LHCphysics5

General formalizm to quantify the new physics effects:

S, T parameters

Assumption: "new" physics contributes mainly to the vacuum polarization of the electroweak gauge bosons

(this assumption is based on "experience" with models)

$$\tilde{\Pi}^{\rho\sigma}(k) = \left(g^{\rho\sigma}k^2 - k^\rho k^\sigma\right)\Pi(k^2)$$

This structure is fixed by Lorentz covariance and gauge invariance

In perturbative theories one can calculate $\tilde{\Pi}^{\rho\sigma}(k)$ perturbatively;

More general formalism (as for the Higgs model):

Gauge bosons couple to the currents of the $SU_L(2)\times U(1)$ symmetry which contain the fermionic and the Higgs part:

$$g W^a_\mu j_L^{a\mu} + g' B_\mu j_Y^\mu = g\left(W_\alpha^+ j_L^- + W_\alpha^- j_L^+\right) +$$

$$+ e A_\mu j_{EM}^\mu + \frac{g}{\cos\theta_W} Z_\mu \left(j_L^{3\mu} - \sin^2\theta_W\, j_{EM}^\mu \right)$$

$$j^\mu_{L,EM,Z} = j^{\mu(f)}_{L,EM,Z} + j^{\mu(H)}_{L,EM,Z} + j^{\mu(NEW)}_{L,EM,Z}$$

The gauge field propagator then reads:

$$\langle 0 | (A_\mu^{(0)} + A_\mu^{(1)})(A_\nu^{(0)} + A_\nu^{(1)}) | 0 \rangle =$$

$$= \langle 0 | A_\mu^{(0)} A_\nu^{(0)} | 0 \rangle + \langle 0 | A_\mu^{(1)} A_\nu^{(1)} | 0 \rangle =$$

$$= D_{\mu\nu} + \frac{P_{\mu\rho}}{k^2} g^2 \underbrace{\langle 0 | j_\Phi^\rho \, j_\Phi^\sigma | 0 \rangle}_{\substack{\text{this is called a vacuum} \\ \text{polarization tensor} \\ \Pi^{\rho\sigma}(k)}} \frac{P_{\sigma\nu}}{k^2}$$

Last lecture

$$j_L^{\pm(H)} = v\,\partial_\mu \pi^{\pm} + \cdots$$ → Goldstone bosons

$$j_Z^{(H)} \sim v\,\partial_\mu \pi^3 + \cdots$$

and, therefore,

$$\langle 0 | j^{\pm\,\mu}_{L,Z} | \pi^{\pm,3} \rangle \sim v\, k_\mu\, e^{-ikx}$$

Similarly as for U(1) case, we can discuss now the full propagator for the gauge bosons, starting with free propagator $1/k^2$ and including interactions

We remember that the Goldstone boson contributes

$$g^2 \langle 0 | j^\rho_\Phi | \chi \rangle \langle \chi | j^\sigma_\Phi | 0 \rangle = k^\rho k^\sigma g^2 v^2 / k^2$$

Hence $\Pi_W(k^2) = \dfrac{g^2 v^2}{k^2}$ $\qquad \Pi_Z(k^2) = \dfrac{g^2 v^2}{\cos^2\theta_W \, k^2}$

The full gauge boson propagator is

$$G_{\mu\nu} = \frac{-i P_{\mu\nu}}{k^2} + \frac{(-i P_{\mu\rho})}{k^2} i\Pi^{\rho\sigma}(k) \frac{(-i P_{\sigma\nu})}{k^2} =$$

$$= \frac{-i P_{\mu\nu}}{k^2} \left(1 + \Pi(k^2)\right)$$ (in one loop)

Summing up the whole series we get

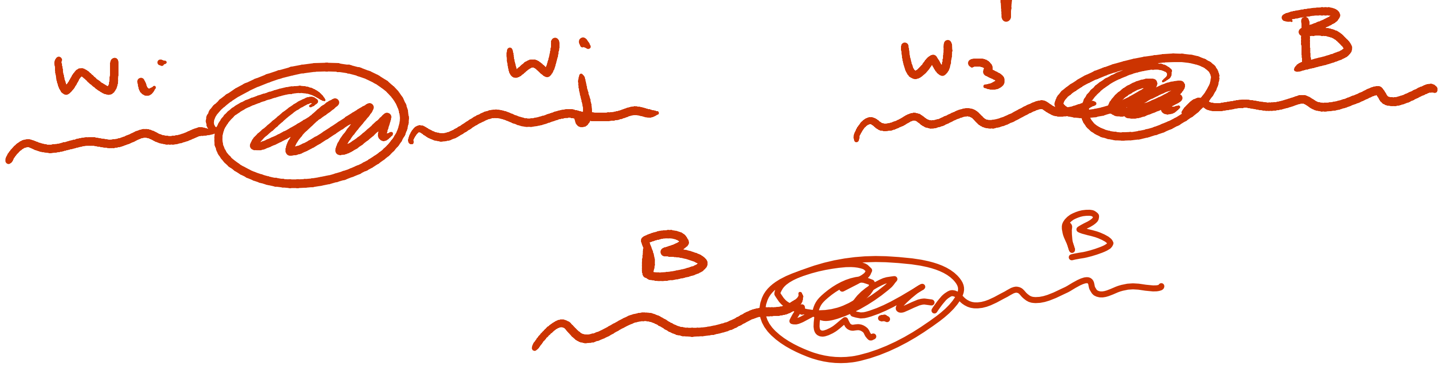
$$\boxed{G_{\mu\nu} = \frac{-i P_{\mu\nu}}{k^2 (1 - \Pi(k^2))}}$$

The Goldstone pole is the origin of mass

The photon should remain massless; it does not get a contribution from the Goldstone pole:

$$\tilde{\Pi}_{\gamma\gamma}(k^2) = k^2\,\Pi_{\gamma\gamma}(k^2) = 0 \quad \text{for} \quad k^2 = 0$$

Electroweak vacuum polarizations



or, equivalently,

$$\tilde{\Pi}_{\gamma\gamma}(k^2=0) = 0$$

$$\tilde{\Pi}_{Z\gamma}(k^2=0) = 0$$

$$\tilde{\Pi}_{W^+W^-}(k^2) = 0$$

(charge conservation)

$$W_\pm = W_1 \pm i W_2$$

$$\tilde{\Pi}_{W^+W^-}(k^2) = \tilde{\Pi}_{11}(k^2) - \tilde{\Pi}_{22}(k^2) + i(W_1 W_2 + W_2 W_1) \Rightarrow$$

$$\tilde{\Pi}_{11}(k^2) = \tilde{\Pi}_{22}(k^2) \qquad \tilde{\Pi}_{ij} = 0 \quad (i \neq j)$$

General formalism :

tilde is omitted

$$i g^{\mu\nu}\,\Pi_{XY}(k^2) + k^\mu k^\nu \text{ terms} \equiv$$

$$\int d^4x\, e^{-ikx} \langle 0 | J^\mu_X(x) J^\nu_Y(0) | 0 \rangle$$

$(XY) = (11), (33), (3Q), (QQ)$

(taken as independent)

We can define (notation of Peskin, Takeuchi)

$$\Pi_{XY}(k^2) = \Pi_{XY}(0) + k^2\, \Pi'_{XY}(k^2)$$

Note that $\Pi'_{XY}(k^2)$ is equal to

$d\Pi_{XY}/dk^2$ only at $k^2 = 0$

Remember that $\Pi_{QQ}(0) = 0$

$\Pi_{3Q}(0) = 0$

Vacuum polarizations (including the coefficients)

$\tilde{\Pi}_{\gamma\gamma}(k^2) = e^2\Pi_{QQ}$, $\tilde{\Pi}_{Z\gamma} = \frac{e^2}{sc}(\Pi_{3Q} - s^2\Pi_{QQ})$

$\tilde{\Pi}_{ZZ}(k^2) = \frac{e^2}{s^2c^2}(\Pi_{33} - 2s^2\Pi_{3Q} + s^4\Pi_{QQ})$

$\tilde{\Pi}_{WW} = \frac{e^2}{s^2}\Pi_{11}$

and

$$\Pi_{QQ}(k^2) = k^2\,\Pi'_{QQ}(k^2)$$

$$\Pi_{3Q}(k^2) = k^2\,\Pi'_{3Q}(k^2)$$

Keeping terms up to k^2, we have 6 parameters

$$\Pi_{\gamma\gamma} \approx k^2\,\Pi'_{\gamma\gamma}(0)\,, \quad \Pi_{3\gamma} \approx k^2\,\Pi'_{3\gamma}(0)$$

$$\Pi_{33} \approx \Pi_{33}(0) + k^2\,\Pi'_{33}(0)$$

$$\Pi_{11} \approx \Pi_{11}(0) + k^2\,\Pi'_{11}(0)$$

We fix 3 combinations of those parameters by using

$$\alpha^{-1} = 137$$

$$G_F = 1.166 \times 10^{-5} (\mathrm{GeV})^{-2}$$

$$m_Z = 91 \ \mathrm{GeV}$$

The three combinations that remain are such that the UV cancel out (because the UV divergences are already included in α, G_F, m_Z)

These are

$$\alpha S \equiv 4e^2 \left[\Pi'_{33}(0) - \Pi'_{3Q}(0)\right]$$

$$\alpha T \equiv \frac{e^2}{s^2 c^2 m_Z^2} \left(\Pi_{11}(0) - \Pi_{33}(0)\right]$$

$$\alpha U \equiv 4e^2 \left[\Pi'_{11}(0) - \Pi'_{33}(0)\right]$$

(In many models U is small and, instead, more important are $O(k^4)$ terms $Y \sim \Pi''_{YY}$, $W \sim \Pi''_{33}$, $X \sim \Pi''_{3Y}$)

Why (and when) those parameters are finite in SM ?

a) the oblique corrections can be renormalized using α, G_F, m_Z (but not all radiative corrections)

Another argument in favour of this choice: chiral symmetry, custodial symmetry

But S, T are not necessari finite with new physics (may depend on cut-off)

Using previous formulae one can easily check that

$$\alpha T \equiv \frac{e^2}{s^2 c^2 M_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right] =$$

$$= \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} = \rho - 1$$

$$\alpha S = 4e^2\left[\Pi'_{33}(0) - \Pi'_{3\gamma}(0)\right] = 4e^2\,\Pi'_{3Y}(0)$$

The logic for using experimental data

1) Write $S = S_{ref} + \Delta S$

$T = T_{ref} + \Delta T$

where S_{ref}, T_{ref} are calculated in the SM; all the parameter are know from experiment, except the Higgs boson mass; for S_{ref}, T_{ref} take, for instance,

$m_H = 114$ GeV

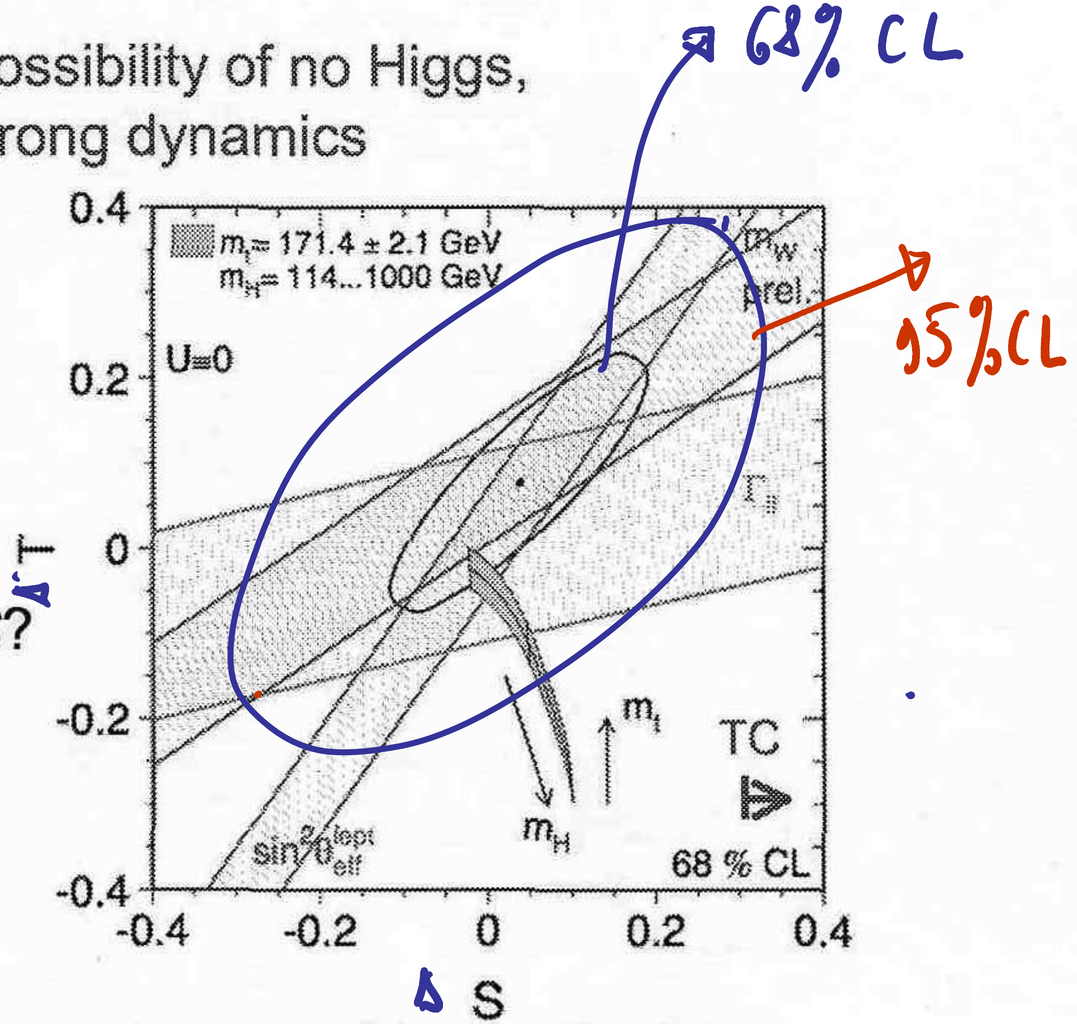
2) Fit ΔS and ΔT to precision (LEP) electroweak data by comparing with experiment the theoretical predictions with S & T included in the calculation

One gets the limits on ΔS & ΔT as below:

Still open is the possibility of no Higgs, and new strong dynamics

- flavour?
- EW precision data?

$$\Delta S \approx \frac{1}{6\pi}\left(n_{TF}N_{TC} + \ln\frac{\Lambda_{TC}}{m_Z}\right)$$



Negative contributions to S?

Hirn-Sanz; Delgado-Falkowski; Agashe-Csaki-Grojean-Reece

Walking at small N_{TC}?

Foadi-Frandsen-Ryttov-Sannino; Piai

Higgsless? EW broken by boundary conditions with KK gauge bosons curing unitarity up to about 10 TeV (or less)

Csaki-Grojean-Pilo-Terning

3) Remembering the SM results as a function of the top mass and the Higgs mass:

$$S_{SM} = -\frac{1}{6\pi}\ln\frac{m_t}{M_W} + \frac{1}{12\pi}\frac{m_H}{M_W}$$

one gets

$$\Delta S \simeq \frac{1}{12\pi}\ln\frac{m_H^2}{m_{H,ref}^2} - \frac{1}{6\pi}\ln\frac{m_t^2}{m_{t,ref}^2}$$

and similarly

$$\Delta T \approx \frac{3}{16\pi s^2 c^2} \left[\frac{m_t^2 - m_{t,\mathrm{ref}}^2}{m_Z^2} \right]$$

$$- \frac{3}{16\pi c^2} \ln \left[\frac{m_H^2}{m_{H,\mathrm{ref}}^2} \right]$$

One can then adjust the comparison with experimental data to new measurements of the top quark mass without performing a new fit to precision LEP data.

Furthermore one can see what happens as a function of the Higgs Boson mass. This is seen at the plot above.

Finally, one can check the effects of the new physics contributions.

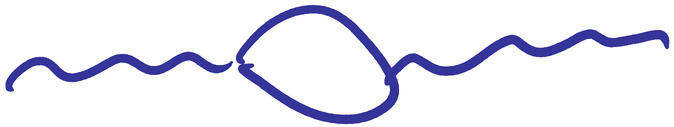
Two possibilities:

1) perturbative new physics
2) non-perturbative new physics

Perturbative physics beyond the SM,

e.g. supersymmetry ⇒ new contributions

to



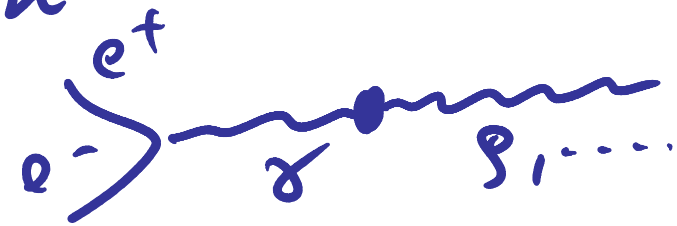
⇒ superpartners

but, if only at the loop level (as in

MSSM), no problem with S & T

Non-perturbative physics, like technicolour, with resonances coupled to W, Z

(similar to resonances in e^+e^- annihilation



)

We can estimate S by using the representation of Π's in terms of $\langle 0 | J J | 0 \rangle$

Let's assume the technicolour interactions conserve weak isospin and parity;

since $J_3^\mu = \frac{1}{2}(J_V^\mu - J_A^\mu)$, $J_Q^\mu = J_V^\mu + \frac{1}{2} J_Y^\mu$,

we get
$$\Pi_{33} = \frac{1}{4}(\Pi_{VV} + \Pi_{AA}), \quad \Pi_{3Q} = \frac{1}{2}\Pi_{VV}$$

Since the vector symmetries are exact, while the axial-vector symmetries are spontaneously broken,

$$\Pi_{VV}(q^2) = q^2 \, \Pi'_{VV}(q^2)$$

$$\Pi_{AA}(q^2) = \underbrace{\Pi_{AA}(0)}_{\text{from the Goldstone pole}} + q^2 \, \Pi'_{AA}(q^2) =$$

$$= v^2 + q^2 \, \Pi'_{AA}(q^2)$$

In terms of Π_{VV} and Π_{AA} we find

$$S = -4\pi\left[\Pi'_{VV}(0) - \Pi'_{AA}(0)\right]$$

We can now use dispersion relations to calculate $\Pi'_{VV}(0)$ and $\Pi'_{AA}(0)$.

Cauchy theorem (under certain assumptions about analiticity structure):

$\Pi \sim$ 

For an analitic function inside C

$$\Pi(t) = \frac{1}{2\pi i} \int_c \frac{\Pi(t')\,dt'}{t'-t}$$

Let's also assume $\Pi(t^*) = \Pi^*(t)$

so $\Pi(t^*) - \Pi(t) = 2i\,\mathrm{Im}\,\Pi$

If $\mathrm{Im}\,\Pi \neq 0$ then $\Pi(t)$ has a cut;
if it's on the real axis then

$$\Pi(t+i0) = \frac{1}{\pi} \int_0^\infty \frac{\mathrm{Im}\,\Pi(t')}{t'-(t+i0)}\,dt'$$

Useful "formula":

$$\frac{1}{x \pm i0} = P\,\frac{1}{x} \mp i\pi\,\delta(x)$$

One assumes (sometimes one can prove, e.g. in perturbation theory) analiticity of various scattering amplitudes in the complex plane, except for the cut on the real axis because $\mathrm{Im}\,\Pi \neq 0$

Unitarity of S - matrix

$$SS^{+} = 1 \qquad (*)$$

Let's define matrix T

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^{(4)}(P_f - P_i) T_{fi}$$

From (*) we get

$$T_{fi} - T^{*}_{if} = i(2\pi)^4 \sum_n \delta^{(4)}(P_f - P_n) T_{fn} T^{*}_{in}$$

$$\left[i(T - T^{+}) = TT^{+} \right]$$

Let's take $i = f$ (initial and final states are the same):

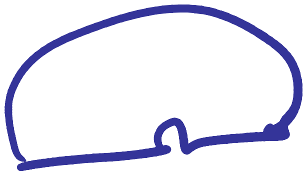
$$2\,\mathrm{Im}\,T_{ii} = (2\pi)^4 \sum_n |T_{in}|^2 \delta^{(4)}(P_i - P_n)$$

$\sim \sigma_{tot}$ from the initial state i

(optical theorem)

$$\mathrm{Im}\,T_{WW} \simeq \sigma_{tot}^{W\to\text{anything}}$$

Secondly, one has to know something about the behaviour for $|t| \to \infty$, to be sure that the integral over the large circle



converges. If not we have to use the dispersion relation with subtraction:

$$\Pi(t) - \Pi(t_0) = \frac{1}{\pi}\int \frac{\operatorname{Im}\Pi(t')}{t'-t} - \frac{1}{\pi}\int \frac{\operatorname{Im}\Pi(t')}{t'-t_0} =$$

$$= (t - t_0) \frac{1}{\pi} \int dt' \frac{\operatorname{Im} \Pi(t')}{(t'-t)(t'-t_0)}$$

better convergence

But then $\Pi(t)$ depends on the subtraction constant $\Pi(t_0)$; interesting link to renormalization procedure in quantum field theory (one can calculate Feynman diagrams by using dispersion relations)

$$A(q^2) = \frac{1}{\pi} \int_0^\infty ds \, \frac{\operatorname{Im} A(s)}{s - q^2}$$

Therefore

$$\Pi_{VV}(q^2) - \Pi_{AA}(q^2) = q^2 \left[\Pi'_{VV}(q^2) - \Pi'_{AA}(q^2) \right]$$

$$- \Pi_{AA}(0) = \frac{q^2}{\pi} \int_0^\infty ds \, \frac{\left[\operatorname{Im} \Pi'_{VV}(s) - \operatorname{Im} \Pi'_{AA}(s) \right]}{s - q^2}$$

$$- F_\pi^2$$

Hence

$$S = \frac{1}{2\pi} \int_0^\infty \frac{ds}{s} \left[\operatorname{Im} \Pi'_{VV} - \operatorname{Im} \Pi'_{AA} \right]$$

How to calculate Im's ?

Resonance saturation:

$$\operatorname{Im} \Pi'_{VV} \sim F_\rho^2 \, \delta(s - m_\rho^2)$$

$$\operatorname{Im} \Pi'_{AA} \sim F_A^2 \, \delta(s - m_A^2)$$

We can calculate F_ρ & F_A in terms of the masses by using the so-called Weinberg sum rules.

Finally $\quad S = 4\pi \left\{ 1 + \frac{m_\rho^2}{m_A^2} \right] \frac{v^2}{m_\rho^2} > 0$

Dispersion relations for the Π's can be proved rigorously (using causality) and they are

called Källen-Lehmann

spectral representation for the product of two currents

$$\int d^4x \, e^{ikx} \langle 0 | T j^a_\mu(x) j^b_\nu(0) | 0 \rangle =$$

$$= i \int_0^\infty ds \, \frac{\rho^{ab}_{\mu\nu}(k, k^2=s)}{s-k^2}$$

where

$$(2\pi)^3 \sum_n \langle 0 | j^a_\mu(0) | n \rangle \langle n | j^b_\nu(0) \delta(k-k_n)$$

$$\equiv \left(g_{\mu\nu} - k_\mu k_\nu / s \right) \rho^{ab}(s)$$

(for spin 1 intermediate states)

$$\equiv k_\mu k_\nu \rho^{ab}(s)$$ for spin-zero intermediate states

For resonance saturation we have

$$\rho_V = F_V^2 \, s \, \delta(s - m_\rho^2)$$

$$\rho_A = F_A^2 \, s \, \delta(s - m_A^2)$$

and we recover the previous results

Important final conclusion: technicolor-like theories give positive contribution to S (now look again at the plot!)