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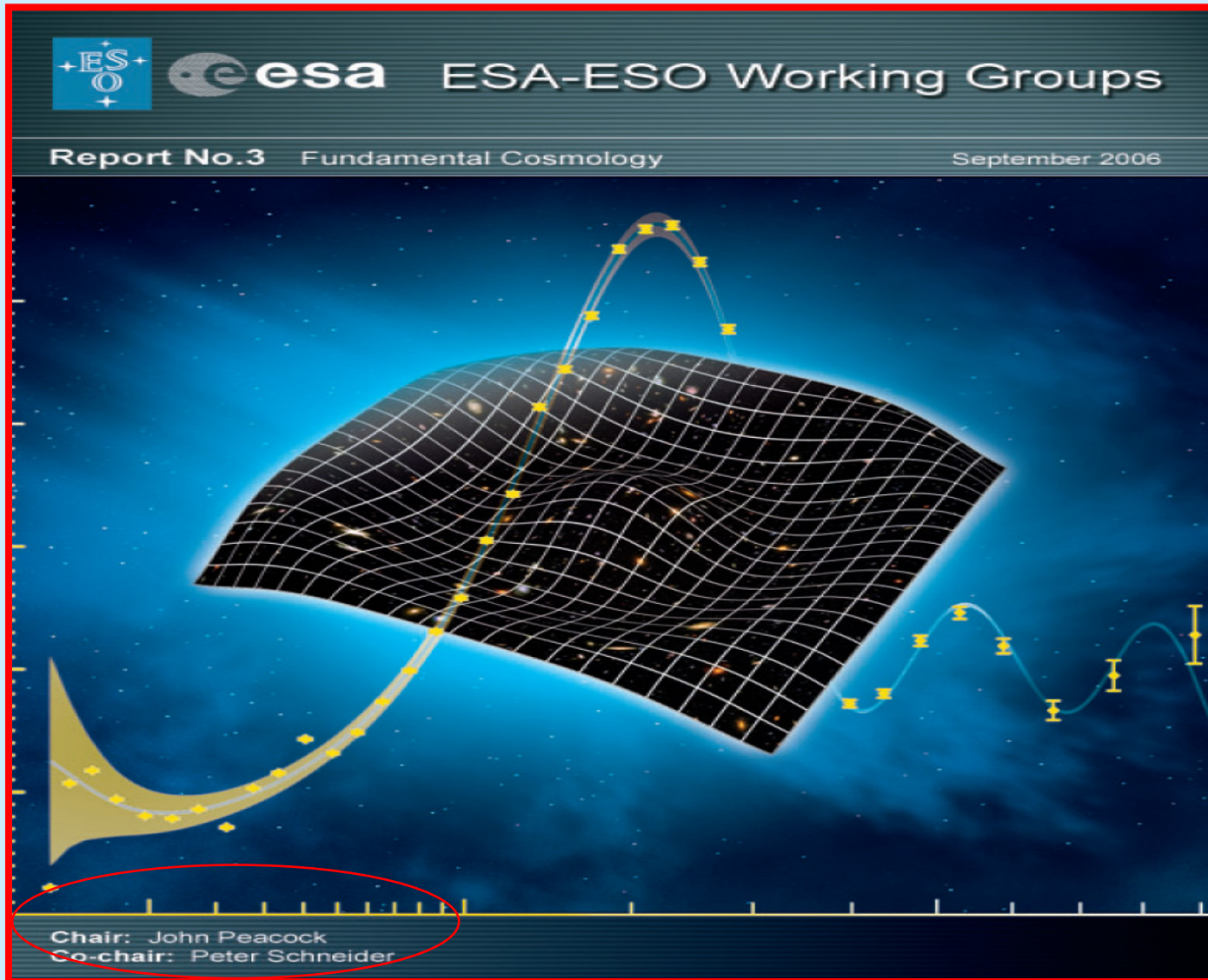
**Activities of the Dept. of
Astrophysics and Cosmology
U. of Silesia**

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Katowice, Poland

Activities of the Department - put in the PERSPECTIVE

1. Dark matter/energy problem -fitting models to observations
2. Dark matter/energy problem as a problem of model selection -- information theoretic model selection approach
3. Astrophysical bound on exotic (non-standard) physics
4. Gravitational wave experiments in cosmological context
5. Last stages of stellar evolution
6. Model - theoretical approaches to Quantum Gravity

What is the PERSPECTIVE ?



Activities of the Department - put in the PERSPECTIVE

Report by the ESA-ESO Working Group on Fundamental Cosmology

Abstract

In September 2003, the executives of ESO and ESA agreed to establish a number of working groups to explore possible synergies between these two major European astronomical institutions on key scientific issues. The first two working group reports (on Extrasolar Planets and the Herschel–ALMA Synergies) were released in 2005 and 2006, and this third report covers the area of Fundamental Cosmology.

The Working Group's mandate was to concentrate on fundamental issues in cosmology, as exemplified by the following questions: (1) What are the essential questions in fundamental cosmology? (2) Which of these questions can be tackled, perhaps exclusively, with astronomical techniques? (3) What are the appropriate methods with which these key questions can be answered? (4) Which of these methods appear promising for realization within Europe, or with strong European participation, over the next ~ 15 years? (5) Which of these methods has a broad range of applications and a high degree of versatility even outside the field of fundamental cosmology?

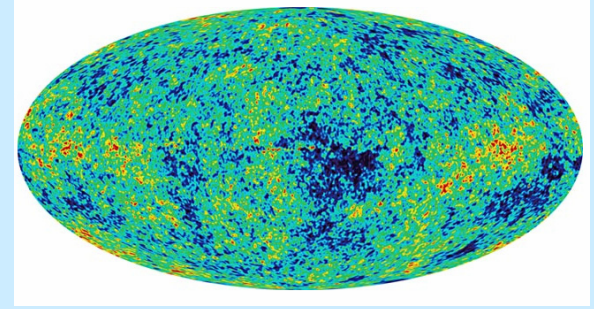
Key Questions:

key questions:

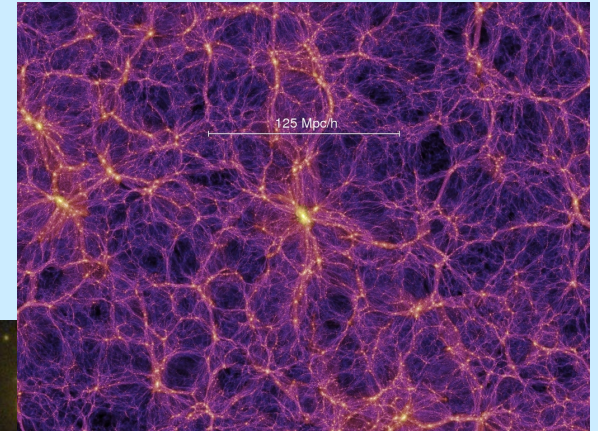
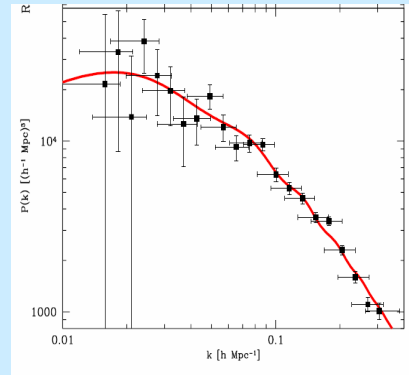
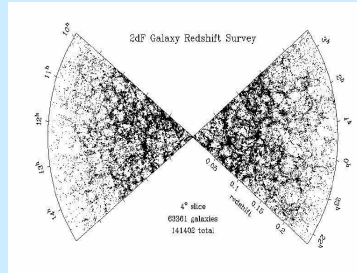
- (1) What generated the baryon asymmetry? Why is there negligible antimatter, and what set the ratio of baryons to photons?
- (2) What is the dark matter? Is it a relic massive supersymmetric particle, or something (even) more exotic?
- (3) What is the dark energy? Is it Einstein's cosmological constant, or is it a dynamical phenomenon with an observable degree of evolution?
- (4) Did inflation happen? Can we detect relics of an early phase of vacuum-dominated expansion?
- (5) Is standard cosmology based on the correct physical principles? Are features such as dark energy artefacts of a different law of gravity, perhaps associated with extra dimensions? Could fundamental constants actually vary?

Pillars of Modern Physical Cosmology

1. CMBR

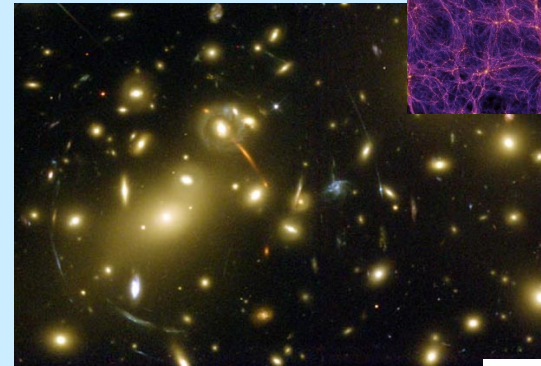


2. LSS

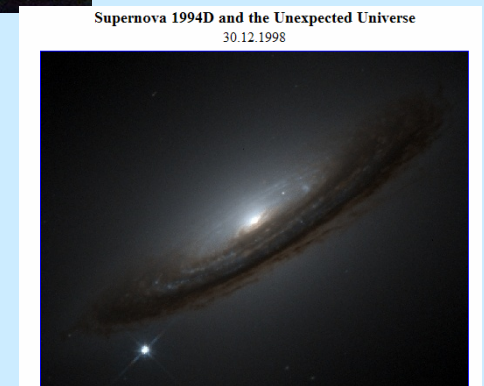


3. Gravitational lensing

4. Quasar spectra (La)



5. High redshift SN Ia



Credit: [High-Z Supernova Search Team](#), HST, NASA

Activities of the Department

M. Biesiada

- Dark energy in the Universe - tests and bounds
SNIa, BAO, acoustic peaks in CMBR
(partly in collaboration with Cracow, OAUJ)

Strong lensing systems as alternative test

- DE problem as a problem of model selection - many competing models with little theoretical guidance which is right ; Which model (or a class thereof) is the most supported by the data ?

(partly in collaboration with Cracow, OAUJ)

GENERALIZED CHAPLYGIN GAS MODELS TESTED WITH TYPE Ia SUPERNOVAE

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ABSTRACT

The generalized Chaplygin gas (GCG), with the equation of state $p = -A/\rho^\alpha$, was recently proposed as a candidate for dark energy in the universe. In this paper we confront the GCG with Type Ia supernova (SN Ia) data using available samples. Specifically, we have tested the GCG cosmology in three different classes of models with (1) $\Omega_m = 0.3$ and $\Omega_{\text{Ch}} = 0.7$, (2) $\Omega_m = 0.05$ and $\Omega_{\text{Ch}} = 0.95$, and (3) $\Omega_m = 0$ and $\Omega_{\text{Ch}} = 1$, as well as a model without prior assumptions on Ω_m . The best-fit models are obtained by minimizing the χ^2 function. We supplement our analysis with confidence intervals in the (A_0, α) -plane by marginalizing the probability density functions (pdf's) over the remaining parameters assuming uniform priors. We have also derived one-dimensional pdf's for Ω_{Ch} obtained from joint marginalization over α and A_0 . The maximum value of such a pdf provides the most probable value of Ω_{Ch} within the full class of GCG models. The general conclusion is that SN Ia data give support to the Chaplygin gas (with $\alpha = 1$). However, a noticeable preference for A_0 -values close to 1 means that the α dependence becomes insignificant. This is reflected in one-dimensional pdf's for α that turned out to be flat, meaning that the power of the present supernova data to discriminate between various GCG models (differing by α) is weak. Extending our analysis by relaxing the prior assumption of the flatness of the universe leads to the result that even though the best-fit values of Ω_k are formally nonzero, they are still close to the flat case. Our results show clearly that in GCG cosmology, distant (i.e., $z > 1$) supernovae should be brighter than in the Λ CDM model. Therefore, one can expect that future supernova experiments (e.g., *SNAP*) having access to higher redshifts will eventually resolve the issue of whether the dark energy content of the universe could be described as a Chaplygin gas. Moreover, it would be possible to differentiate between models with various values of the α -parameter and/or discriminate between GCG, Cardassian, and Λ CDM models. This discriminative power of the forthcoming mission has been demonstrated on simulated *SNAP* data.

Subject headings: cosmology: theory — distance scale — supernovae: general

Perspectives

Hubble diagrams on GRB data - via Ghirlanda relation

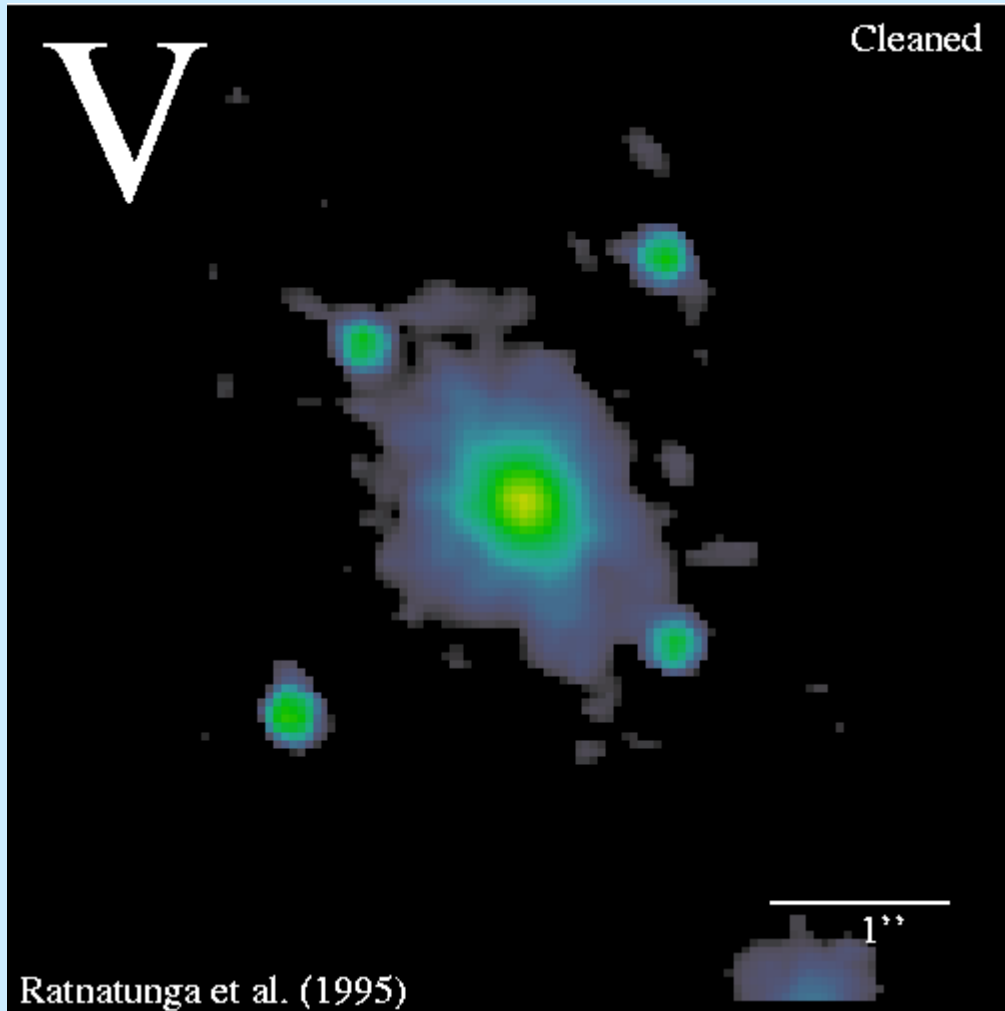
Possible collaboration --- T. Bulik CAMK/OA UW

Strong lensing systems as a probe of dark energy in the universe

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(Received 3 November 2005; published 23 January 2006)

Current advances in observational cosmology suggest that our Universe is flat and dominated by dark energy. Out of many particular models of dark energy present in the literature we focus on four: quintessence, quintessence with time varying equation of state, braneworld model and generalized Chaplygin gas model. In this paper we discuss the utility of strong lensing systems for providing additional constraints on dark energy models. In particular, we use an Einstein cross gravitational lensing system HST 14176 + 5226 to confront its measured characteristics with background cosmologies invoked in the context of dark energy. The image separations in the system depend on angular distances to the lens and to the source, which in turn are determined by background cosmology. This opens a possibility to constrain cosmological model provided that we have good knowledge of the lens model. We demonstrate that recent measurements of velocity dispersion in the lensing galaxy made by Subaru telescope seem to be consistent with independently obtained bounds on parameters of cosmological models considered. The method we describe is based on angular diameter distances and could become a valuable tool of cosmological model inference complementary to Hubble diagram technique based on luminosity distance.



Lensing system HST 14176+5226 in V band

$$z_L = 0.809$$

$$z_S = 3.4$$

$$\theta_E = 1.''489$$

SIE Lens model

$$\theta_E = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{LS}}{D_S}$$

Idea:

Take the expression for the Einstein ring

$$\theta_E = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{LS}}{D_S}$$

From angular measurements of images θ_1 and θ_2

$$\theta_1 = \theta_E \sqrt{1-\epsilon}$$

$$\theta_2 = \theta_E \sqrt{1+\epsilon}$$

From the SIE model fitting:
 $e=0.4$

$$\theta_E = 1.''489$$

Determined by the cosmological model

$$D_{ls} = \frac{1}{1+z_s} \frac{c}{H_0} \int_{z_l}^{z_s} \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_{Ch} (A_0 + (1-A_0)(1+z')^{3(1+\alpha)})^{\frac{1}{1+\alpha}}}}$$

SUBARU

LSD

$$\sigma_v = 290 \pm 8 \text{ km/s}$$

Quintessence

$$w < -0.67$$

Dynamical scalar field -

$$w(z) = w_0 + w_1 z$$

$$w_0 > -0.1$$

$$w_1 < -1.2$$

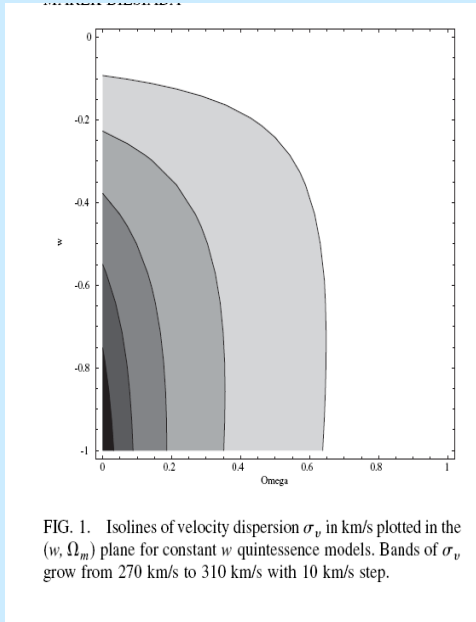


FIG. 1. Isolines of velocity dispersion σ_v in km/s plotted in the (w, Ω_m) plane for constant w quintessence models. Bands of σ_v grow from 270 km/s to 310 km/s with 10 km/s step.

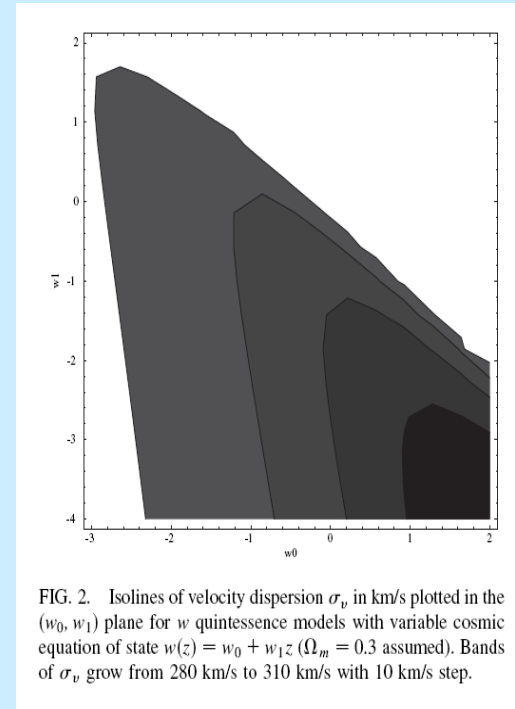
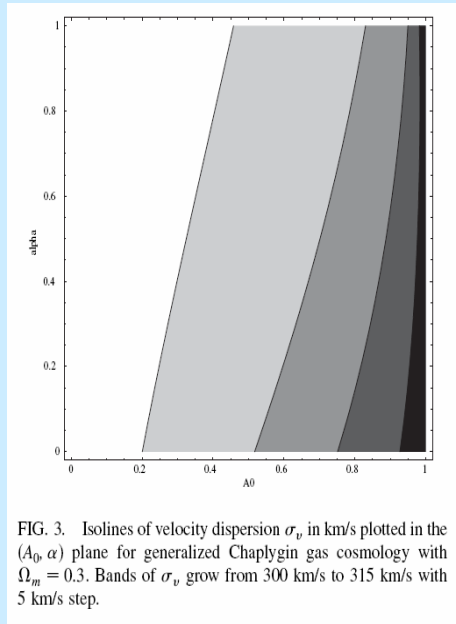


FIG. 2. Isolines of velocity dispersion σ_v in km/s plotted in the (w_0, w_1) plane for w quintessence models with variable cosmic equation of state $w(z) = w_0 + w_1 z$ ($\Omega_m = 0.3$ assumed). Bands of σ_v grow from 280 km/s to 310 km/s with 10 km/s step.



Chaplygin Gas

models with $A_0 \approx 1$ preferred
i.e. equivalent to Λ CDM

Brane world

$$1.09H_0^{-1} < r_c < 1.39H_0^{-1}$$

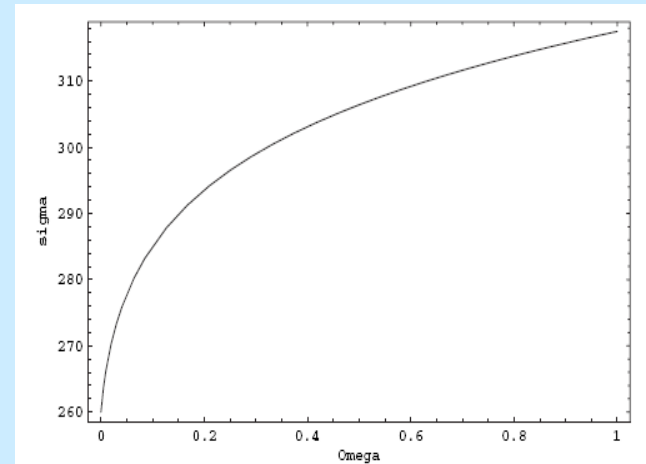


FIG. 4. Plot of $\sigma_v(\Omega_m)$ relation in braneworld model with flat spatial section.

Perspectives

Apply this for a sample of strong lensing systems from CLASS survey

Pros.

Method complementary to Hubble diagram
- different systematics

Cons.

Dependence on the lens model - mass density profile

Information theoretic model selection applied to supernovae data

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Abstract. Current advances in observational cosmology suggest that our Universe is flat and dominated by dark energy. There are several different theoretical ideas invoked to explain the dark energy with relatively little guidance of which one of them might be right. Therefore the emphasis of ongoing and forthcoming research in this field shifts from estimating specific parameters of the cosmological model to the model selection.

In this paper we apply an information theoretic model selection approach based on the Akaike criterion as an estimator of Kullback–Leibler entropy. Although this approach has already been used by some authors in a similar context, this paper provides a more systematic introduction to the Akaike criterion. In particular, we present the proper way of ranking the competing models on the basis of Akaike weights (in Bayesian language: posterior probabilities of the models). This important ingredient is lacking from alternative studies dealing with cosmological applications of the Akaike criterion.

Of the many particular models of dark energy we focus on four: quintessence, quintessence with a time varying equation of state, the braneworld scenario and the generalized Chaplygin gas model, and test them on Riess’s gold sample.

As a result we obtain that the best model—in terms of the Akaike criterion—is the quintessence model. The odds suggest that although there exist differences in the support given to specific scenarios by supernova data, most of the models considered receive similar support. The only exception is the Chaplygin gas which is considerably less supported. One can also note that models similar in structure, e.g. Λ CDM, quintessence and quintessence with a variable equation of state, are closer to each other in terms of Kullback–Leibler entropy. Models having different structure, e.g. Chaplygin gas and the braneworld scenario, are more distant (in the Kullback–Leibler sense) from the best one.

Keywords: dark energy theory, classical tests of cosmology

Four classes of cosmological models invoked to explain accelerated expansion of the Universe

- Cosmological constant Λ
- Quintessence - dynamical scalar fields
- Chaplygin Gas
- Multidimensional models - braneworld scenarios

Table 1. Expansion rates $H(z)$ in four models tested.

Model	Cosmological expansion rate $H(z)$ (the Hubble function).
Λ CDM	$H^2(z) = H_0^2(\Omega_m (1+z)^3 + \Omega_\Lambda)$
Quintessence	$H^2(z) = H_0^2(\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w)})$
Var Quintessence	$H^2(z) = H_0^2(\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w_0-w_1)} \exp(3w_1 z))$
Chaplygin Gas	$H(z)^2 = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_{Ch} \left(A_0 + (1-A_0)(1+z)^{3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} \right]$
Braneworld	$H(z)^2 = H_0^2 \left[(\sqrt{\Omega_m (1+z)^3 + \Omega_{r_c}} + \sqrt{\Omega_{r_c}})^2 \right]$

Note: The quantities Ω_i represent fractions of critical density currently contained in energy densities of respective components (like clumped pressure-less matter, Λ , quintessence, Chaplygin gas or brane effects).

Which model is the best?

- The one that is the most supported by data.
- Model selection based on information theory
- Starting point: Kullback - Leibler Information \rightarrow AIC (Akaike Information Criterion)
- Also, BIC and Bayesian methods

Cosmological models fitted to SN Ia data

Table 1. Values of best fitted parameters of four models tested.

Model	Best fit model parameters (with 1σ ranges)
Λ CDM	$\Omega_m = 0.31 \pm 0.04$
Quintessence	$\Omega_m = 0.49 \pm 0.06, w = -2.40 \pm 1.12$
Var. quintessence	$\Omega_m = 0.48 \pm 0.14, w_0 = -2.48 \pm 1.38, w_1 = 1.88 \pm 2.59$
Chaplygin gas	$\Omega_m = 0.31 \pm 0.04, A_0 = 1.00 \pm 0.035, \alpha = 0.002 \pm 0.088$
Braneworld	$\Omega_m = 0.21 \pm 0.03$

Comparative ranking

Information theoretic model selection applied to supernovae data

Table 4. Ranking of cosmological models fitted to SNIa data according to AIC, BIC and χ^2/dof criteria.

Ranking	AIC	BIC	χ^2/dof
1	Quintessence	Λ CDM	Quintessence
2	Λ CDM	Braneworld	Var. quintessence
3	Var. quintessence	Quintessence	Λ CDM
4	Braneworld	Var. quintessence	Braneworld
5	Chaplygin gas	Chaplygin gas	Chaplygin gas

Perspectives

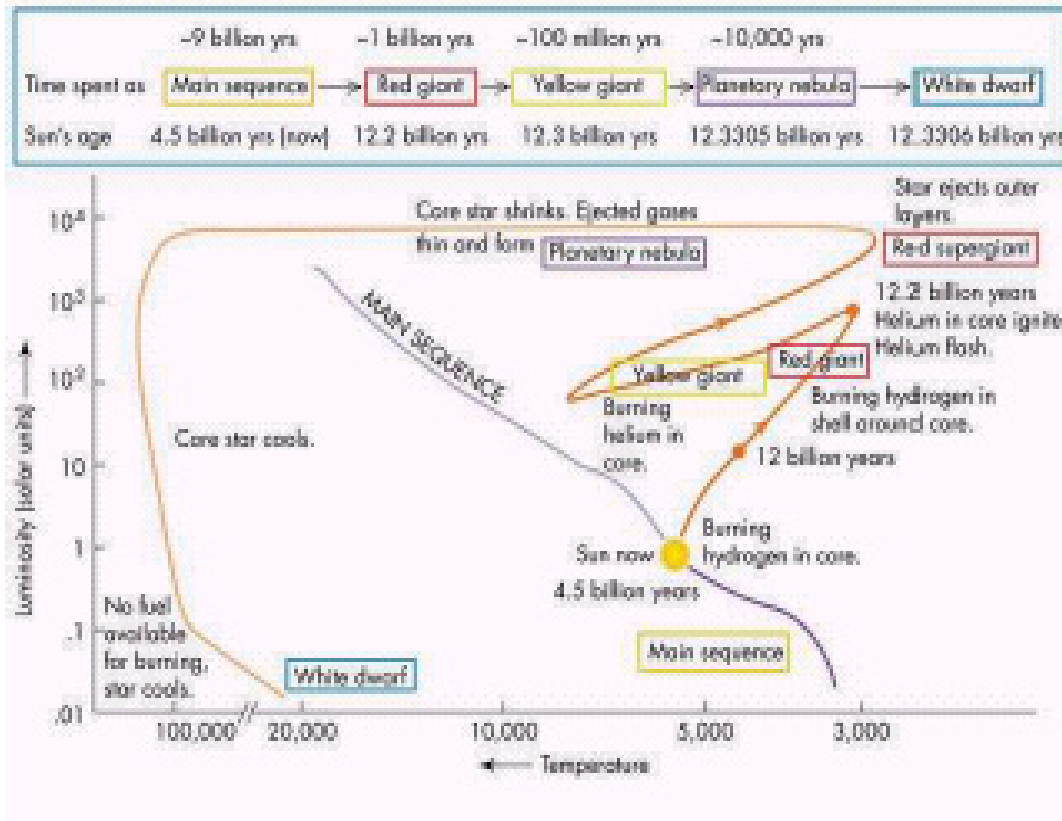
1. Test different models with different priors
e.g. topology prior - Boud Roukema
2. Model selection based on different tests: CMBR, BAO, LSS power spectrum, (ambitious task of self-consistency) GRBs, SN Ia, lensing, Alcock-Paczynski test for clusters

Astrophysical bounds on Dark Matter candidates

i d e a

- **weakly interacting particles** (axions, Kaluza-Klein gravitons, etc.) can be produced in stellar interiors and escape freely
- they become an **additional channel** of energy loss from stellar interiors (WIMPS too !)
- **new channel** of energy loss would **modify** stellar evolution

e.g. Raffelt G., *Annu.Rev.Nucl.Particle Sci.*, **49**, 1999



Scheme of Evolutionary Track of a Star

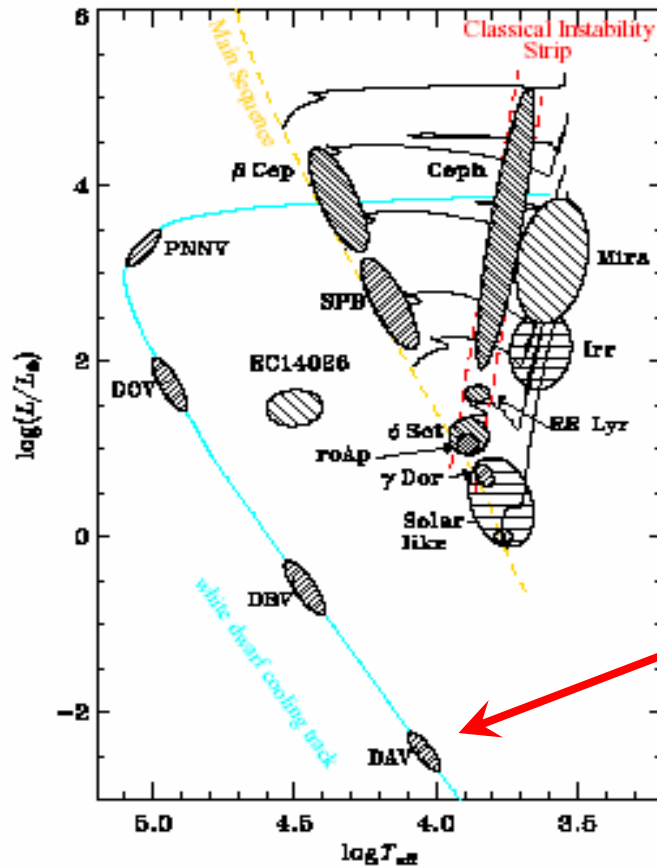


Fig. 1. H-R diagram showing the different classes of pulsating stars. In this article, we are interested in the DAV's.

Instability strips on H-R diagram

ZZ Ceti

What have we done with G117-B15A ?

Biesiada & Malec *PhysRevD* **65**, 2002

• we have used this approach to constrain the **compactification mass scale M_s** (in **$n=2$** large extra dimensions)

Arkani-Hammed, Dimopoulos & Dvali (1998) model

the upper 2σ limit on P_{OBS} translates into a bound:

$$L_{KK} = 5.86 \times 10^{-75} \frac{T^3 n_e}{\rho \cdot M_S^2} \sum_j n_j Z_j^2 \cdot M < \left(\frac{\dot{P}_{OBS}}{\dot{P}_O} - 1 \right) L_\gamma = 0.308 \cdot L_\gamma$$

White dwarf cooling and large extra dimensions

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(Received 12 March 2001; revised manuscript received 25 July 2001; published 28 January 2002)

Theories of fundamental interactions with large extra dimensions have recently become very popular. Astrophysical bounds from the Sun, red giants, and SN 1987a have already been derived by other authors for the theory proposed by Arkani-Hamed, Dimopoulos, and Dvali. In this paper we consider the G117-B15A pulsating white dwarf (ZZ Ceti star) for which the secular rate at which the period of its fundamental mode increases has been accurately measured and claimed that this mode of G117-B15A is perhaps the most stable oscillation ever recorded in the optical band. Because an additional channel of energy loss (Kaluza-Klein gravitons) would speed up the cooling rate, one is able to use the aforementioned stability to derive a bound on theories with large extra dimensions. Within the framework of the theory with large extra dimensions proposed by Arkani-Hamed, Dimopoulos, and Dvali we find the lower bound on string compactification scale $M_s > 14.3 \text{ TeV}/c^2$ which is more stringent than solar or red-giant bounds.

DOI: 10.1103/PhysRevD.65.043008

PACS number(s): 97.20.Rp, 04.50.+h, 11.10.Kk

comparison with other bounds

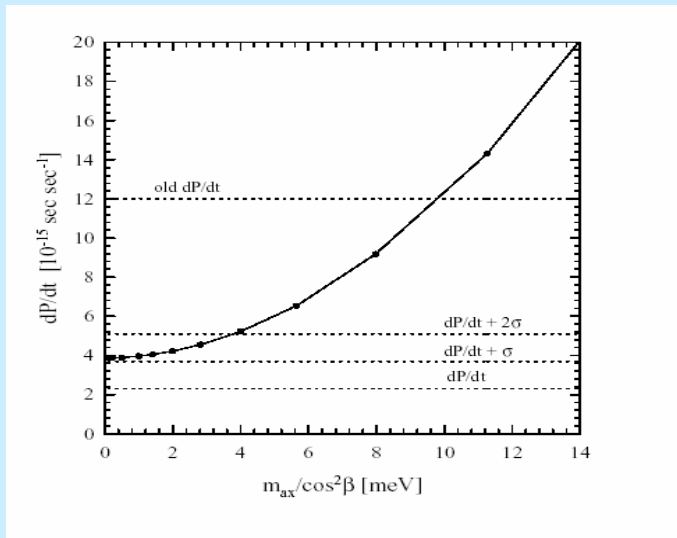
- LEP $M_s > 1 \text{ TeV}/c^2$
- The Sun $M_s > 0,3 \text{ TeV}/c^2$
- Red Giants $M_s > 4 \text{ TeV}/c^2$
- SN1987A $M_s > 30-130 \text{ TeV}/c^2$
- White Dwarf $M_s > 14,3 \text{ TeV}/c^2$

What have the others done with G117-B15A ?

Corsico et al. *New Astron.* 6, 2001

- used G117-B15A to constrain the **mass of an axion**
- evolutionary and pulsational codes with axion emissivity added
- obtained bound to axion mass

$$m_{ax} \leq 4 \cos^2 \beta \text{ meV}$$



Another issue - Varying G

- renewed debate over the issue whether the fundamental constants of nature (G , c , h or e) can vary with time

MOTIVATION

- Dirac's Large Number Hypothesis
- Brans-Dicke Theory
- Theories with higher dimensions, superstring theories, M-theory etc.
- Claims that fine structure constant might vary

Webb & Murphy 2001

- Gravity constant G :
historically the first considered as varying

A new white dwarf constraint on the rate of change of the gravitational constant

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ABSTRACT

In this paper we derive a bound on the rate of change of the gravitational constant G based on observations of the pulsating white dwarf G117-B15A. This star is a ZZ Ceti pulsator which has been extensively studied with astroseismological techniques for the last three decades. The most recent determination of $\dot{P} = (2.3 \pm 1.4) \times 10^{-15} \text{ s s}^{-1}$ for the 215.2-s fundamental mode agrees very well with predictions of the best-fitting theoretical model. The rate of change of the oscillation period can be explained by two effects: the cooling (dominant factor) and change of gravitational binding energy (residual gravitational contraction). Since the white dwarfs are pulsating in g-modes, the frequencies of which are related to the Brunt–Väisälä frequency (explicitly dependent on G), observational determination of the change of the period (more precisely the difference between the observed and calculated \dot{P}) can be used to set an upper bound on the rate of change of G . In light of the current data concerning G117-B15A we derive the following bound: $|\dot{G}/G| \leq 4.10 \times 10^{-10} \text{ yr}^{-1}$. We also demonstrate that a varying gravitational constant G does not modify the cooling of white dwarfs in a significant way, at least at the luminosities where white dwarfs are pulsationally unstable.

Key words: gravitation – stars: individual: G117-B15A – stars: oscillations – white dwarfs.

1 INTRODUCTION

There is a renewed debate in the literature over the issue of whether the quantities known as the constants of nature (such as G , c , h or e) can vary with time (Bekenstein 1982; Moffat 1993a,b; Barrow 2002; Barrow et al. 2002; Sandvik et al. 2002). One of the reasons for this debate is associated with the advances of string theory and associated ideas that the world we live in may have more than four dimensions.

String theory (Horava & Witten 1996; Witten 1996) is the only known framework which gives hopes of reconciling gravity theory with quantum mechanics and one of its hints is that the coupling constants appearing in low-energy Lagrangians are determined by the vacuum expectations of some a priori introduced massless scalar fields (dilaton or moduli fields). Interest in physical theories with extra spatial dimensions has also experienced considerable revival. In the framework of multidimensional theories (Antoniadis et al. 1998; Arkani-Hamed et al. 1998) the four-dimensional gravity constant, for example, is in fact an effective constant appearing when integrated over additional dimensions.

Another trigger for the interest in varying the fundamental constants is associated with recent observational evidence that the fine

structure constant may indeed vary with time (Webb et al. 1999, 2001).

All this motivates us to take seriously the possibility that (at least some of) the fundamental constants could in fact vary in time. Although from a historical perspective the gravity constant G was the first fundamental constant considered as a dynamical quantity (Dirac 1937), in fact it is not strongly constrained concerning its variability nor even measured with an accuracy comparable to other fundamental constants – the latest CODATA report raised the relative uncertainty of G from 0.013 to 0.15 per cent (Grundlach & Merkowitz 2000). In the past, there were numerous attempts to constrain the possible temporal variability of G as summarized in a recent review paper by Uzan (2003). White-dwarf cooling has already been used by García-Berro et al. (1995) to constrain the temporal variation of G at the level of the white dwarf luminosity function.

This paper is devoted to another constraint on \dot{G}/G obtained from astroseismological considerations of the ZZ Ceti pulsating star G117-B15A. More precisely, recent measurements of the rate of change of period of the 215.2-s pulsational mode (Kepler et al. 2000) turned out to be in a very good agreement with evolutionary models of DAV white dwarfs comprising cooling and gravitational contraction effects. Pulsations of DAV white dwarfs are excited as so-called g-modes, which arise due to the competition between buoyancy and gravity forces acting on an element of matter displaced in a stratified medium. We take advantage of this circumstance to derive a bound on the rate of change of the gravity constant.

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Paper

M. Biesiada & B. Malec
MNRAS 350, 644, 2004

Astroseismology

of

G117-B15A

Nature of oscillations: g-modes, Brunt - Väisälä frequency

$$N^2 = -gA = -g \left(\frac{d \ln \rho}{dr} - \frac{1}{\Gamma_1} \frac{d \ln p}{dr} \right)$$

Here is the dependence on G

Asymptotic form

$$P^2 \approx -\frac{k^2 r^2}{AgI(V+1)}$$

Rate of period change (classically)

$$\frac{d \ln P}{dr} = -a \frac{d \ln T}{dr} + b \frac{d \ln R}{dr}$$

Modification for varying G

$$\frac{d \ln P}{dr} = -a \frac{d \ln T}{dr} + b \left(\frac{d \ln R}{dr} - \frac{d \ln G}{dr} \right)$$

Cooling

Residual contraction

Idea:
observed

$$\left(\frac{\dot{P}}{P}\right)_o$$

agrees with

theoretical

$$\left(\frac{\dot{P}}{P}\right)_c$$

(with some accuracy)

$$\frac{d \ln P}{dt} = a \frac{d \ln T}{dt} + b \left(\frac{d \ln R}{dt} - \frac{d \ln G}{dt} \right).$$

[Theoretical model according to Salaris et al. 1997]

so

$$\left| \Delta \left(\frac{\dot{P}}{P} \right) \right| = \left| \frac{\dot{G}}{G} \right|.$$

We obtain the bound

$$\left| \frac{\dot{G}}{G} \right| \leq 4.10 \times 10^{-10} \text{ yr}^{-1}$$

Perspectives

Use stellar evolution constraints (esp.
Helio/astroseismology & WD cooling) for
Supersymmetric particles

Prof. Manka-Marcisz + Ilona Bednarek (+ PhD Students)

- Equation of state in (proto)neutron stars - field theoretical approach
- **Structure and evolution of strange and hybrid stars**
- some interesting results concerning mass distribution of hybrid stars

Dr Jerzy Król

- Model theoretical approach to Quantum Gravity
- **Toposes as natural environment**

Some current papers

Bednarek I. Mańka R., Properties of a protoneutron star in the Effective Field Theory, *Phys. Rev.* **C73**, 045804 (2006).

Król J., A model for spacetime. The role of interpretations in some Grothendieck topoi, *Found. Phys.* 36, No5, (2006).

Król J., A model for spacetime II. The emergence of higher dimensions and field theory/strings dualities, *Found. Phys.* 36, No12, (2006).