Hubbard model with spin dependent disorder

Krzysztof Byczuk

Institute of Theoretical Physics, Faculty of Physics
University of Warsaw
and
Center for Electronic Correlations and Magnetism, Institute of Physics,
Augsburg University

July 13th, 2011

WYDZIAŁ FIZYKI
UNIWERSYTET WARSZAWSKI

EKM

INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

FNP
Foundation for Polish Science

EUROPEAN UNION
EUROPEAN REGIONAL DEVELOPMENT FUND

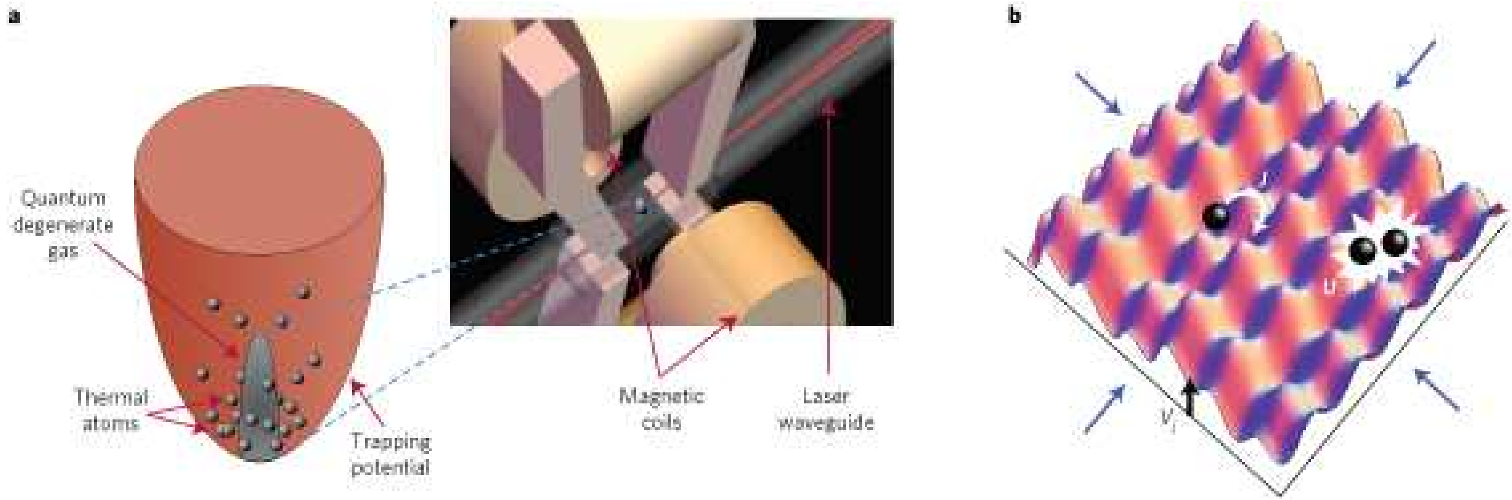
www.physik.uni-augsburg.de/theo3/kbyczuk

www.fuw.edu.pl/pmss

Collaboration and Discussion

Prabuddha B. Chakraborty - Augsburg University
Richard T. Scalettar - University of California
Jan Skolimowski - University of Warsaw
Dieter Vollhardt - Augsburg University

Introduction: Optical lattices

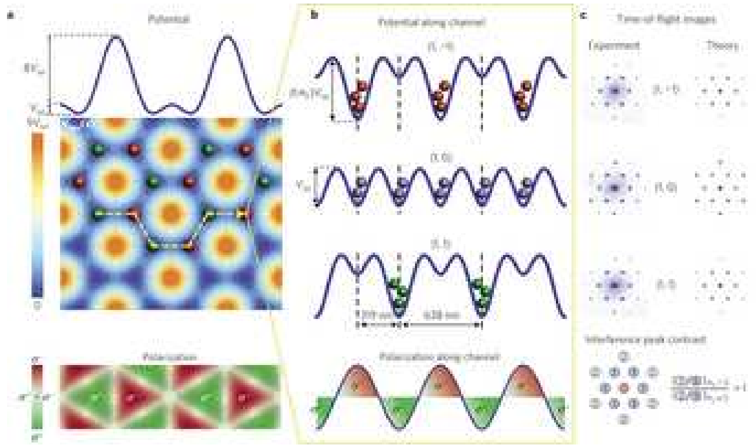


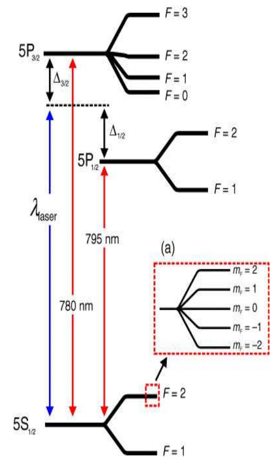
L. Sanchez-Palencia, M. Lewenstein, Nature Phys. 6, 87 (2010)

$$H = \sum_{ij\sigma} J_{ij\sigma} a^\dagger_{i\sigma} a_{j\sigma} + \sum_{i\sigma} V_{i\sigma} a^\dagger_{i\sigma} a_{i\sigma} + \frac{1}{2} \sum_{i\sigma\sigma'} U_{i\sigma\sigma'} n_{i\sigma} n_{i\sigma'}$$

- high tunability and control over system parameters
- (almost) perfect realization of model lattice Hamiltonians
- access to systems not seen in solid-state matter

Introduction: Eg. Hexagonal spin-dependent lattice



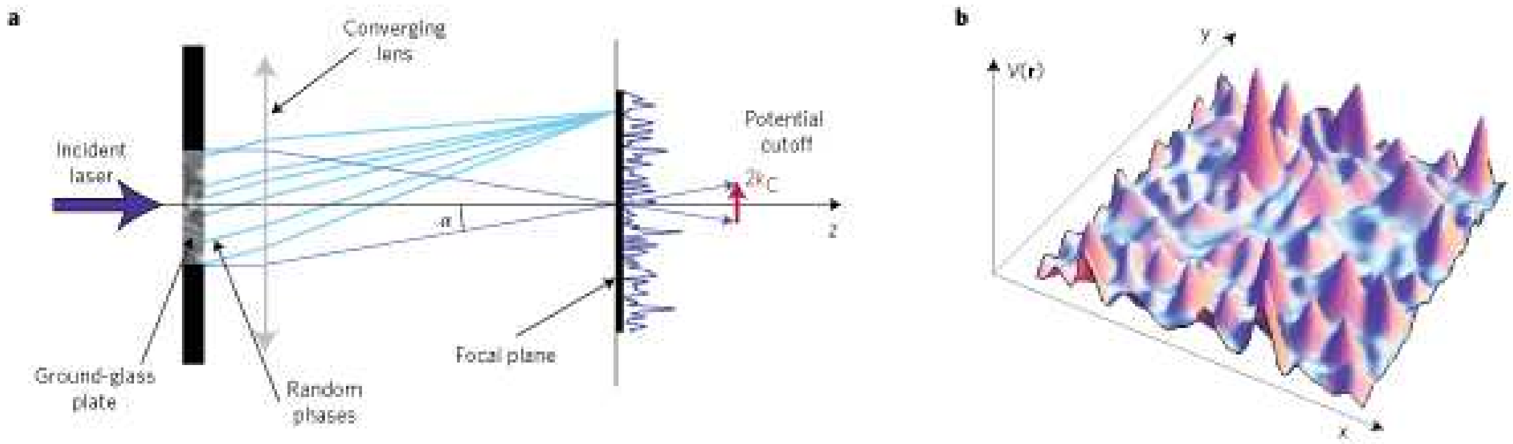


P. Soltan-Panahi et al. , Nature Phys. 7, 434 (2011); D McKay, B DeMarco, NJP 12, 055013 (2010)

$$H = \sum_{ij\sigma} J_{ij\sigma} a^\dagger_{i\sigma} a_{j\sigma} + \sum_{i\sigma} V_{i\sigma} a^\dagger_{i\sigma} a_{i\sigma} + \frac{1}{2} \sum_{i\sigma\sigma'} U_{i\sigma\sigma'} n_{i\sigma} n_{i\sigma'}$$

- "For neighbouring sites along the vertices of the hexagonal lattice, ..., the local polarization alternates between σ^+ and σ^-."
- "As atoms in a light field experience a polarization dependent a.c. Stark shift, the potential at these σ^+ and σ^- sites is different for different atomic Zeeman substates labeled by m_F"

Introduction: Eg. Optical lattices with disorder

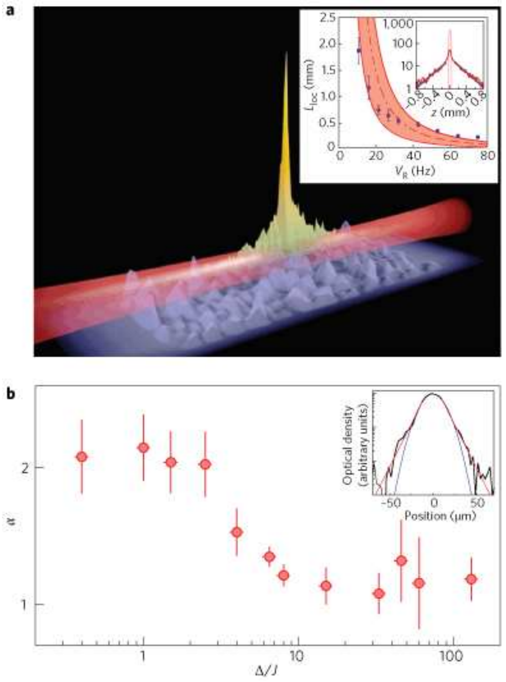


L. Sanchez-Palencia, M. Lewenstein, Nature Phys. 6, 87 (2010)

$$H = \sum_{ij\sigma} J_{ij\sigma} a^{\dagger}_{i\sigma} a_{j\sigma} + \sum_{i\sigma} V_{i\sigma} a^{\dagger}_{i\sigma} a_{i\sigma} + \frac{1}{2} \sum_{i\sigma\sigma'} U_{i\sigma\sigma'} n_{i\sigma} n_{i\sigma'}$$

- Optical speckle potential or superposition of incommensurate periodic fields creates a random (pseudo random) disorder on top of the optical lattice

Introduction: Eg. Anderson localization



G. Roati et al., Nature 453, 895 (2008)

$$|\Psi(x)|^2 \sim e^{-2x/L_{loc}}$$

- Observation of Anderson localization of matter waves made of ^{87}Rb,
V_R is disorder strength

Introduction: Our motivation

Spin Dependent Disorder

- soon available experiments where disorder acts selectively only on one spin component
- not know theoretically too much ...

Let's do theoretical work ...

Hubbard model with local spin dependent disorder

$$H = \sum_{ij\sigma} t_{ij\sigma} a^{\dagger}_{i\sigma} a_{j\sigma} + \sum_{i\sigma} V_{i\sigma} n_{i\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow},$$

with $V_{i\uparrow} = \epsilon_i$ - random variable, $V_{i\downarrow} = 0$.

$$\mathcal{P}(\epsilon_1, ..., \epsilon_n, ...) = \prod_i P(\epsilon_i)$$

and

$$P(\epsilon) = \begin{cases} \frac{1}{\Delta} & |\epsilon| < \frac{\Delta}{2} \\ 0 & |\epsilon| > \frac{\Delta}{2}. \end{cases}$$

We consider now fermions (^{40}K) and leave bosons (^{87}Rb) for later on.

Model A: $\langle n_{\uparrow} \rangle + \langle n_{\downarrow} \rangle = n$ conserved - Today
Model B: $\langle n_{\uparrow} \rangle = n_{\uparrow}$, $\langle n_{\downarrow} \rangle = n_{\downarrow}$ independently conserved - Next time

Hubbard model with spin dependent disorder

Simple trick

$$n_i = n_{i\uparrow} + n_{i\downarrow},$$

$$m_i = n_{i\uparrow} - n_{i\downarrow},$$

leads to Hamiltonian

$$H = \sum_{ij\sigma} t_{ij\sigma} a^\dagger_{i\sigma} a_{j\sigma} + \sum_i \mu_i n_i + \sum_i h_i m_i + U \sum_i n_{i\uparrow} n_{i\downarrow},$$

where we identify local random chemical potential and random magnetic field that are correlated

$$\langle \mu_j h_j \rangle_{\mathrm{dis}} = \frac{\Delta^2}{48} \delta_{ij}.$$

- possible simultaneous response in both density and spin channels

Hubbard model with spin dependent disorder - Dynamical Mean-Field Theory

M. Ulmke, V. Janis, D. Vollhardt, Phys. Rev. B 51, 10411 (1995)

$$Z\{\mathcal{G}^{-1}, \epsilon_i\} = \int Da^* Da\, e^{\mathcal{A}}$$

$$\mathcal{A} = \sum_{n\sigma} a^*_{n\sigma}(\mathcal{G}^{-1} - \epsilon_i \delta_{\sigma\uparrow}) a_{n\sigma} - U \int_0^\beta d\tau n_\uparrow(\tau) n_\downarrow(\tau)$$

with $\mathcal{G}^{-1}_{n\sigma} = G^{-1}_{n\sigma} + \Sigma_{n\sigma}$ and

$$G_{n\sigma} = \int_0^\beta d\tau e^{i\omega_n \tau} \langle \langle a^*_\sigma(\tau) a_\sigma(0) \rangle_{\mathrm{qm,T}} \rangle_{\mathrm{dis}}$$

and generalization for AB lattices with given DOS

No Anderson localization physics

Hubbard model with spin dependent disorder - results

Preliminary results obtained within HF-QMC for

$n = 0.5,\ 0.7$, and 1.0
$U = 0.1,\ 1.0,\ 2.0$, and 3.0
$\Delta = 0.0,\ 1.0,\ 3.0$, and 5.0

model semi-elliptic density of states with bandwidth $W = 2$.

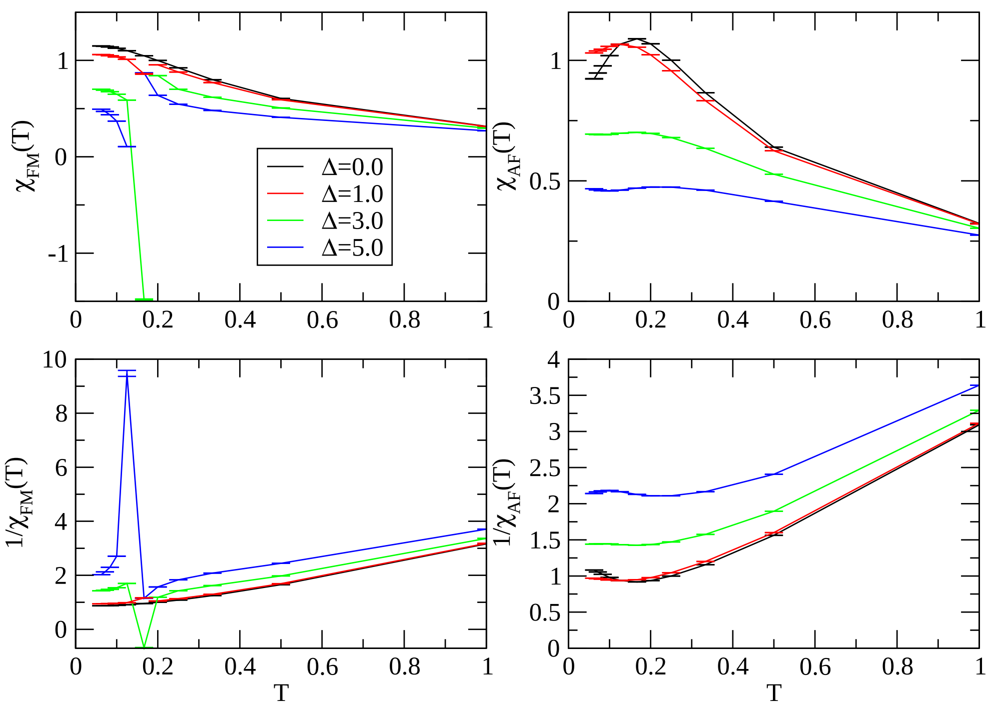
Computed: FM susceptibility, AF susceptibility, compressibility, DOS at Fermi level, double occupancy (local moment), magnetization as functions of temperatures.

$$d = \langle n_{i\uparrow} n_{i\downarrow} \rangle = \langle n_{i\uparrow} + n_{i\downarrow} \rangle - \frac{2}{3} \langle \mathbf{S}_i^2 \rangle$$

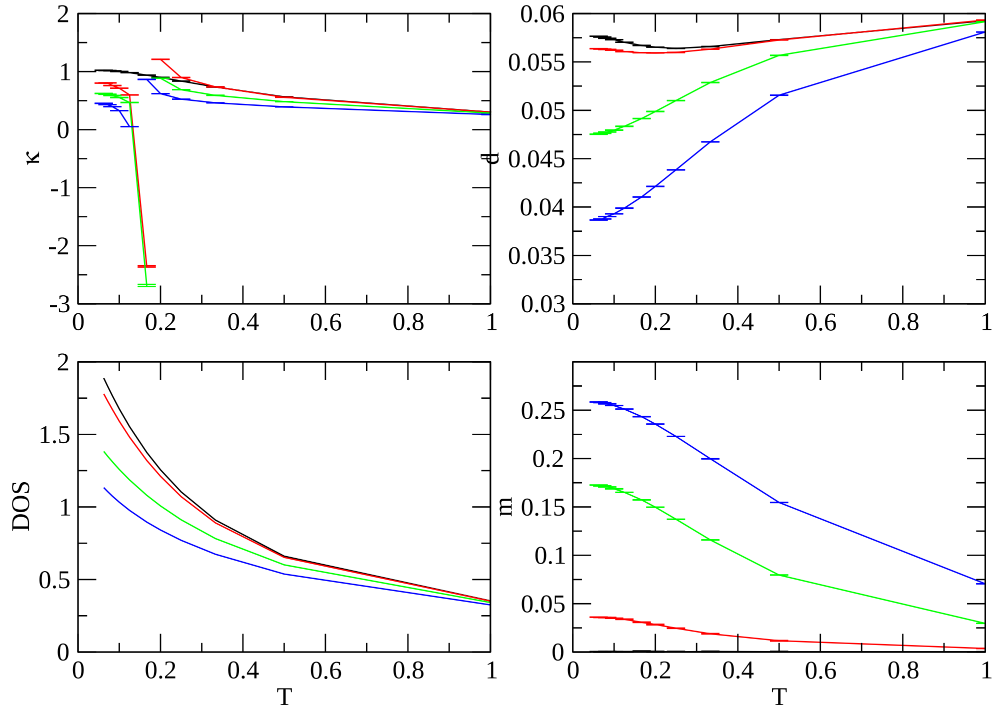
Hubbard model with spin dependent disorder - results

Almost non-interacting system away from half-filling

n=0.5, U=0.1, W=2.0



n=0.5, U=0.1, W=2.0

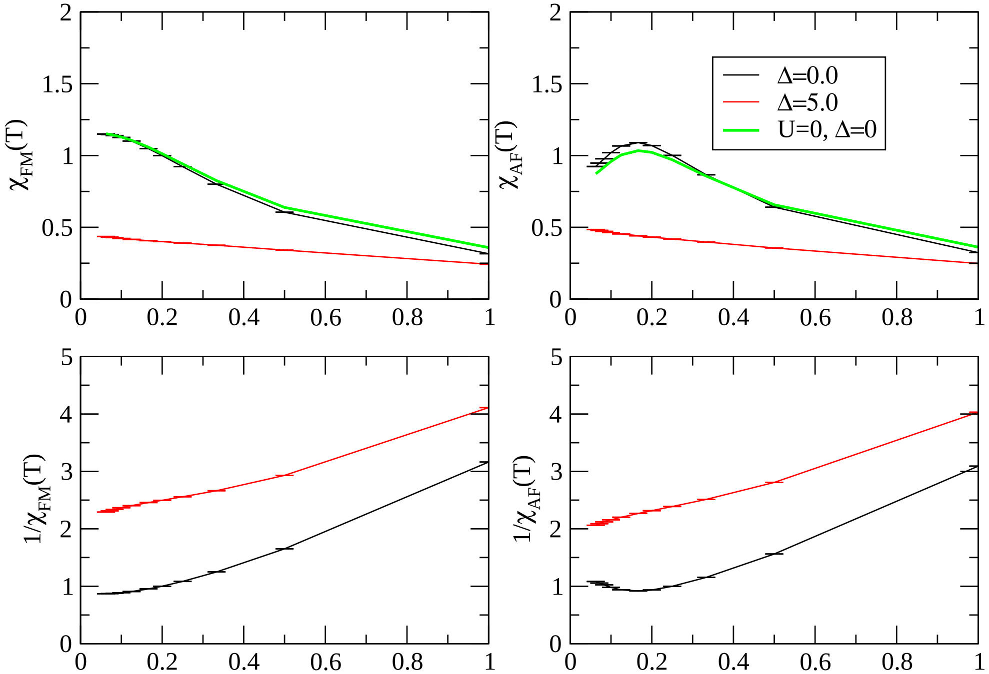


- divergence in FM susceptibility and compressibility
- negative sign of FM susceptibility and compressibility
- no singularity in other thermodynamic quantities
- finite magnetization, increasing local moments

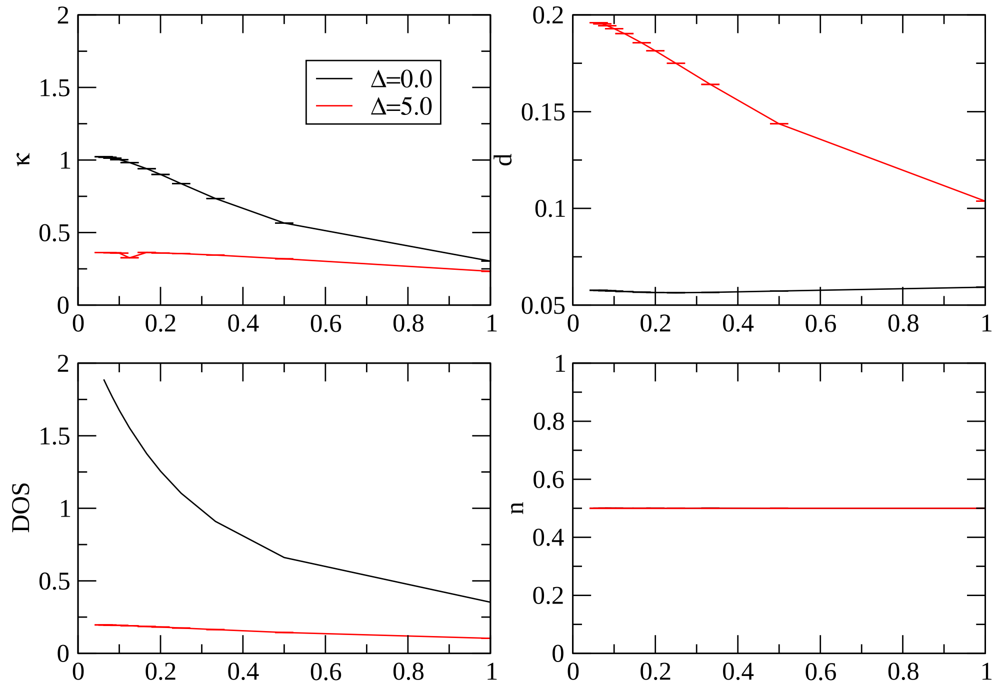
Hubbard model with normal disorder - comparison

Almost non-interacting system away from half-filling

n=0.5, U=0.1



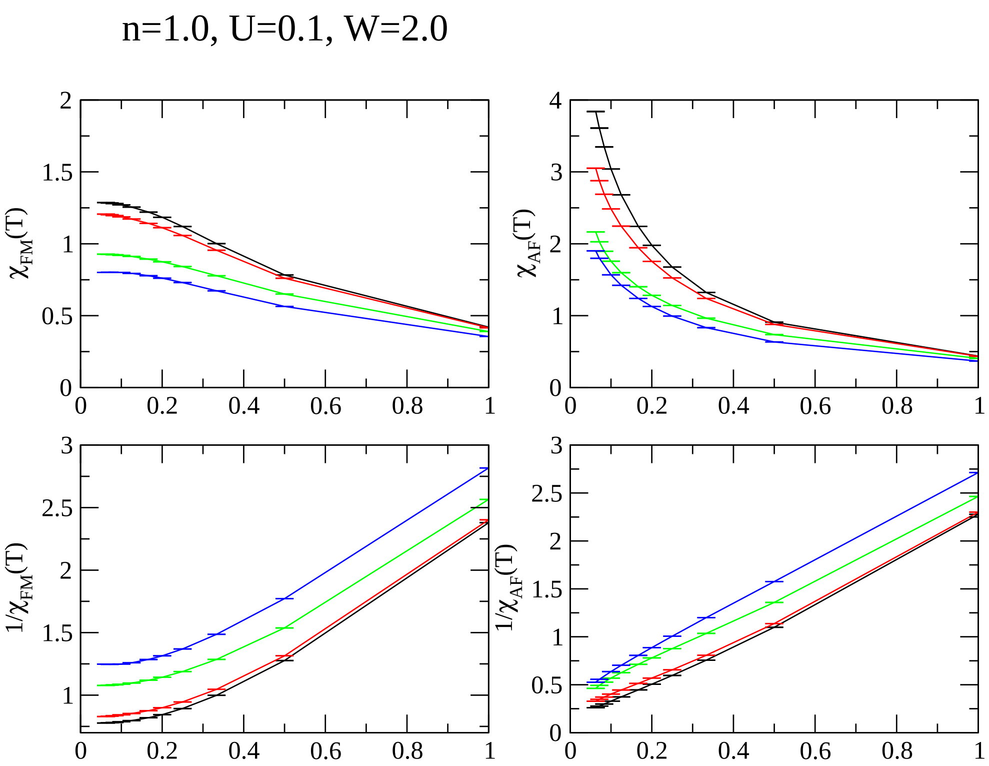
n=0.5, U=0.1

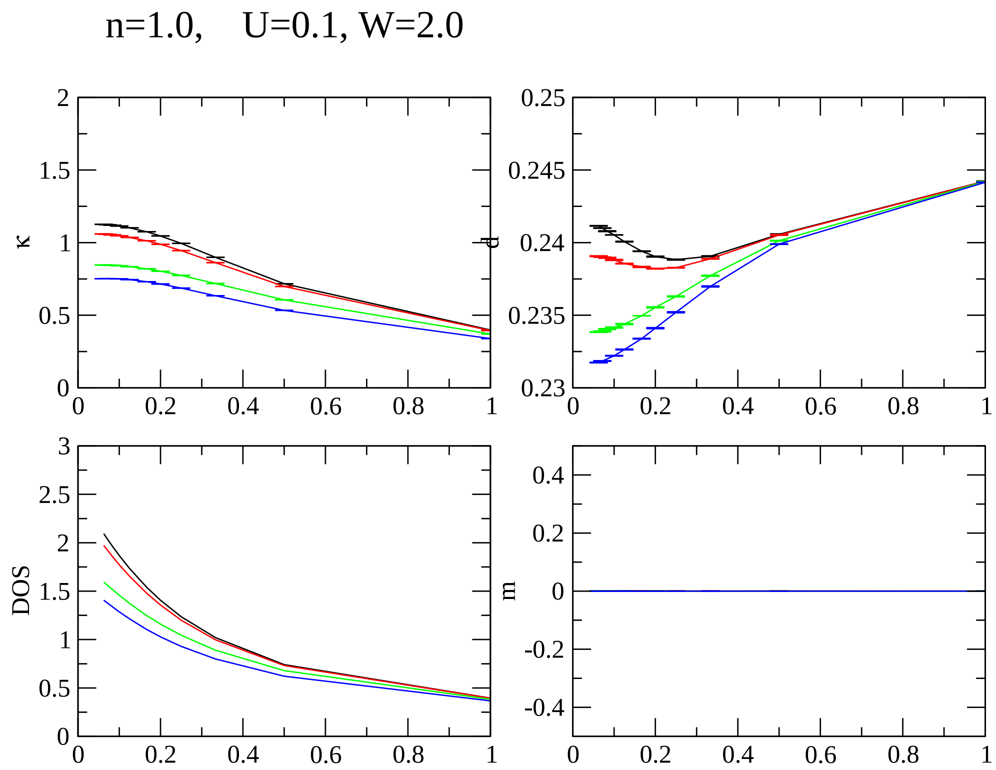


- no singularity in FM susceptibility and (?) compressibility
- no singularity in other thermodynamic quantities
- zero magnetization, decreasing local moments

Hubbard model with spin dependent disorder - results

Almost non-interacting system at half-filling





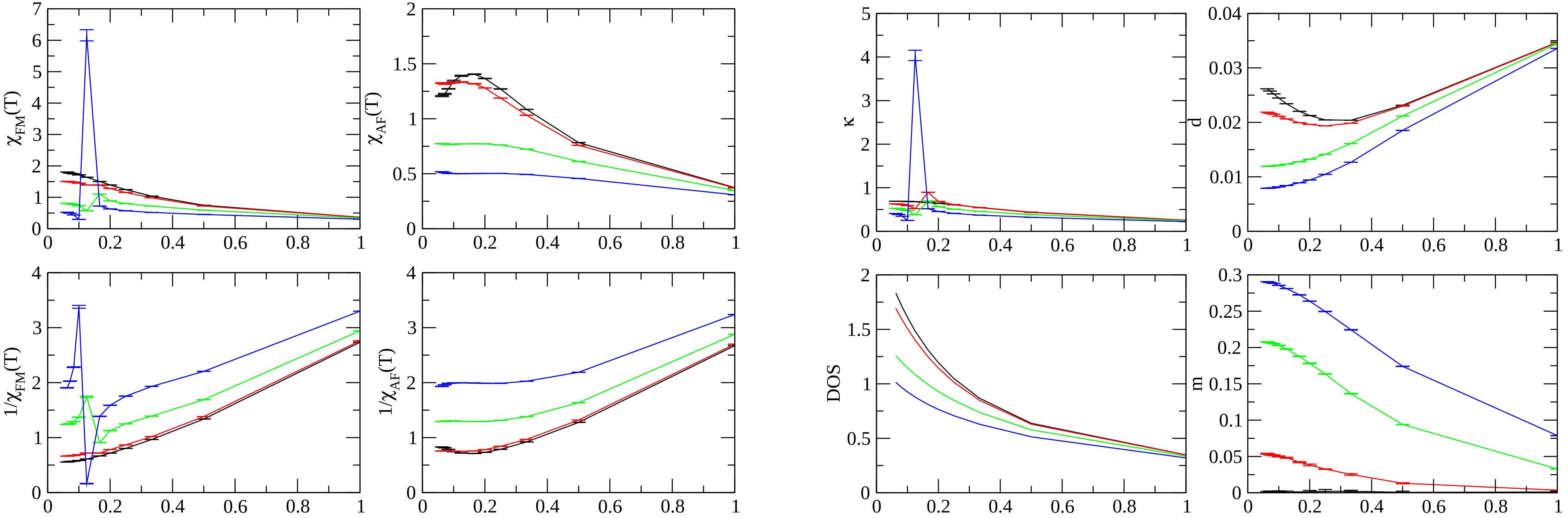
- singularity in AF susceptibility killed by disorder
- no singularity FM susceptibility and compressibility
- no singularity in other thermodynamic quantities
- zero magnetization, increasing local moments

Hubbard model with spin dependent disorder - results

Intermediate interacting system away from half-filling

n=0.5, U=1.0, W=2.0

n=0.5,   U=1.0, W=2.0

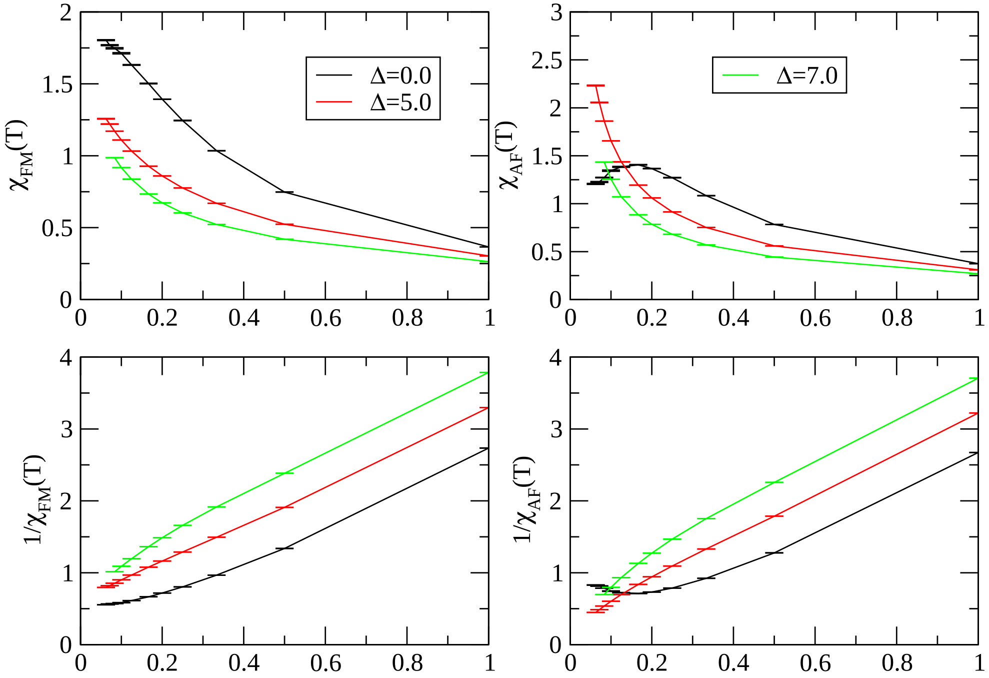


- divergence in FM susceptibility and compressibility
- negative sign of FM susceptibility and compressibility
- no singularity in other thermodynamic quantities
- finite magnetization, increasing local moments

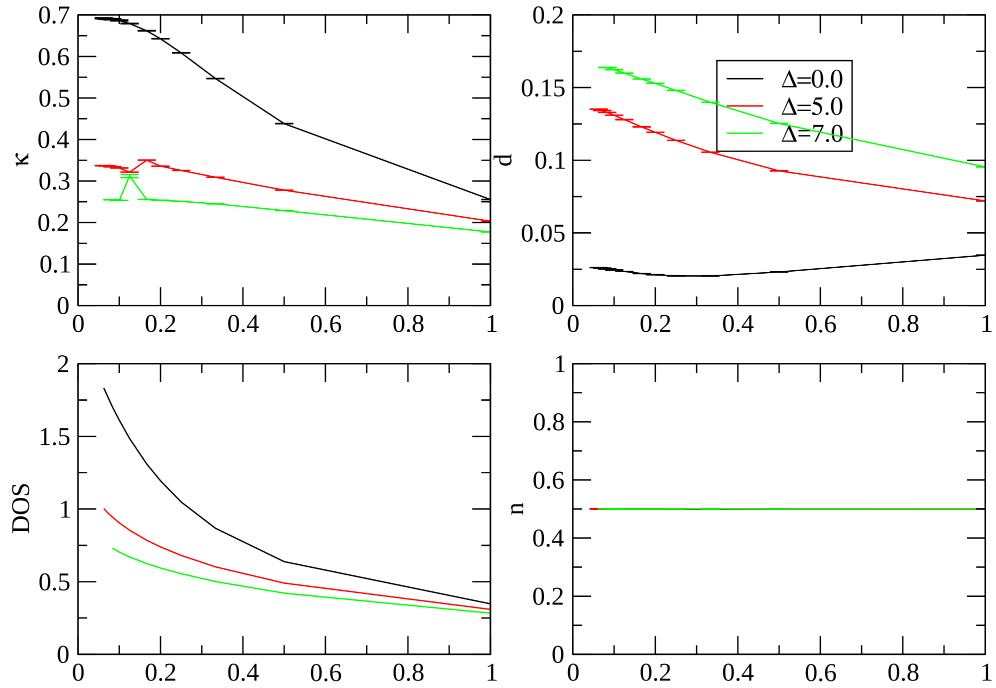
Hubbard model with normal disorder - comparison

Intermediate interacting system away from half-filling

n=0.5, U=1.0



n=0.5, U=1.0, W=2.0

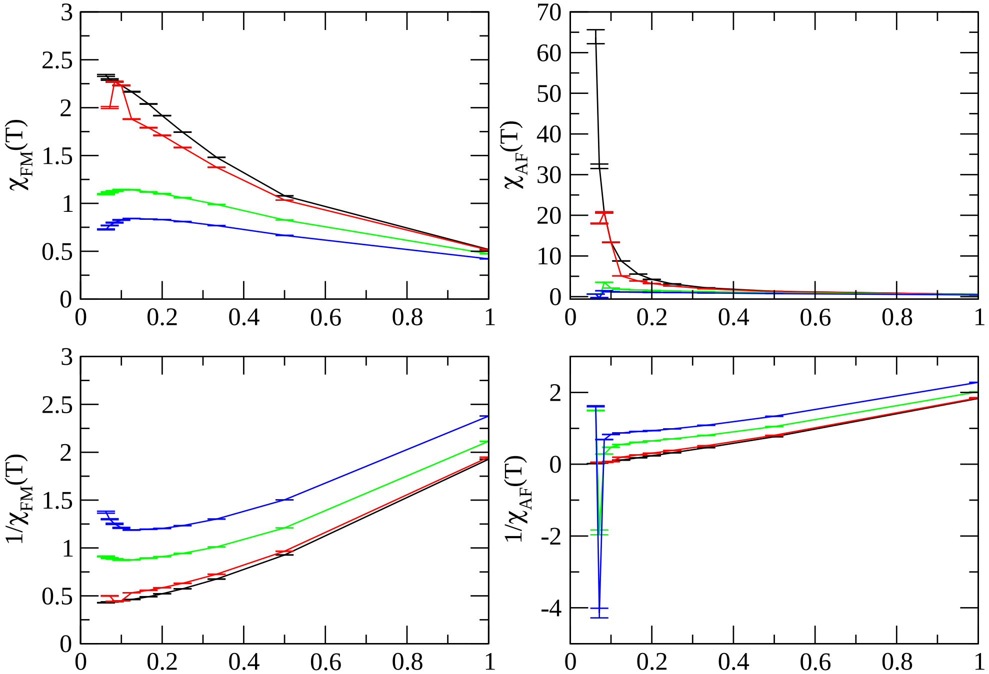


- no singularity in FM susceptibility and (??) compressibility
- no singularity in other thermodynamic quantities
- zero magnetization, decreasing local moments

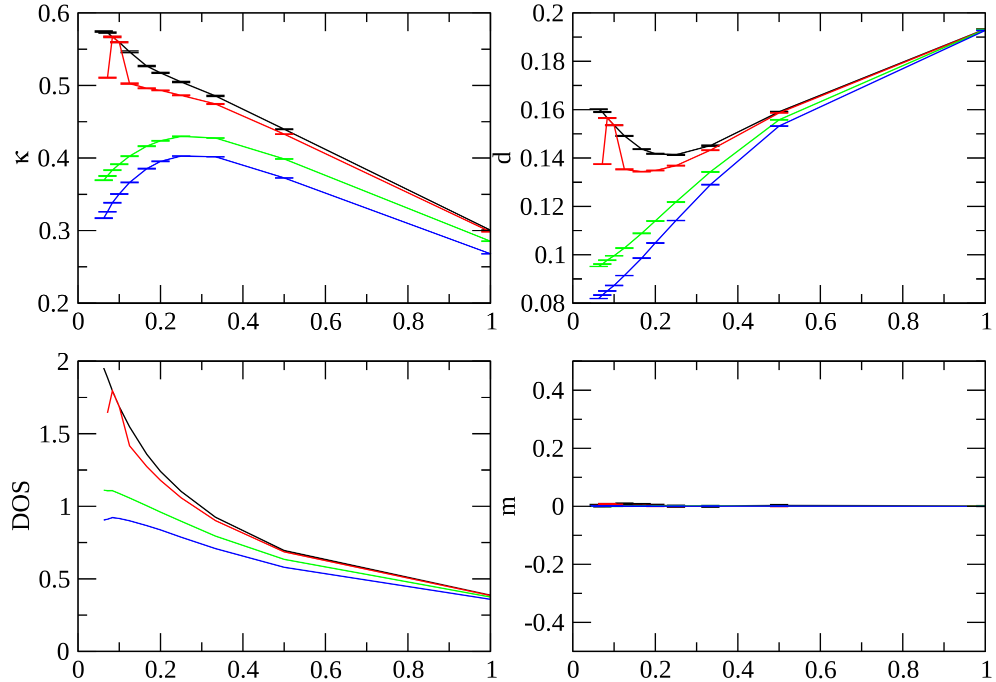
Hubbard model with spin dependent disorder - results

Intermediate interacting system at half-filling

n=1.0, U=1.0, W=2.0



n=1.0, U=1.0, W=2.0

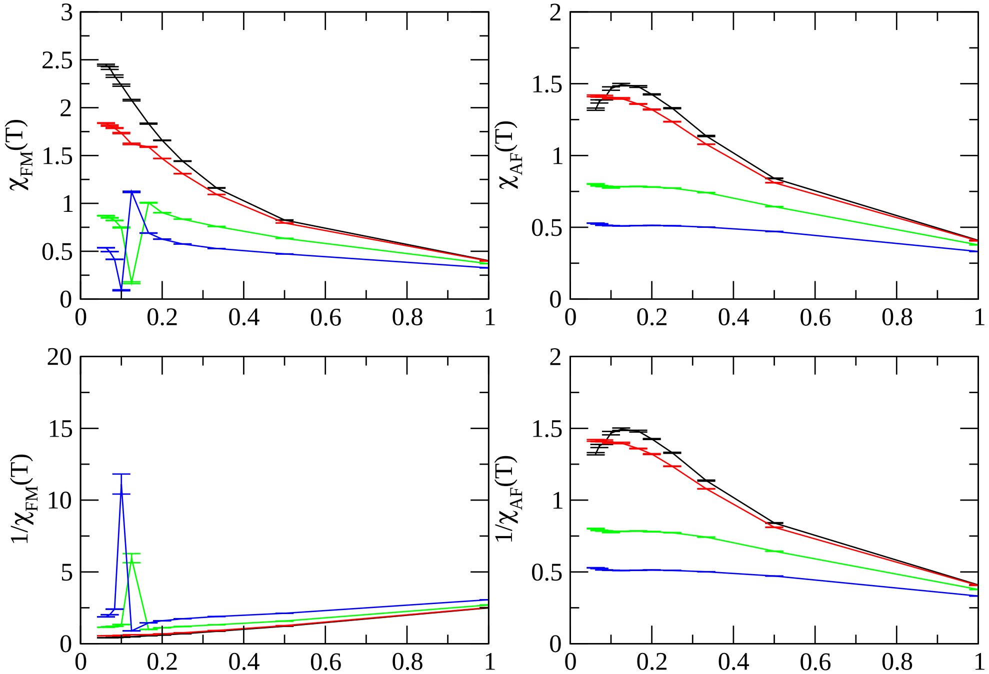


- divergence in AF susceptibility
- negative sign of AF susceptibility
- no singularity in other thermodynamic quantities
- zero magnetization, increasing local moments

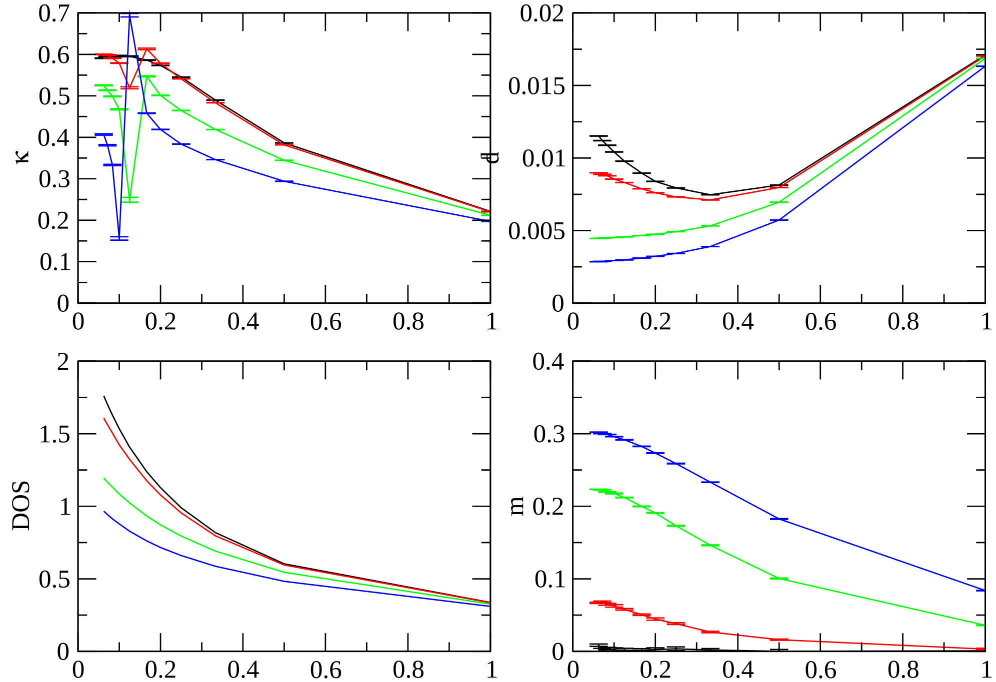
Hubbard model with spin dependent disorder - results

Strongly interacting system away from half-filling

n=0.5, U=2.0, W=2.0



n=0.5, U=2.0, W=2.0

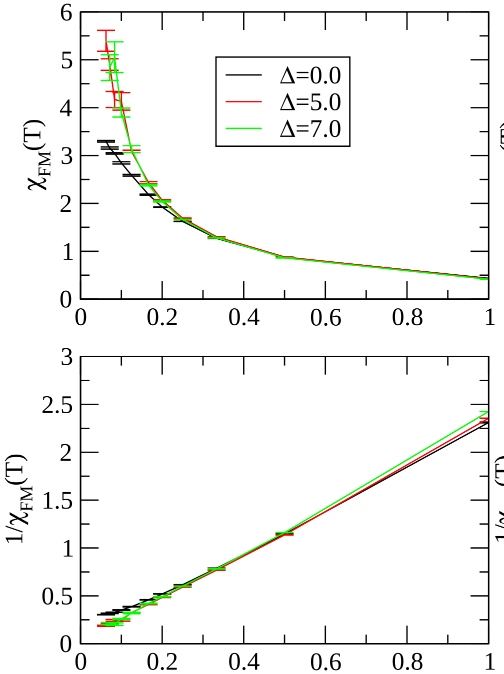


- divergence in FM susceptibility and compressibility
- negative sign of FM susceptibility and compressibility
- no singularity in other thermodynamic quantities
- finite magnetization, increasing local moments

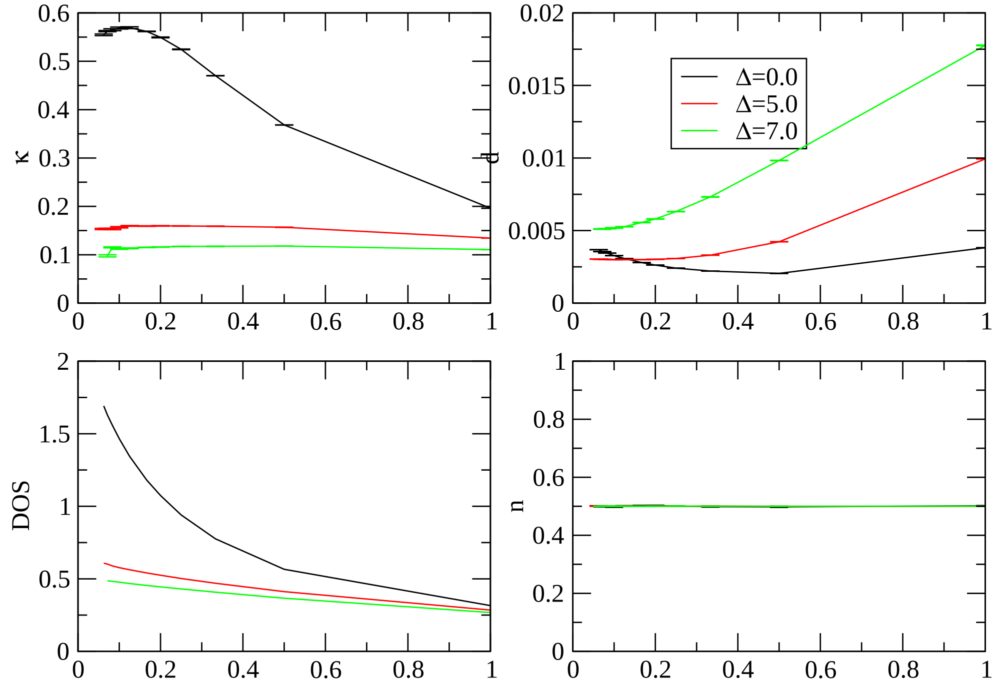
Hubbard model with normal disorder - comparison

Strongly interacting system away from half-filling

n=0.5, U=4.0, W=2.0



n=0.5, U=4.0, W=2.0

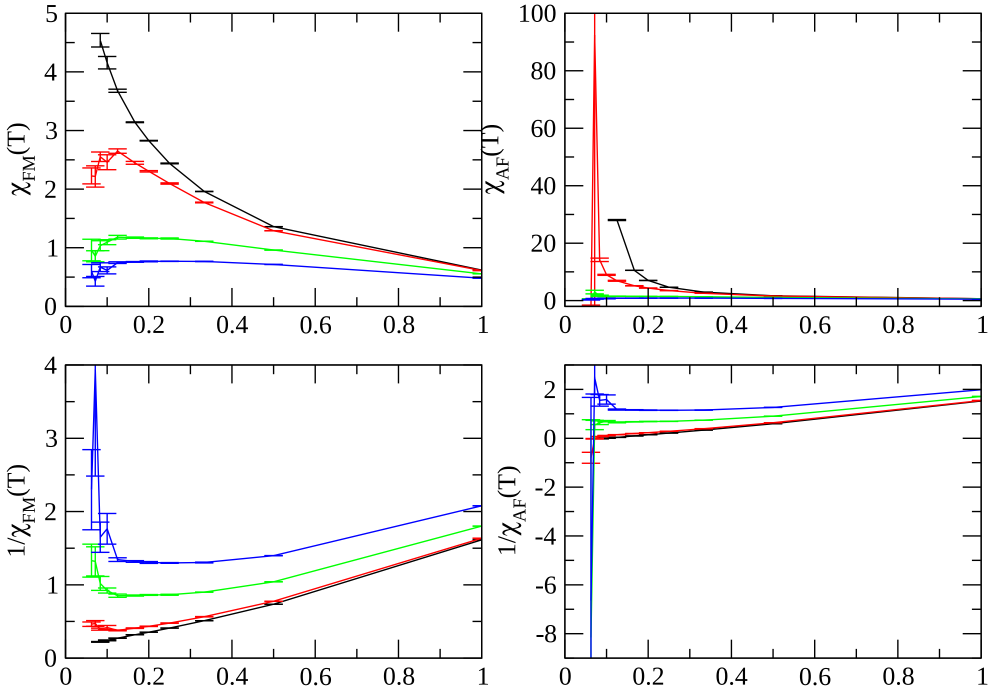


- singularity in FM susceptibility and AF susceptibility
- susceptibility crossing
- no singularity in other thermodynamic quantities
- zero magnetization, decreasing/increasing local moments

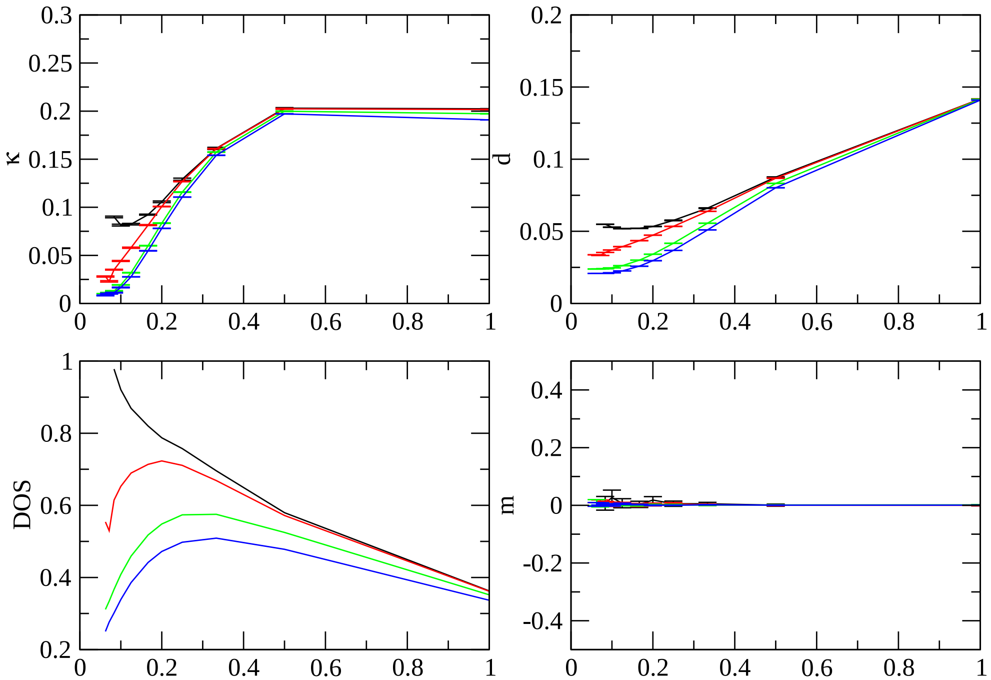
Hubbard model with spin dependent disorder - results

Strongly interacting system at half-filling

n=1.0, U=2.0, W=2.0



n=1.0, U=2.0, W=2.0



- divergence in AF susceptibility
- negative sign of AF susceptibility
- no singularity in other thermodynamic quantities
- zero magnetization, increasing local moments

Conclusions and outlook

1. Wanted better understanding
2. Why susceptibilities/compressibility change signs
3. First order transitions, instabilities toward which phases
4. Any suggestions