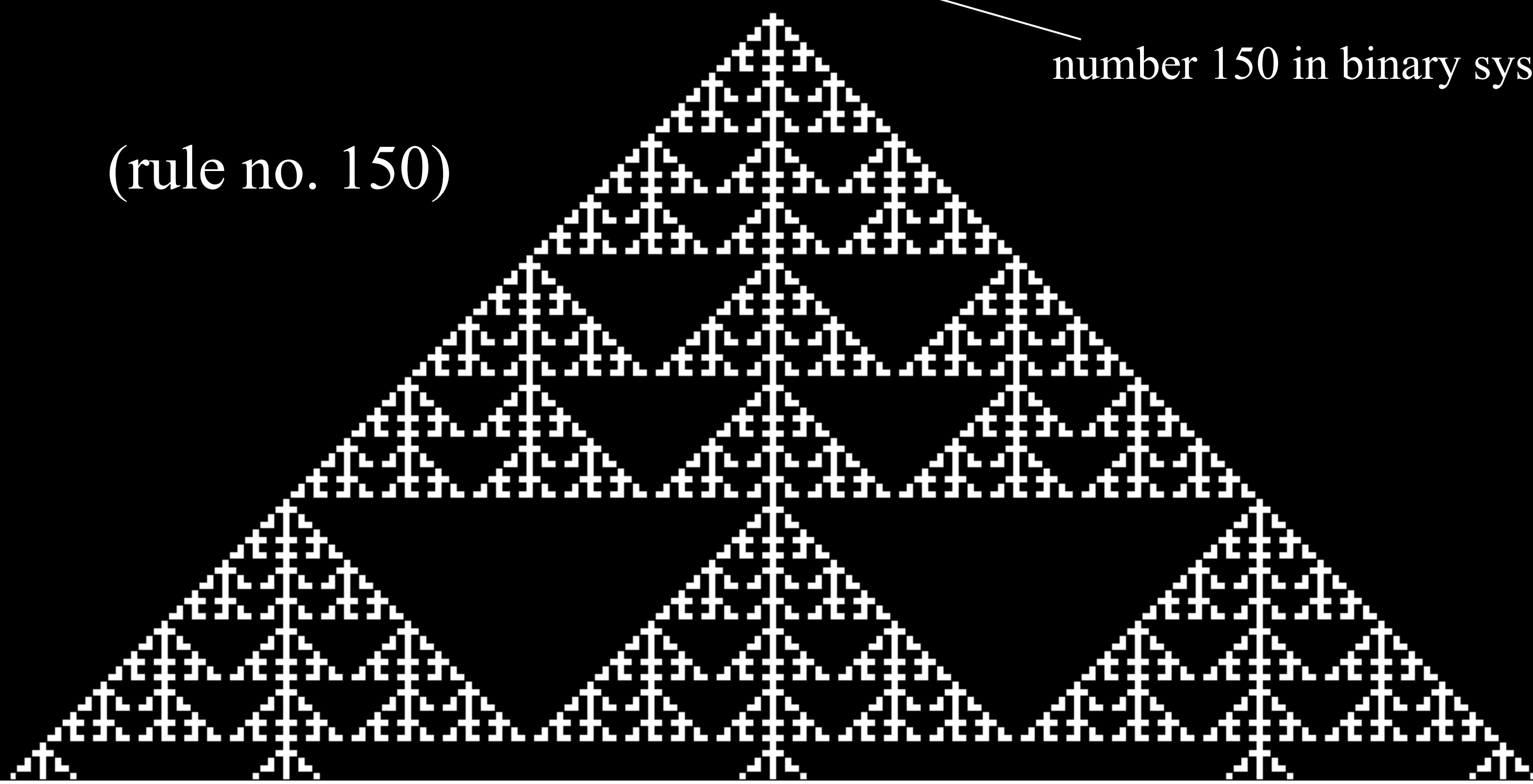


256 different rules!

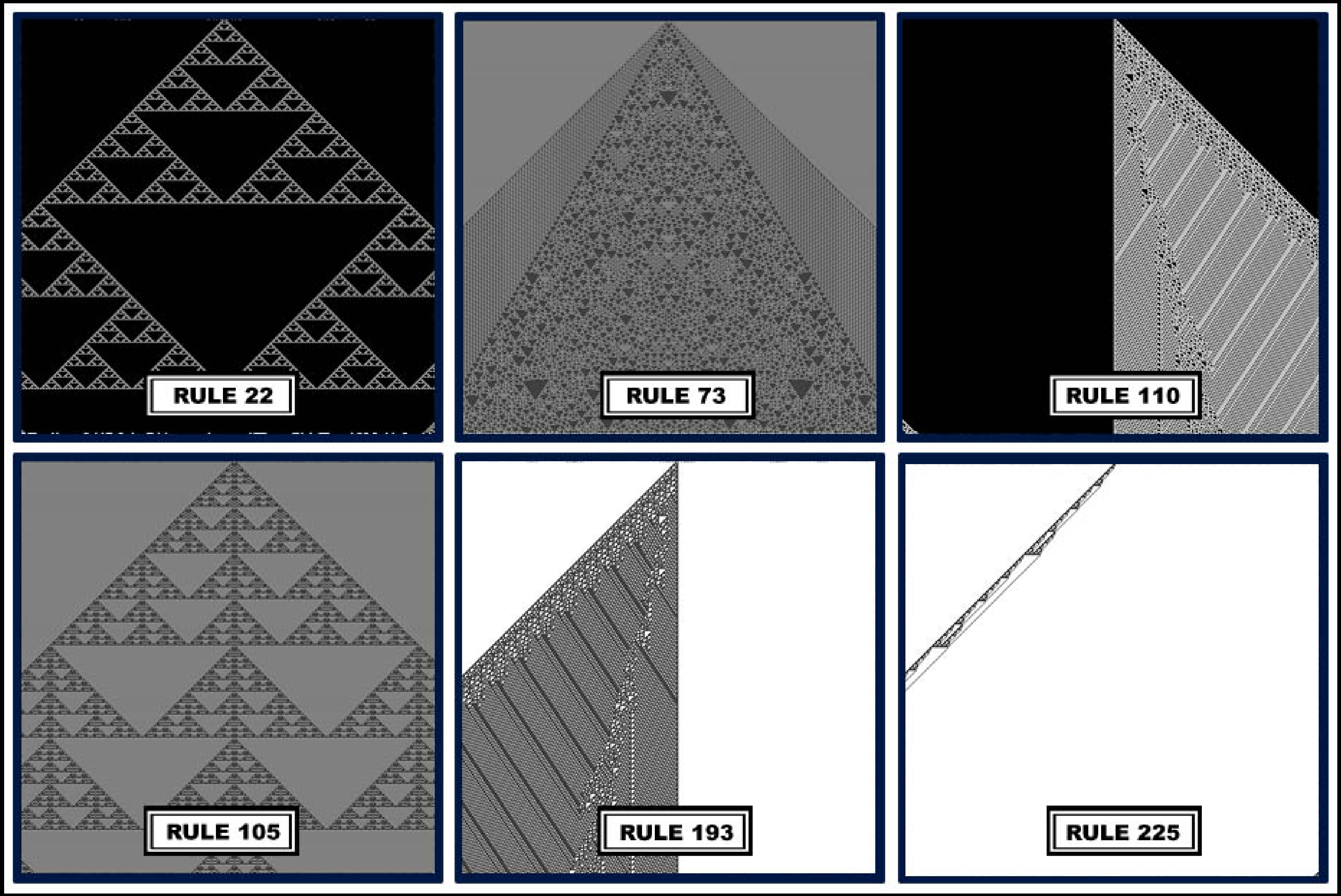
111	110	101	100	011	010	001	000
1	0	0	1	0	1	1	0

number 150 in binary system

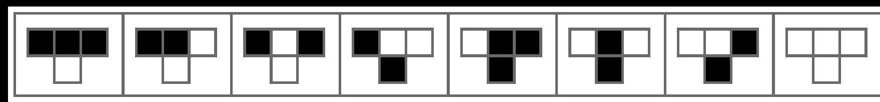
(rule no. 150)



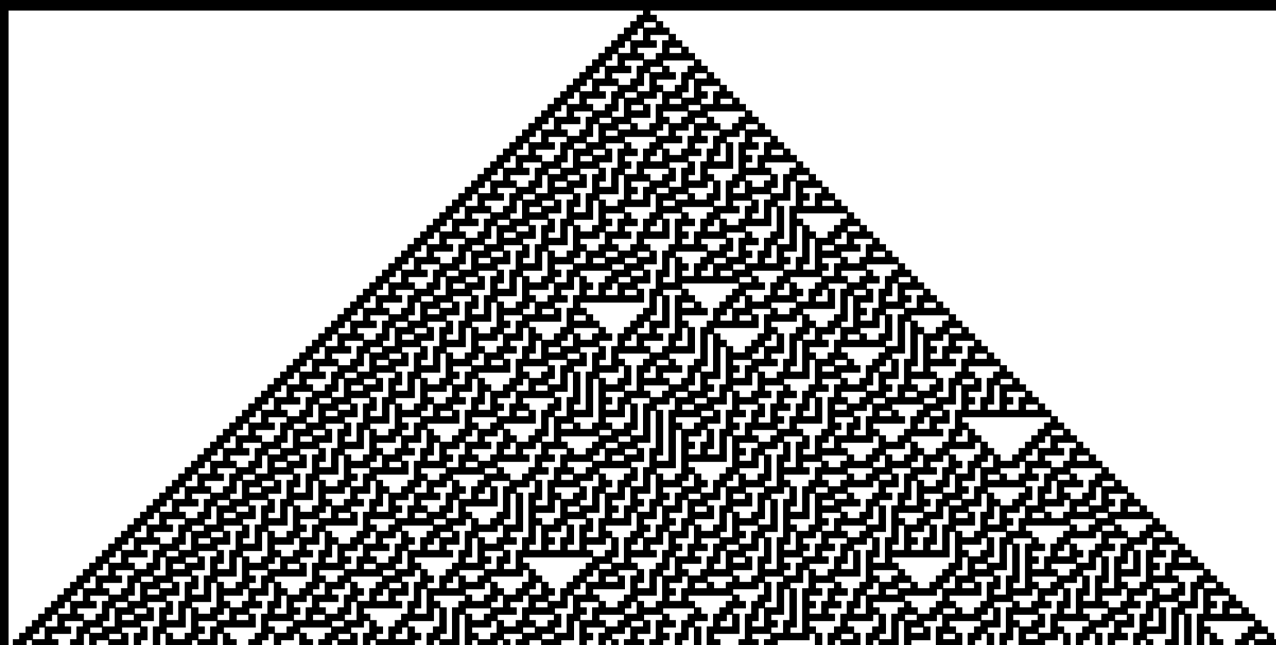
Further examples



Rule 30

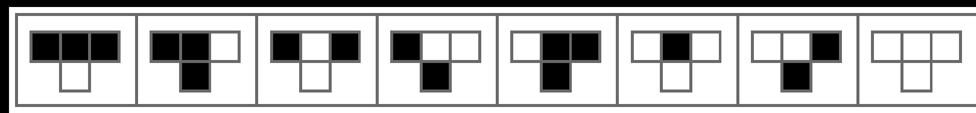


$$c_i(t+1) = c_{i-1}(t) + c_i(t) + c_{i+1}(t) + c_i(t)c_{i+1}(t) \pmod 2$$



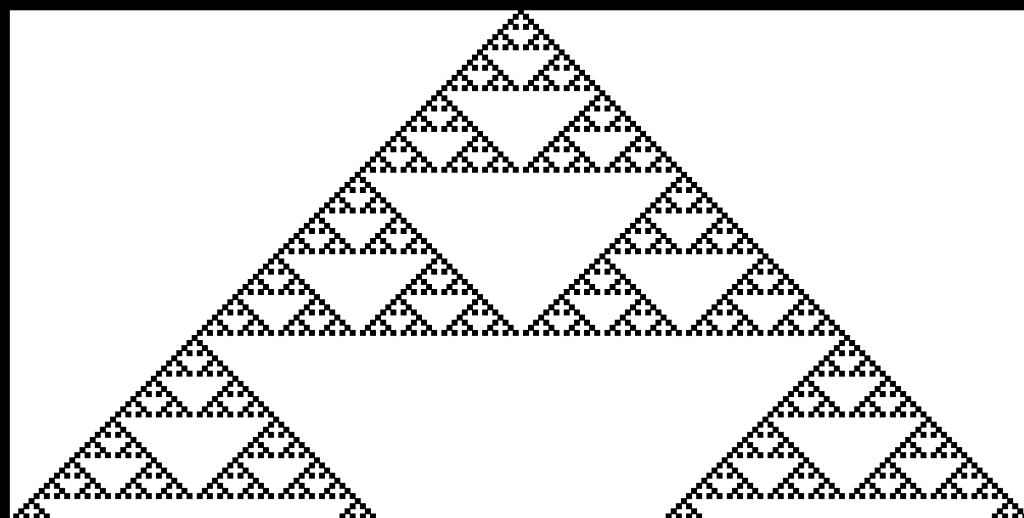
central column: 1001110011... ⟵ perfectly random

Rule 90



$$c_i(t+1) = c_{i-1}(t) + c_{i+1}(t) \pmod 2$$

Sierpiński gasket
D=1.585

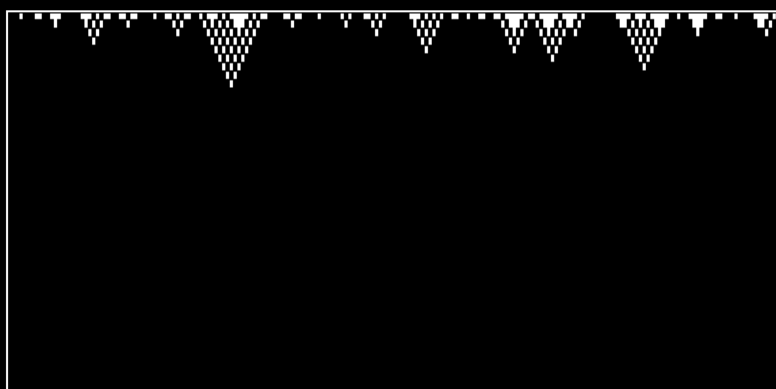


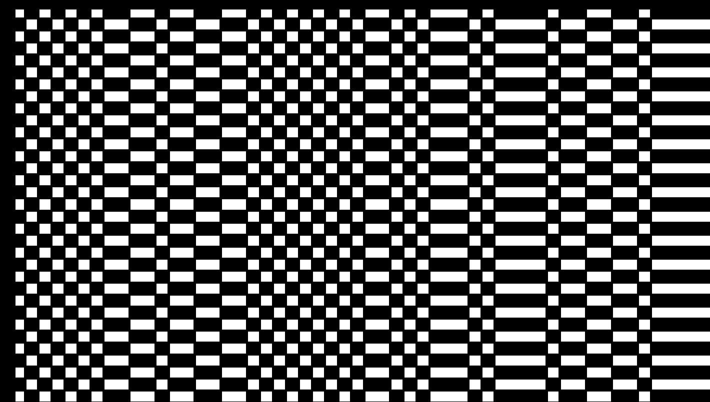
Pascal triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

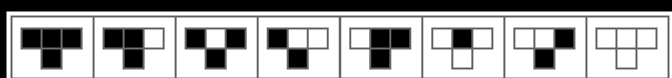
rule 90 shows the positions of odd coefficients

Irreversibility



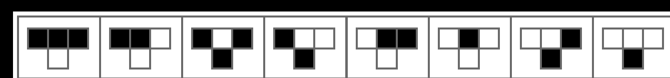


rule 250



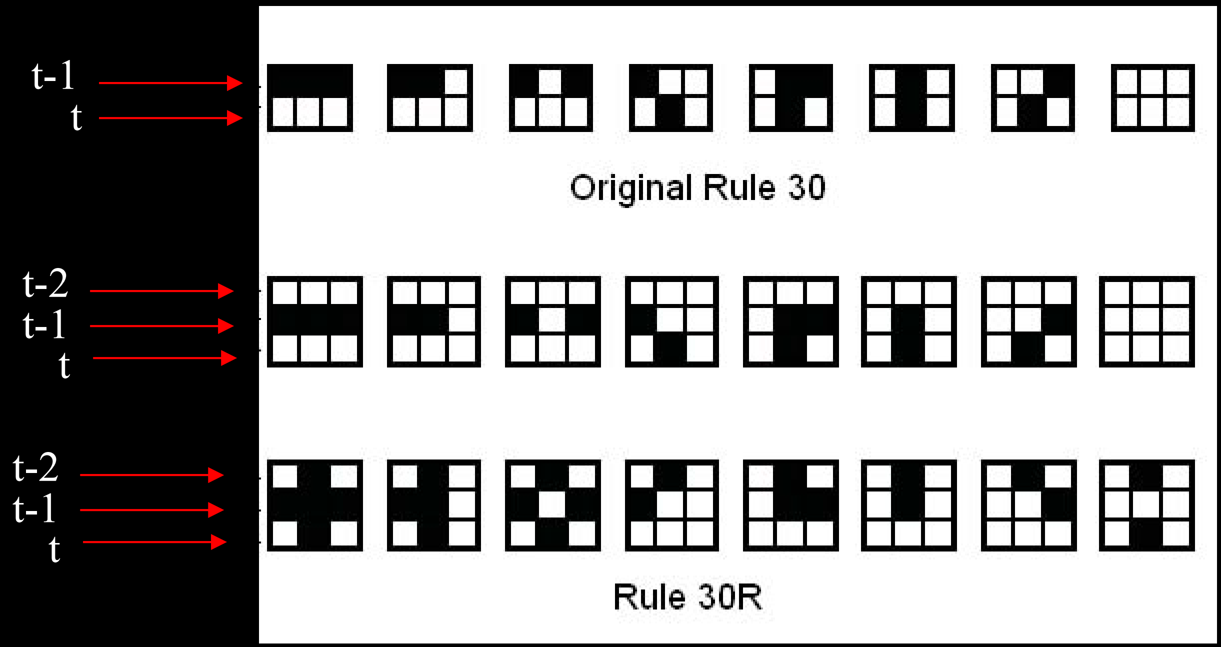
everything turns to black – irreversible

rule51



each cell reverses its color in every step – trivially reversible

Construction of reversible automata



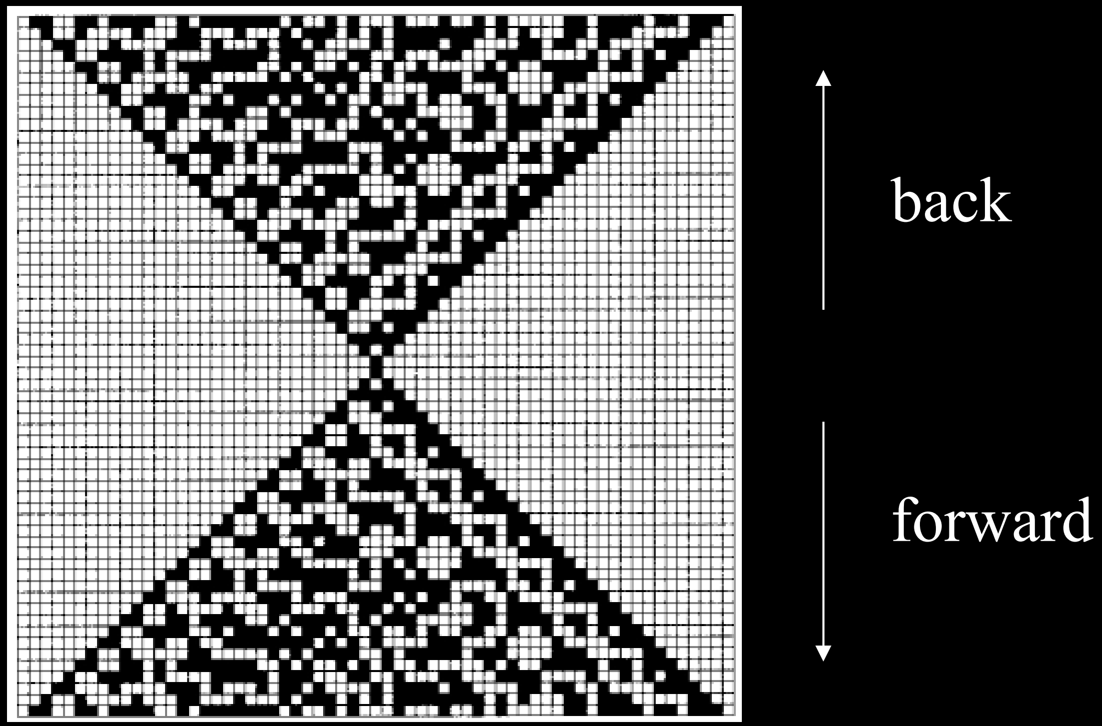
$$c_i(t+1) = \varphi(c_{i-1}(t), c_i(t), c_{i+1}(t))$$ → original rule

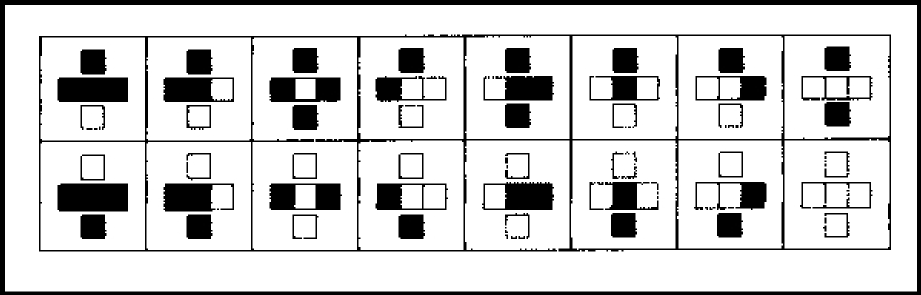
$$c_i(t+1) = \varphi(c_{i-1}(t), c_i(t), c_{i+1}(t)) + c_i(t-1) \pmod 2$$ → reversible rule

evolution back in time: $$c_i(t-1) = \varphi(c_{i-1}(t), c_i(t), c_{i+1}(t)) + c_i(t+1) \pmod 2$$

identical!

Example





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Irreversibility – cont'd

In irreversible automata the number of accesible configurations of the cells decreases in time. For 10 cells we would have to look at 1024 possible configurations, so let us consider a simpler system with 9 configurations and the following rules:

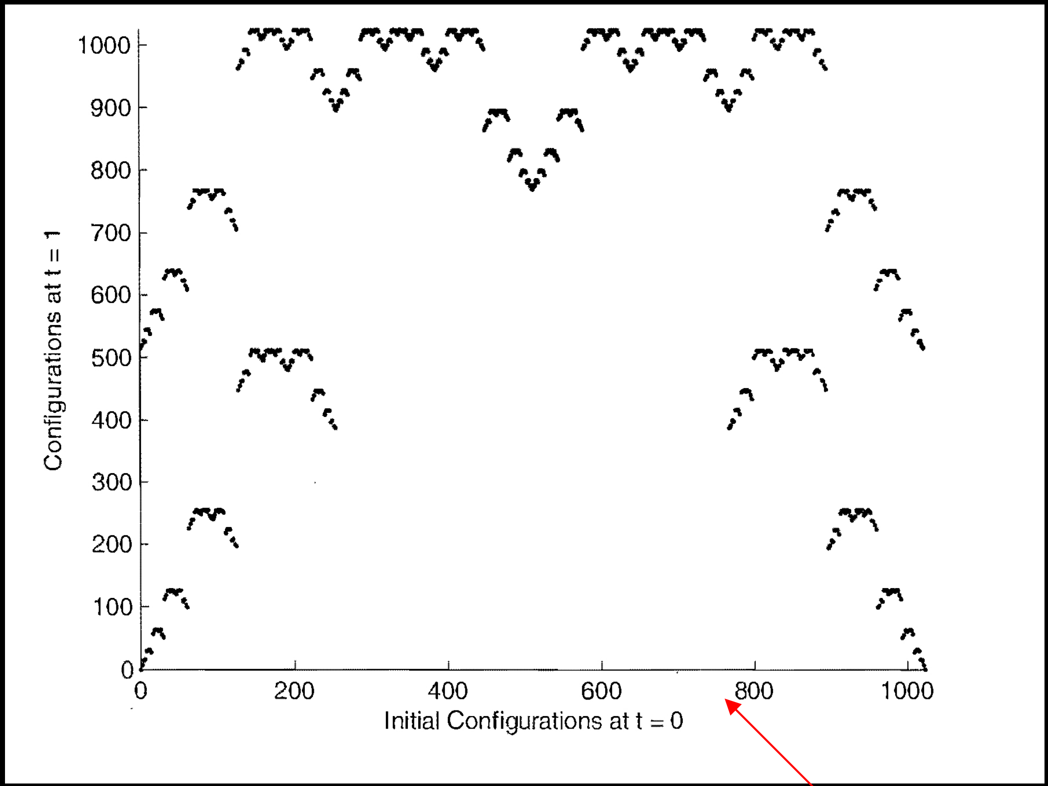
$a \rightarrow b$ $b \rightarrow c$ $c \rightarrow d$ $d \rightarrow e$ $e \rightarrow f$ $f \rightarrow g$ $g \rightarrow h$ $h \rightarrow g$

Garden of Eden

(unreachable by the dynamics)

	a	b	c	d	e	f	g	h
1	b	c	d	e	f	g	h	g
2	c	d	e	f	g	h	g	h
3	d	e	f	g	h	g	h	g
4	e	f	g	h	g	h	g	h
5	f	g	h	g	h	g	h	g
6	g	h	g	h	g	h	g	h

Example – rule 126



after the first step 217 configurations have remained

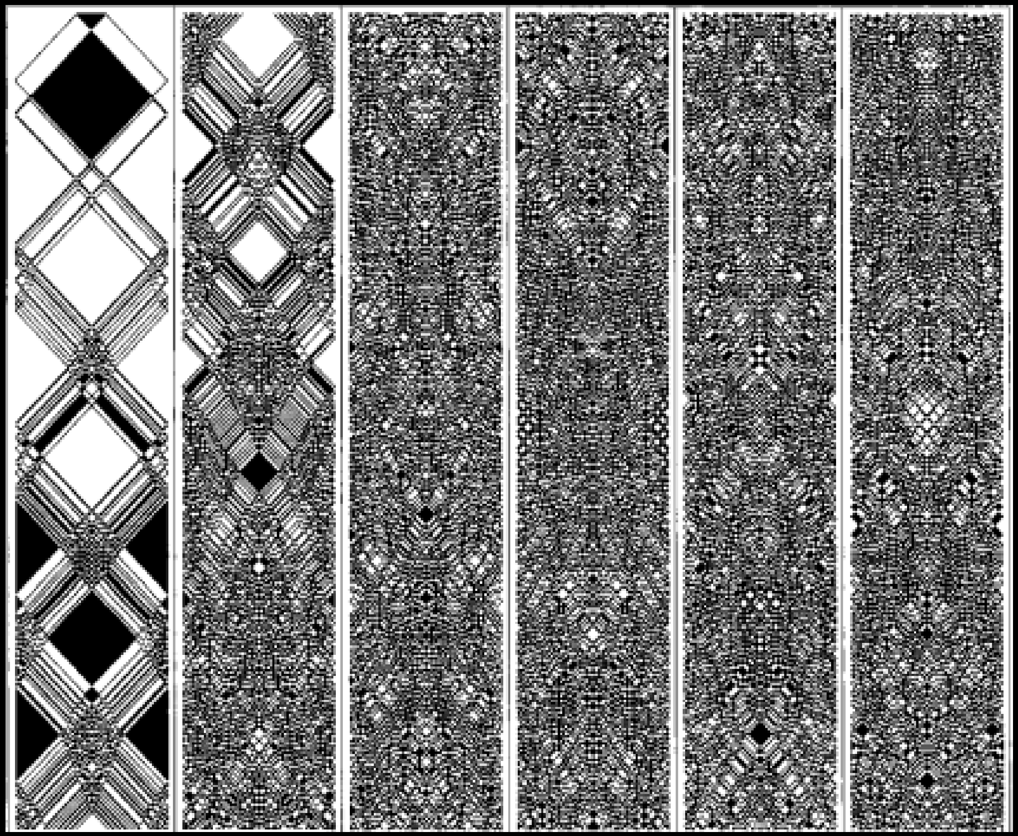
1024 initial configurations (10 cells)

the final fate of those1024 configurations:

- 124 have reached a fixed point
- 100 have landed in an orbit of period 2
- 520 – orbit of period 4
- 280 – period 8

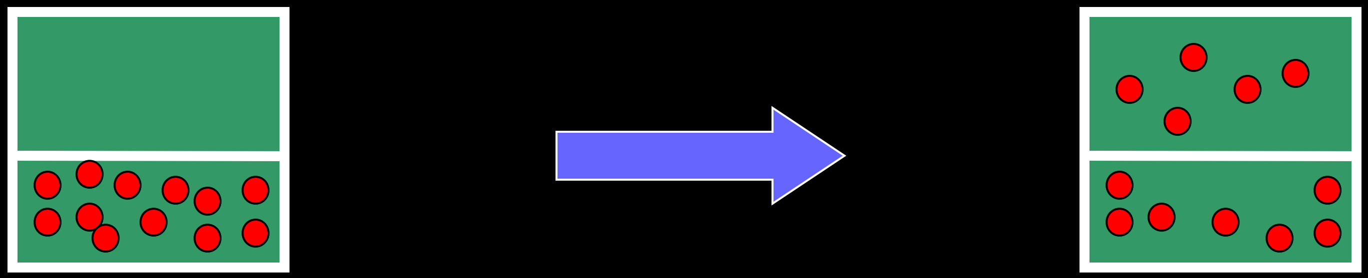
81 *different* final states

Mysterious automaton 122R...



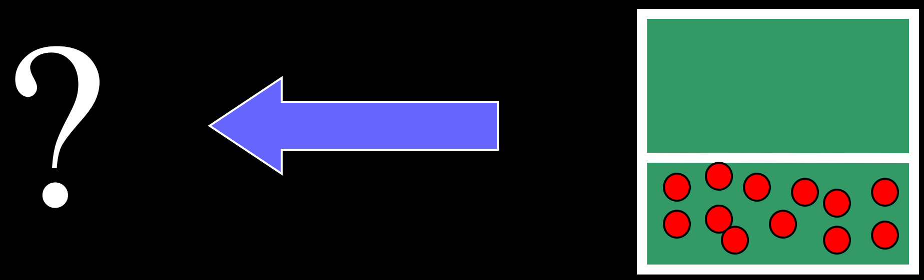
reversible automaton with irreversible evolution?

Macroscopic analogy...

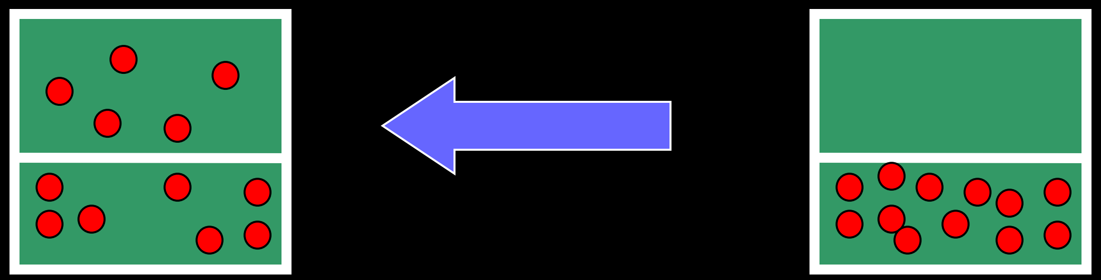


many-body systems: irreversible macroscopic behaviour in the system governed by reversible equations of classical mechanics ...

What happens if we go back in time ?

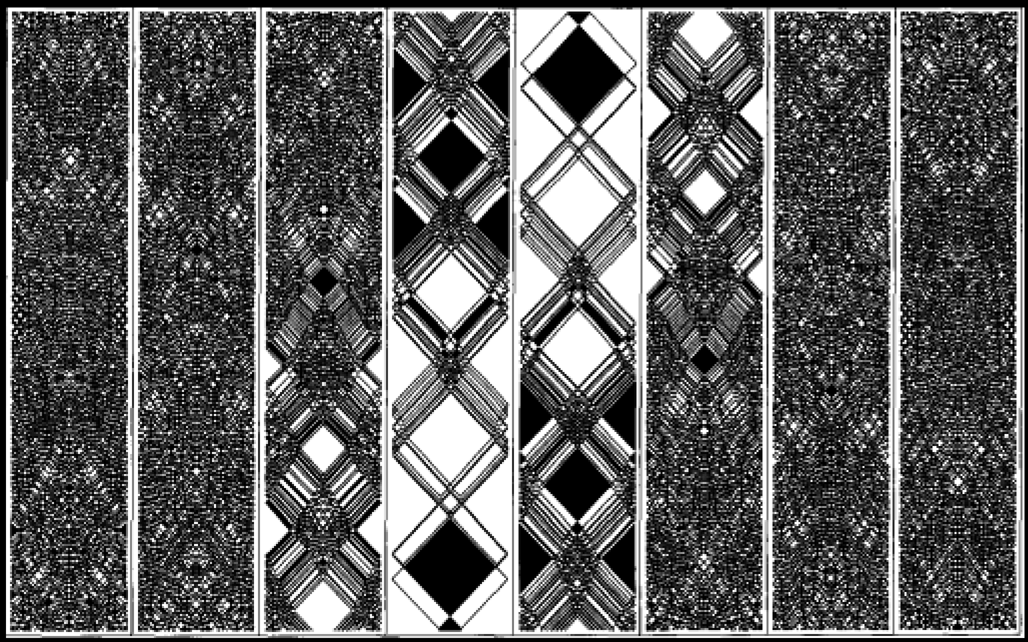


What happens if we go back in time ?



!!!

122R back in time...



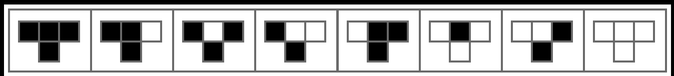
disorder grows in this direction as well!

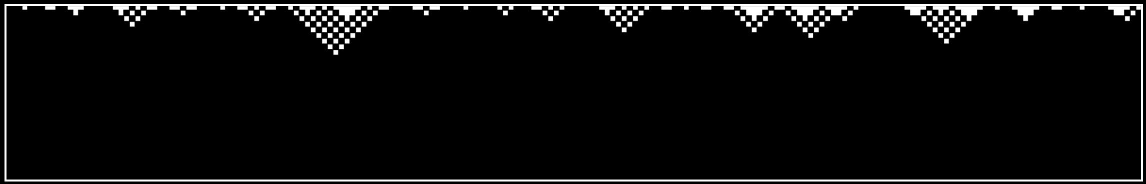
(fully consistent with reversibility)

Classification of 1d automata

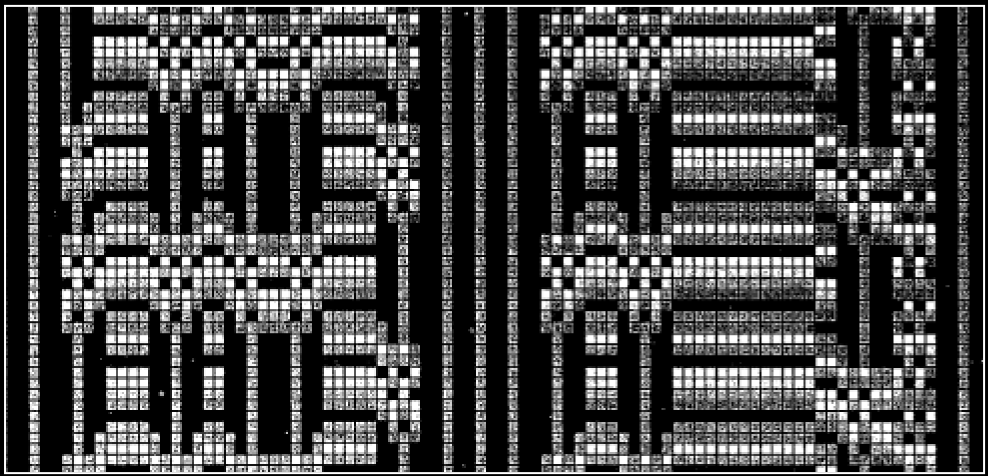
Class I – und up in a uniform state (only 0s or 1s)

eg. rule 250:





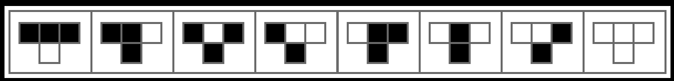
Class II – end up in a fixed point or a periodic orbit



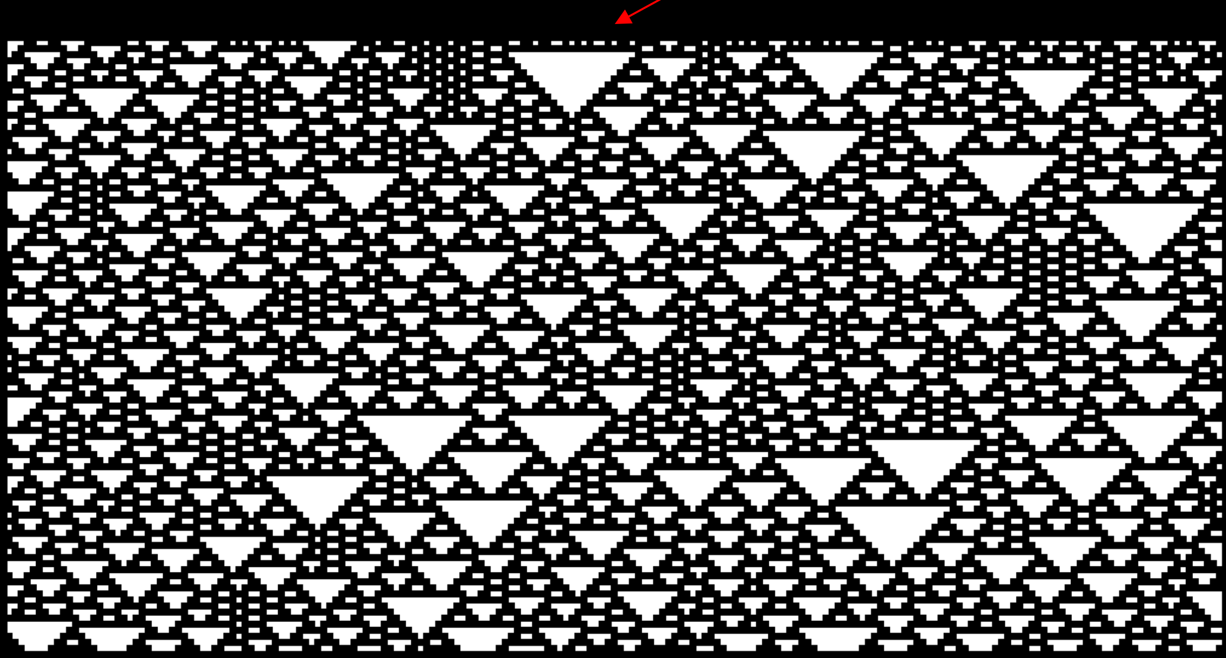
Class III

Automata with a chaotic behaviour (although deterministic)

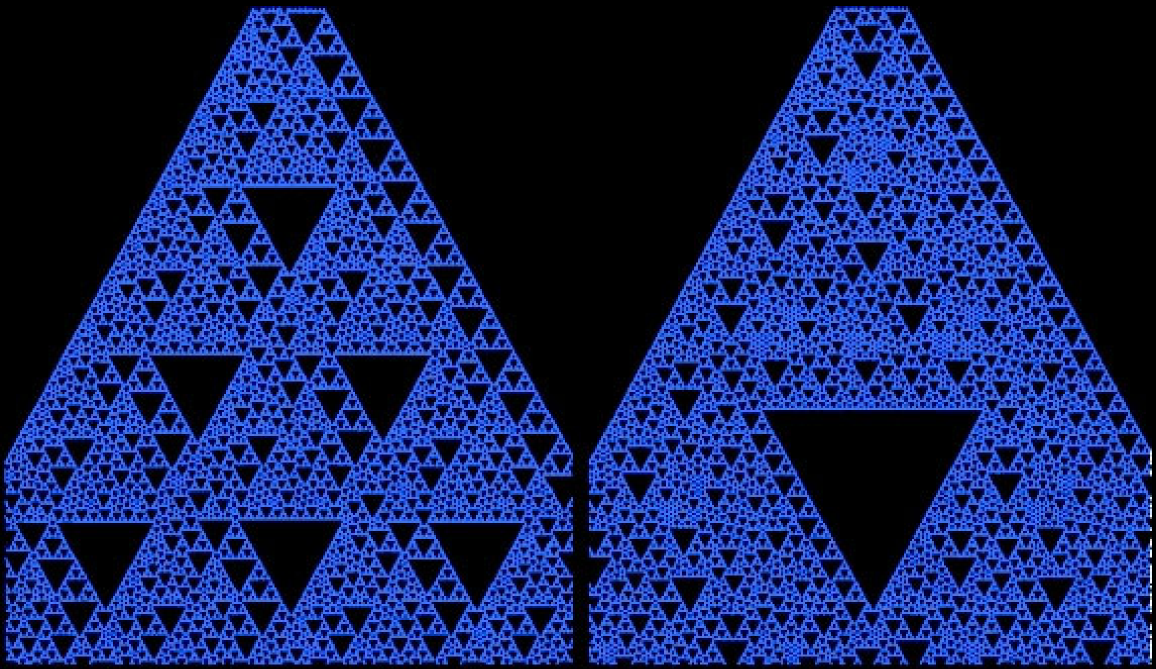
Example: rule 126



random initial conditions

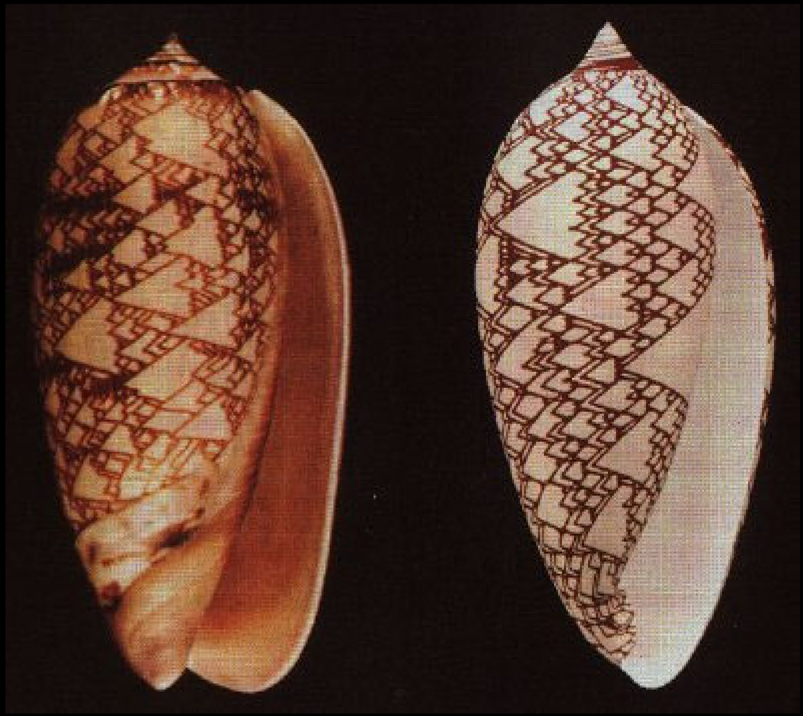


Sensitivity to initial conditions



Evolution according to the rule 126 with initial conditions which differ by a state of a single cell

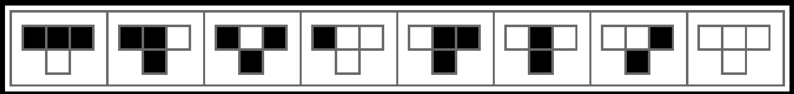
Class III automata on shells...

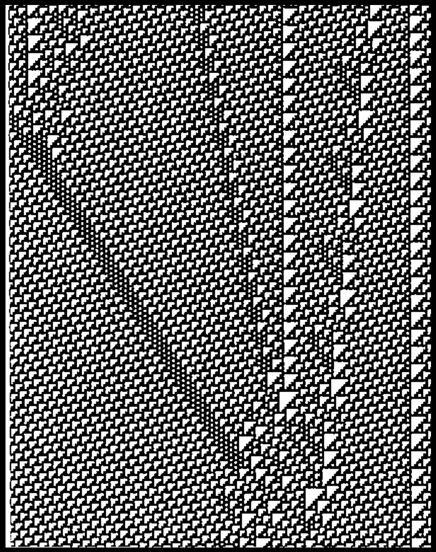


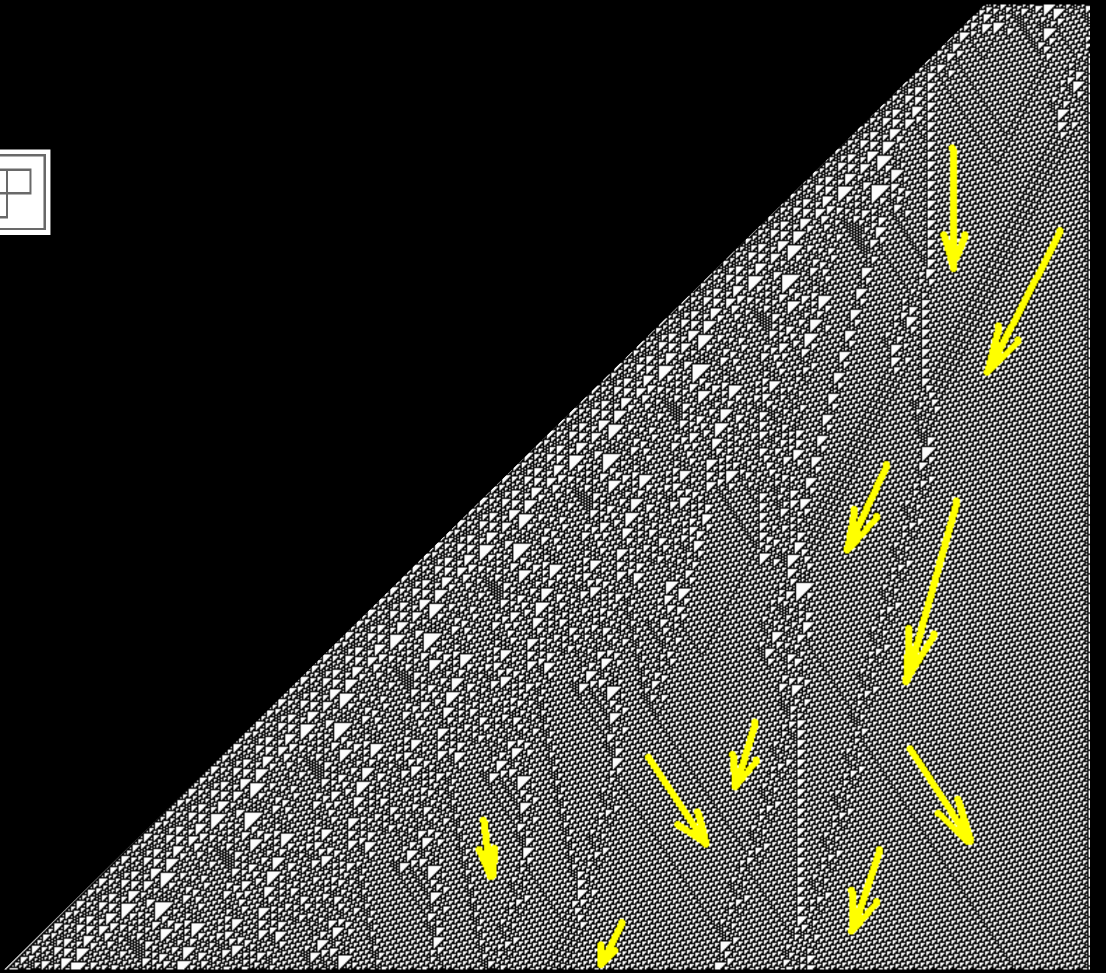
Class IV

Automata 'on the edge of chaos', in which the organized structures appear, which then propagate and interact between themselves

Example: rule 110







Automata as calculators

initial state – input

final state - output

An example – parity checking by the automaton 132: