Viscous Fingering Patterns in Rectangular Grid Geometry

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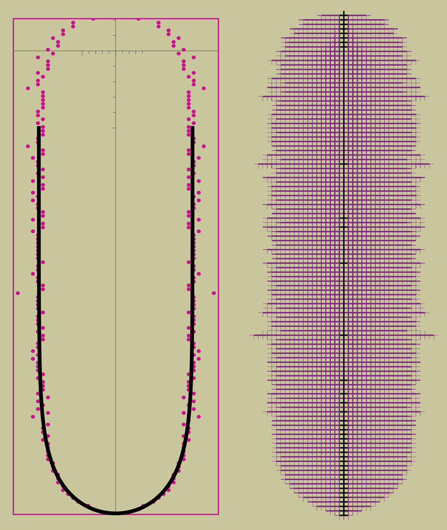
Introduction

Viscous fingering experiments were performed by injecting a less viscous liquid to displace a more viscous one in a regular, rectangular network of channels. This geometry promotes the formation of anisotropic, dendrite-like structures, which then compete with each other for the available flow. This may lead to the appearance of a scale-free, hierarchical growth patterns. We analyze this system both experimentally (in polycarbonate microfluidic channels) and numerically (using a a resistor-network model) for miscible and immiscible fluids, identifying different growth regimes.

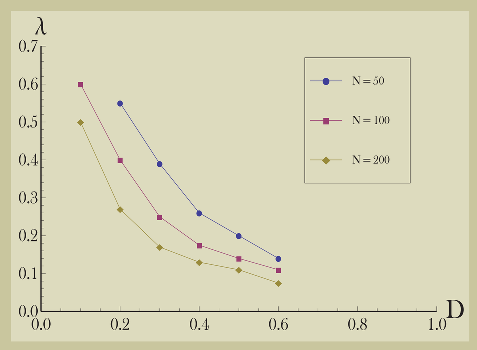
Single Finger

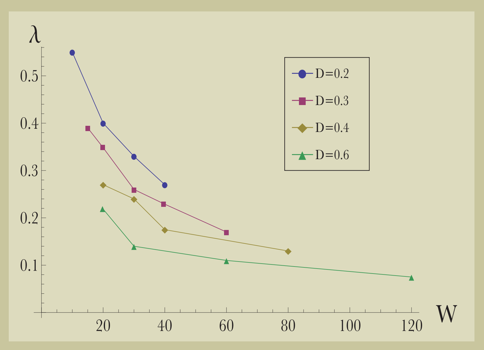
Single thick finger (less viscous fluid injected by a single channel) was examined. The shape of a finger was approximated with a formula describing the shape of Saffman-Taylor fingers.

Width of the finger depends on geometry of the grid and the system size. Values smaller than 0.5 were acquired, which is a lower limit for classical Saffman-Taylor fingers.

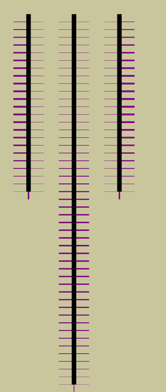


$$y(x) = \frac{1-\lambda}{\pi}\mathrm{Log}\left(\frac{1}{2} + \frac{1}{2}\mathrm{Cos}\left(\frac{\pi x}{\lambda}\right)\right)$$

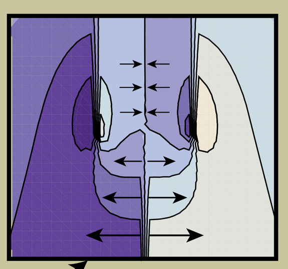


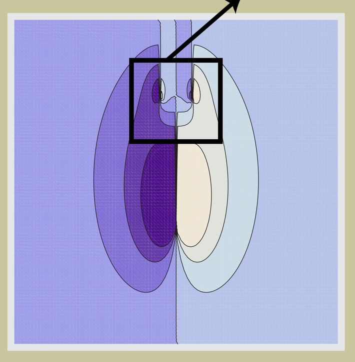


Interactions Between Fingers

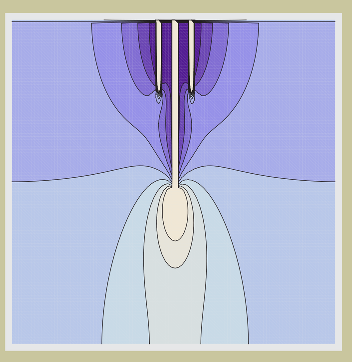


Shorter fingers are deprived of flow by longer one which grows faster. Positive FEEDBACK occurs and longer fingers are privileged. On the other hand we observe a change in the direction of flow in horizontal channels, which may cause tearing off of a longer finger and formation of a SPEAR-like head which moves separately.





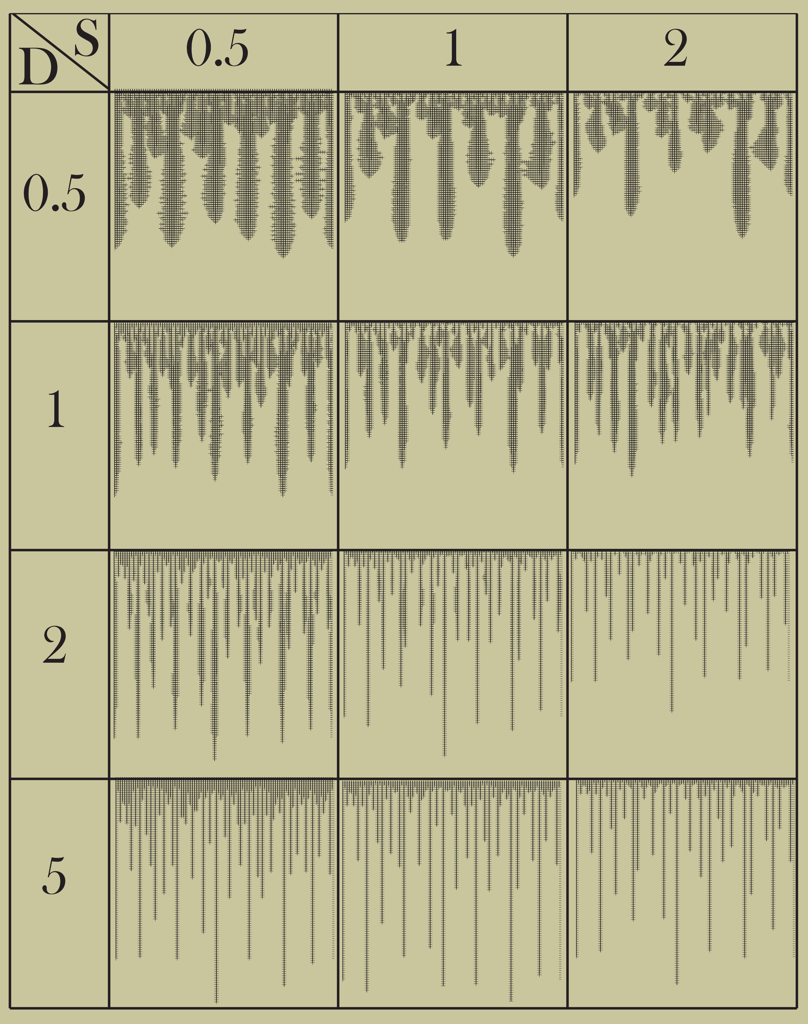
flow through horizontal channels

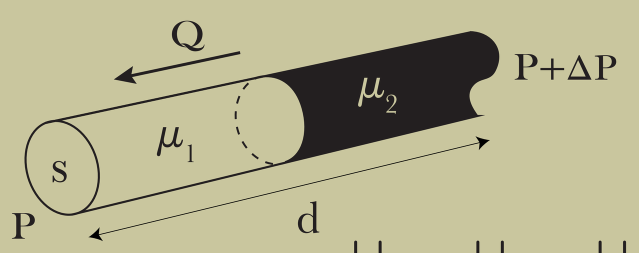


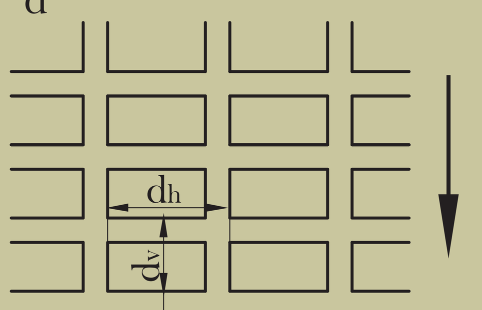
flow through vertical channels

Numerical Simulation
immiscible fluids

$$\Delta P \sim \frac{d\mu}{s^2} Q$$





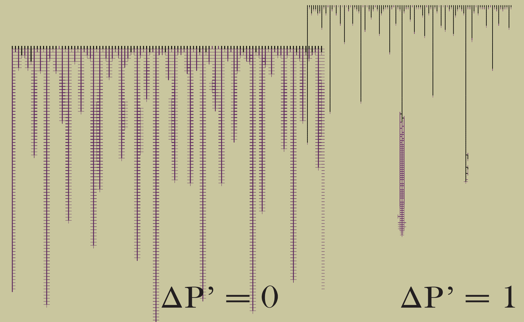


RESISTORS NETWORK model was used since flow is in direct proportion to pressure. Different shapes of dendrites were obtained. Surface tension could be taken into account by adding extra pressure difference ΔP' on the boundary of two fluids.

$D = d_h/d_v$

$S = s_h/s_v$

$\mu_1/\mu_2 = 500$

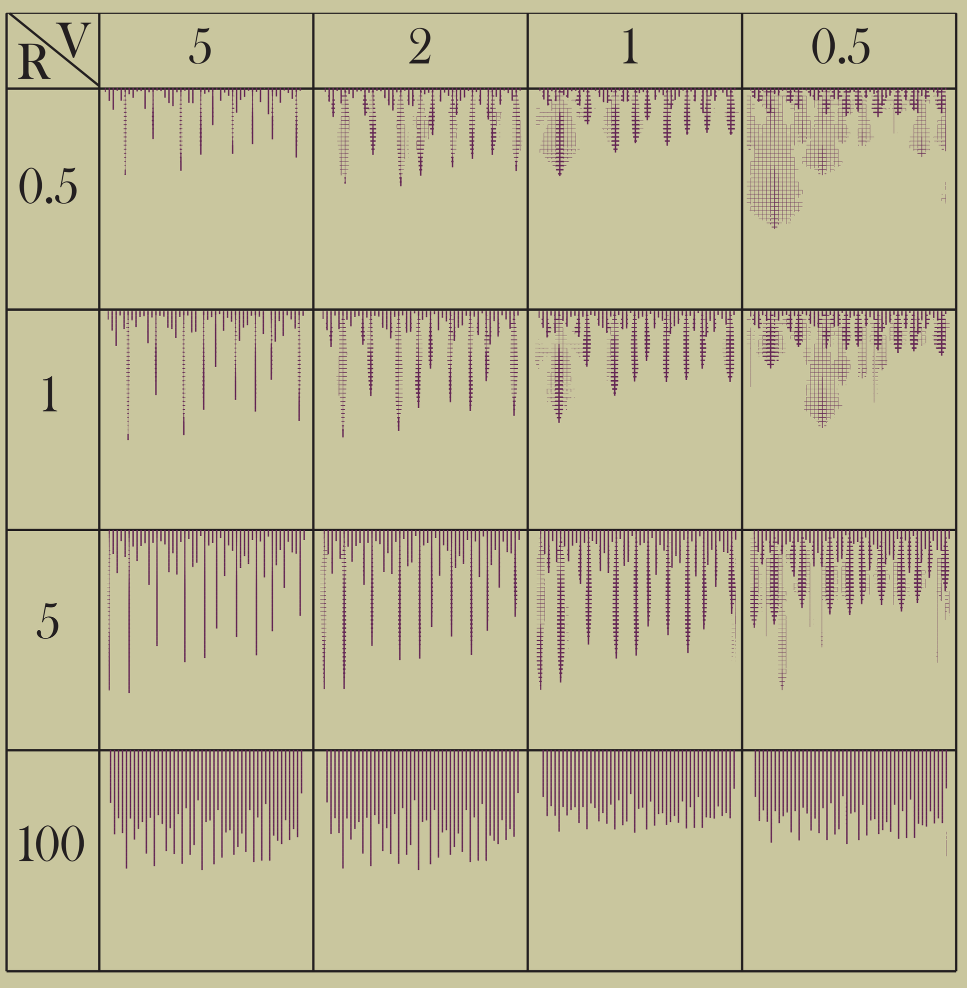


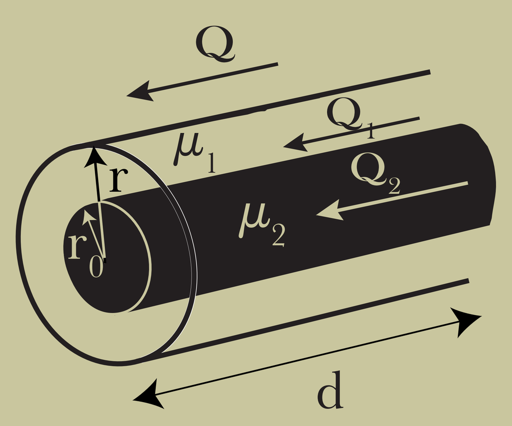
Numerical Simulation
miscible fluids

$$\Delta P = \frac{6\, d\mu_1\mu_2}{\mu_1 r^3 - \mu_2 r^3 + \mu_2 r_0^3} Q$$

$$\Delta P = \frac{12 d\,\mu_1}{(r - r_0)^2 (r + 2r_0)} \quad Q_1$$

$$\Delta P = \frac{12 d\,\mu_1\mu_2}{r\,(2\mu_1 r^2 + 3\mu_2(r_0^2 - r^2))} \quad Q_2$$





Resistor network model was applied, but this time two liquids could move PARALLEL to each other in the same channel.

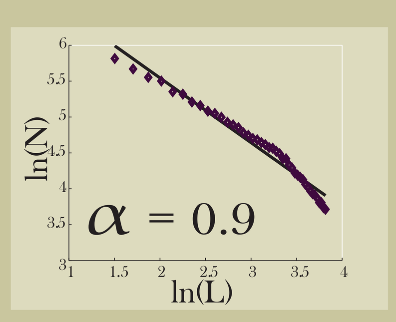
Grid geometry is best described in terms of relative resistance and volume of vertical and horizontal channels.

$R = D/S^2 \quad V = DS$

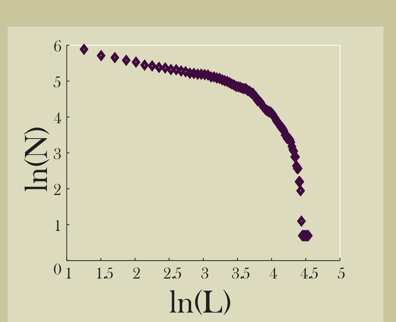
Distribution of Lengths

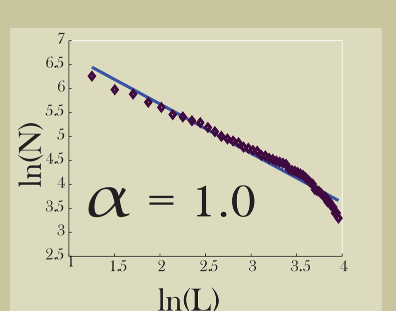
$N(L) \sim L^{\alpha}$ number of fingers longer than L

simulation

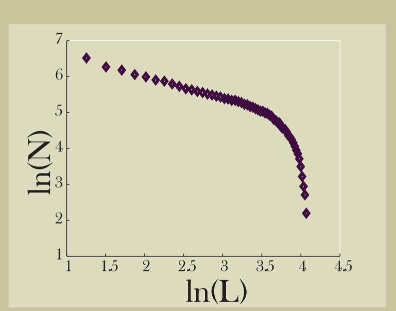


D = 3
S = 1



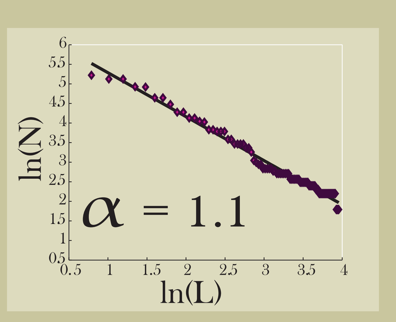


D = 5
S = 1



immiscible fluids miscible fluids

experiment



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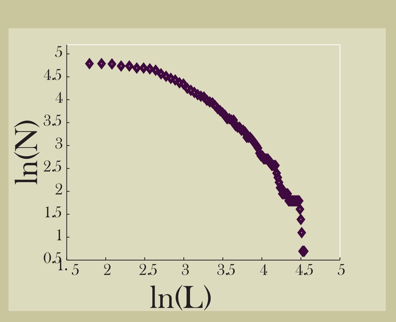
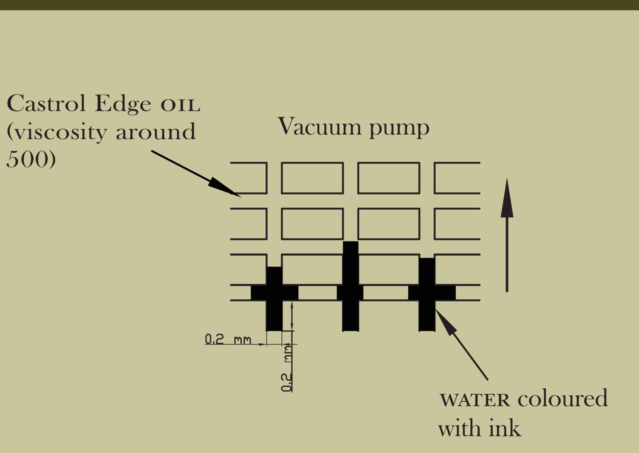
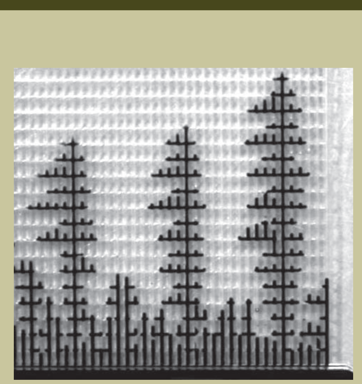
S = 1

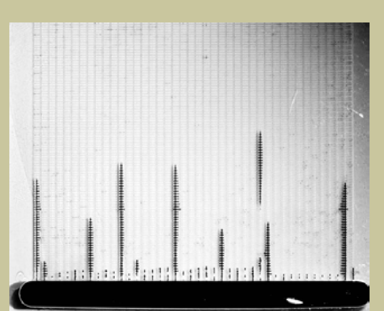


Experiment
immiscible fluids



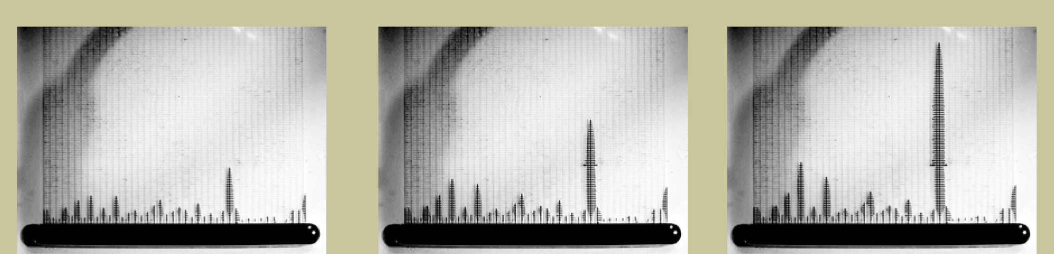


D = 0.5 S = 1



D = 3 S = 1

Three types of polycarbonate framework were used to perform an experiment. Different shapes of DENDRITES were obtained, depending on the geometry of the grid. If pressure per one channel was too high, heads of the dendrites had a tendency to separate.

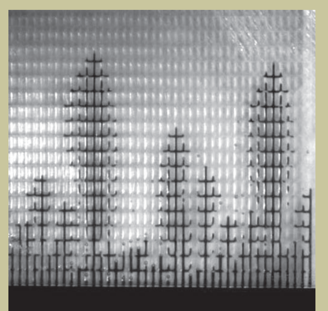


D = 5 S = 1

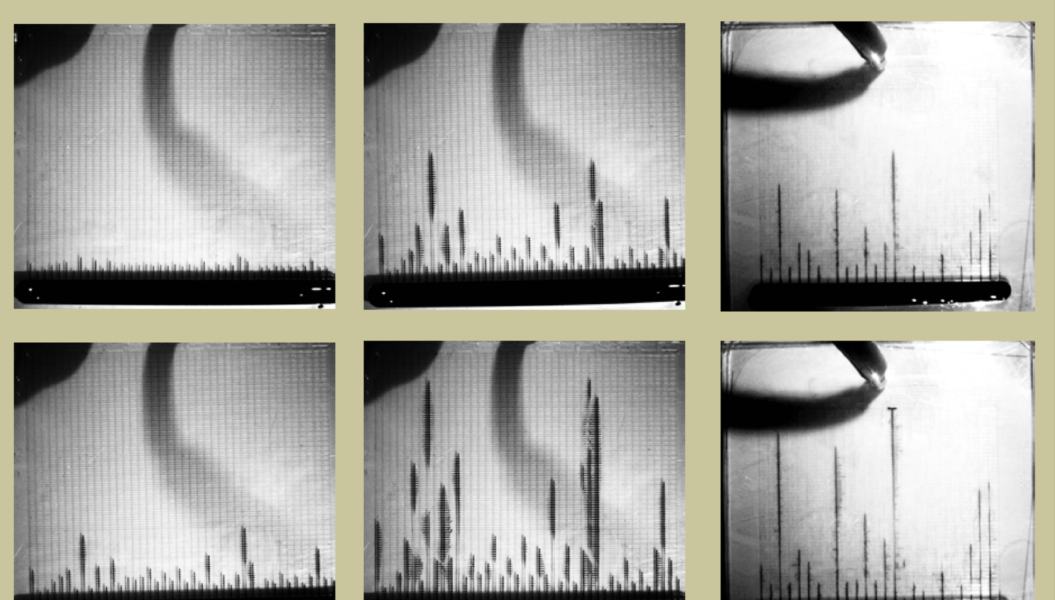
Experiment
miscible fluids

An experiment was carried out on the same experimental setup as in the previous case. Instead of oil, GLYCEROL was used as the more viscous fluid (viscosity around 500).

Observe the process of tearing off the heads of particular dendrites.



D = 0.5
S = 1



D = 3 S = 1 D = 5 S = 1

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