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# Extensive quantities in thermodynamics

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## Abstract

A literature survey shows little consistency in the definitions of the term ‘extensive quantity’ (a.k.a. extensive property) as used in thermodynamics. A majority assumes that extensive quantities are those that are proportional to mass. Taking the mathematical meaning of proportional and taking the ‘mass’ to be that of the system or subsystem, it is shown that the proportionality assumption is only correct for a few extensive quantities under condition of constant composition. A large subset of extensive quantities are completely independent of mass; for most systems extensive quantities are not proportional to mass, but mass is the (extensive) constant of proportionality. The definition by IUPAC, based on the additivity of extensive quantities, is the preferred baseline for discussing this subject. It is noted however, that two types of additivity need to be distinguished and that a few intensive quantities are also additive. This paper leaves several interesting questions open to further scrutiny.

Keywords: extensive quantity, extensive property, proportionality, additivity, extensivity

(Some figures may appear in colour only in the online journal)

## 1. Introduction

The description of any thermodynamic system or process is expressed in terms of quantities and the relationships between them. Quantities can be categorized into two types: intensive and extensive. Denbigh (1971) distinguishes them as follows: ‘the extensive properties are those which are additive, in the sense that the value of the property for the whole of a body is the sum of the values for all of its constituent parts’. DeVoe (2012) adds: ‘sometimes a more restricted definition of an extensive property is used: The property must be not only additive, but also proportional to the mass or the amount when intensive properties remain constant. According to this definition, mass, volume, amount, and energy are extensive, but surface area is not.’ Zemansky and Dittman (1981), however, do without additivity and without constant intensive properties and write: ‘extensive coordinates are proportional to the mass’. So, there are two definitions for an ‘extensive property’ and this creates a problem for surface area: it is extensive according to the additivity rule (AR), but not according to the proportionality rule (PR). As mass obeys AR, all quantities that are proportional to mass will also obey AR, but

additivity does not imply proportionality to mass. Note that DeVoe's restricted definition is circular when intensive is understood as non-extensive.

The following text gives the results of a scan of literature on thermodynamics for definitions for extensivity (section 2). A close look at the concept of proportionality leads to a classification of extensive quantities based on their relationship with mass; more quantities are identified that are not proportional to mass but are extensive according to the AR (section 3). Next, distinction is made between two forms of additivity (section 4). Conclusions with recommendations are given at the end of this paper (section 5). The text in the separate boxes below (boxes 1 through 5) contain issues that are of interest but that might obscure the main line of reasoning if not kept separate. A number of general aspects are stated in box 1. Throughout the text, the symbols  $Z$  and  $z$  will be used to indicate extensive and intensive variables respectively in general. (Other symbols: see table 2).

### Box 1. General observations and assumptions

Several general aspects are of interest to the issue of extensive quantities in thermodynamics

- (1) Redlich (1970) assumes that if a quantity  $Z$  is extensive for one system it will be extensive for all physical systems: extensivity is taken as an inherent property of a quantity. The thermodynamic literature seems to accept this as an inherent truth. It implies that if  $U(x, y)$  is extensive, so is  $U(v, w)$ .
- (2) Landsberg (1961) assumes that all quantities are either extensive or intensive. This is unlike Redlich (1970) who states that 'it is obviously wrong to say only extensive and intensive variables exist'. If extensive quantities are defined as proportional to mass, some quantities like surface area are neither intensive nor extensive (see also box 3).
- (3) Torre (2012) assumes that the ratio of two extensive variables is intensive (i.e.  $Z/Z' = z$ ). This assumption is obvious when both  $Z$  and  $Z'$  are proportional to mass, then  $(Z/Z')$  will not be proportional to mass.
- (4) Torre (2012) assumes that the sum of two extensive quantities is extensive. This implies that when the total energy  $E = \sum E_j$ , where the summation is over all forms of energy  $E_j$ , either all or none are extensive. Similar implications follow from the first law  $\Delta U = Q + W$ , and the definition of enthalpy  $H = U + pV$ .
- (5) Every energy transfer or transformation process can be formulated as  $z dZ$  where (intensive)  $z$  and (extensive)  $Z$  are conjugates of each other. For example:  $p dV$ ,  $T dS$ ,  $A_p d\xi$ ,  $\gamma dA$  etc (see table 2 for symbols). In addition, equilibrium is represented as  $z' = z''$ , the equality of intensive variables. Hence the distinction between intensive and extensive quantities provides insight into the energetic structure of the system.

## 2. Literature survey

In order to get a feel for what DeVoe (2012) means by 'sometimes' in 'sometimes a more restricted definition is used', textbooks were examined to see how they defined 'extensive'. The results have been collected in table 1, where the columns represent: [A] references sorted by year of publication, [B] nomenclature, [C] definition of extensive, and [D] dependency on other quantities, all discussed below. An odd definition encountered for 'extensive' is discussed in box 4.

### [A] References

A total of 40 texts were found that define 'extensive property' in one way or another: 34 of them specify *thermodynamics* or *physical chemistry* as the subject. The others include dictionaries,

**Table 1.** Distribution of definitions of ‘extensive quantity’ gathered from the literature (for explanation of the abbreviations used see the text in section 2 under [A] through [D]).

[A] References (sorted by year)	[B] Nomenclature	[C] Definition	[D] Dependency
Lewis and Randall (1923)	P	AR-	(AR for two identical systems)
Tolman (1938, p 614)	M	PR	$p$ (amount of substance)
Afanasjewa-Ehrenfest (1956, p 3)	pm	AR	
Joel (1966, p 2)	P	PR	$d$ (mass)
Zeleznik and Gordon (1967, p 4)	P	PR	$p$ (amount of material)
Redlich (1970, p 154)	P	AR-	(quotes Lewis & Randall)
Denbigh (1971, p 7)	P	AR	
Young (1973, p 489)	Q	PR	$p$ (amount of matter)
Adamson (1973, p 381)	V	PR	$p$ (amount of the system)
Reiss (1974, p 46)	P	PR	$p$ (mass, extent)
James (1976, p 175)	P	A&P	$p$ (quantity of material)
Zemansky and Dittman (1981, p 45)	C	PR	$p$ (mass)
Atkins (1982, p 76)	P	PR	$d$ (amount of substance)
Tykodi (1982, p 557)	V	PR	$p$ (mass)
Callen (1985, p 10)	pm	AR	
Smith and VanNess (1987, p 26)	P	PR	$p$ (quantity of material)
Astarita (1989, p 15)	Q	A&P	$d$ (mass)
Bortfeldt & Kramer (1991)	Q	AR	
US-DOE (1992, p 4)	P	PR	$p$ (mass)
IUPAC Green Book (1993, p 7)	Q	AR	(referred by Gold Book (2012))
Potter and Somerton (1994, p 4)	P	A&P	intensive = not $d$ (mass)
Warn and Peters (1996, p 60)	F	A&P	$d$ (size)
Perrot (1998)	P + Q	PR	$p$ (dimension), $d$ (mass)
Moran and Shapiro (1998, p 5)	P	AR	$d$ (size, extent)
Tschoegl (2000, p 6)	P + F	PR	$d$ (extent, amount)
Kaufman (2001, p 5)	P	PR	$p$ (size = volume or amount)
Seddon and Gale (2001, p 6)	P	PR	$p$ (mass)
Schaber (2002, p 21)	V	A&P	$p$ (mass)
Parker (2003)	P	A&P	$d$ (quantity of material)
Cengel and Boles (2006, p 12)	P	PR	$p$ (size, extent)
Turns (2006, p 41)	P	PR	$d$ (extent = substance)
Spakovsky and Waitz (2006, p 22)	P	AR	
Koper (2007, p 119)	V	A&P	$p$ (volume, amount)
Borgnakke and Sonntag (2008, p 16)	P	PR	$p$ (mass)
Luethi (2009, p 13)	P	AR	
Brewster (2009, p 119)	P	A&P	intensive = not $p$ (mass)
Lemm (2010, p 5)	Q	PR	$p$ (particle number)
Dinçer and Kanoglu (2010, p 10)	P	PR	$d$ (mass)
Swendsen (2011)	O	PR	$p$ (size = particle number N)
DeVoe (2012, p 28–29)	P	A&P	$p$ (mass, amount)
total of 40 references	25P	21PR	$18p+11d$

one thesis, and Tolman’s *Principles of Statistical Mechanics*. With three exceptions, the texts were published in the period 1966–2012.

#### [B] Nomenclature

Of the 40 references, 25 use the noun ‘property’ (P) with the adjective ‘extensive’, whereas 6 use ‘quantity’ (Q). The other nouns used include: variable/variable of state (V), factor (F), parameter (pm), coordinate (C), magnitude (M) or observable (O). In this paper, it has been assumed that these words can be used without distinction. The usefulness of some distinction is addressed in box 2.

**Box 2. Properties, quantities, variables, and parameters**

The literature listed does not distinguish clearly between the nouns *property*, *quantity*, *variable*, and *parameter* (and others) when used with extensive. However, some properties (such as ‘colour’ or ‘state of aggregation’) are not quantitative and hence cannot be extensive. Some other quantities (such as work and heat) are not considered to be ‘properties’, but are extensive nevertheless. In the sciences, the term ‘extensive quantities’ might therefore be preferable to ‘extensive properties’. The noun ‘property’ could then be used either in a more restricted sense, e.g. density, boiling point, and specific heat are material properties, or in a more general sense, e.g. some quantities have the property of being extensive: they have extensivity.

Quantities are either constants or variables; both intensive and extensive variants exist.

In daily life, the word ‘quantity’ is associated with mass, weight and volume, but in science also applies to temperature, pressure, and other intensive properties.

**[C] Definitions**

The AR is mentioned a total of 19 times (AR+A&P), with only eight stating it without restrictions (AR). Ten references use both definitions (A&P), whereas only Redlich (1970) and DeVoe (2012) note that there is a conflict. Out of the 40, 21 refer only to proportionality or dependency; this includes books that have been widely used in thermodynamics, such as Zemansky and Dittman (1981), Atkins (1982), Smith and VanNess (1987), Cengel and Boles (2006). A practical test for additivity is given in box 3, which suggests that force  $F$  is extensive and time  $t$  is intensive.

**Box 3. The extensivity test**

DeVoe (2012): ‘if we imagine the system to be divided by an imaginary surface into two parts, any property of the system that is the sum of the property for the two parts is an extensive property/any property that has the same value in each part and the whole is an intensive property’.

Dividing the system is a simple test to decide whether a quantity  $Z$  is extensive or intensive: divide the system into two parts (some say equal or identical parts); the intensive quantities are unaffected and the extensive quantities are divided/halved. Note however, that the piston of a system (which can be a subsystem) can be halved in different ways, while the surface area will not be halved (see ‘additivity’ in section 4). Note that the entropy is not halved either (Gibbs paradox).

The division-test can help to decide whether force  $F$  is extensive or intensive: if we divide a cannonball in flight into two parts, the velocities  $v$  remain equal, the (gravitational) force  $F$  is divided: hence  $F$  is extensive and velocity is intensive: height is unaffected, as is time. Note that this result is at odds with Helm (1898) who classified ‘Kraftkomponente’ (forces  $F$ ) as intensive and ‘Wegkomponente’ (distance  $x$ ) as extensive. Note that forces are additive.

**[D] Type of dependency**

Thirty of 40 references stated that extensive quantities are dependent on other quantities, but they do not agree on either the quantities concerned or the nature of the dependency.

Firstly, it should be noted that the terms ‘size’ (mentioned five times), ‘extent’ (4×), ‘quantity’ (3×) and ‘dimension’ (1×) are not physical quantities (no symbols, no units), and proportionality to them is therefore virtually meaningless. ‘Amount’ (9×) is vague, unless it is clear that ‘amount of substance’ is meant (2×) (Swendsen (2011) defines ‘size’ as ‘number

of particles'; Tschoegl (2000) identifies 'extent' with length, area and volume, and 'amount' with both mass and charge).

The references do not agree on whether extensive properties depend on mass ( $13\times$ ), amount ( $9\times$ ), volume ( $3\times$ ), or number of particles ( $2\times$ ); 18 of them state direct proportionality ( $p$ ), where 11 allow a more general dependence ( $d$ ).

In short, the collected literature shows little coherence in either the nomenclature or the definition of an extensive quantity. The data do not suggest any shifts in preferences over time and by extrapolation it is to be expected that at present 'an extensive property is proportional to mass' is the leading paradigm in thermodynamics.

#### Box 4. The Landsberg definition

Landsberg (1961) gave a definition that 'a quantity/function is extensive when it is homogeneous of degree 1'. Several aspects of extensivity are addressed by Dunning-Davies- and Landsberg (1985) and Landsberg (1999), but they leave many questions open. One example is the question of how or when is a 'surface area' homogeneous of degree 1? Kinetic energy does not satisfy this criterion, and mass for closed systems does not either.

Being homogeneous of degree  $n$  is not an inherent property of a physical quantity (like extensivity), but a property of a relation with other variables (function), whereby some of the functions (for the same variable) may be heterogeneous and others homogeneous of various degrees. A constant (e.g. the mass of the system) can be extensive, but need not be a function, or at best a homogeneous function of degree 0. It should be noted that Euler (who first introduced homogeneous functions) did not distinguish between intensive and extensive variables: this distinction is first made by Helm (1898). The Landsberg definition needs further scrutiny because it occasionally is referred to following the definition of extensive quantity. For example by Reiss (1974) and Tykodi (1982), who both use the PR.

### 3. Proportionality

To evaluate the different definitions encountered in the previous paragraph we need to take a closer look at the concept of proportionality and attempt to classify a wide range of additive quantities with respect to their relation to mass (table 2). The following aspects (a/h) need consideration:

- The extensivity-test in box 3 supplies an effective means to decide whether a quantity is intensive or extensive. However, because in the division all additive (extensive) quantities are halved, all additive (extensive) properties seem to be proportional to each other. This does not mean there actually exists any mathematical relation between these quantities in a given system. In other words: the test in box 3 is a test on additivity, not on proportionality.
- Because every quantity is proportional to itself, the proof that mass itself is an extensive quantity must come from the AR.
- To evaluate what is implied by 'proportional to mass', we need to look at the definition of proportionality and examine its implications.

Proportionality is understood here to mean a mathematical relationship: *if  $y$  is directly proportional to  $x$ , then  $y/x$  is a constant*<sup>1</sup>.

Proportionality has the properties of reciprocity and transition (see box 5).

Hence, if some extensive quantity  $Z$  is proportional to mass  $m$ , it means that  $Z/m = k$ , where  $k$  is an (intensive) constant of proportionality and where  $Z$  and  $m$  are taken to be

<sup>1</sup> <http://mathworld.wolfram.com>

variable quantities. Note that quantities that are truly proportional to each other are never independent.

- (d) As suggested by the statement by DeVoe, ‘sometimes’ a conditional proportionality is used in thermodynamics. The true proportionality defined under c. is to be distinguished from the relation  $Z/m = z$  where  $z$  is an intensive variable or function. We will indicate this as conditional proportionality, because it is proportional only under the condition  $z = \text{constant}$ . Without distinction between true and conditional proportionality, contradictions may arise as exemplified in box 5. Note that in the thermodynamic literature no such distinction is made.

Also note that the conservation of mass and energy imply that the additivity and extensivity of mass and energy are unconditional.

**Box 5. *Reductio ad absurdum*: ‘proof’ that temperature is extensive**

For a monatomic ideal gas, the internal energy  $U = m(3RT/2M)$ .

Premise 1:  $U$  is proportional to mass  $m$ .

Premise 2:  $U$  is proportional to temperature  $T$ .

Apply transitivity of proportion: if  $c$  is proportional to  $a$  and to  $b$ , then  $a$  is proportional to  $b$ .

Conclusion 1: ( $T$ ) is proportional to ( $m$ ). Apply definition of extensive variable as proportional to mass ( $m$ ). Conclusion 2: temperature ( $T$ ) is an extensive variable.

Since the conclusion is false, at least one premise must be wrong: for all practical purposes  $m$  is the constant of proportionality. The error arises from the improper use of the word ‘proportional’.

- (e) For almost all thermodynamic systems, the mass  $m$  is a constant rather than a variable. The First Law when given as  $\Delta U = Q + W$  implies that the system is closed. Hence in this situation the extensive quantity is not proportional to mass, but rather mass is a constant of proportion, and mass is not a variable of state. In the survey in table 1, 11 references use a less specific ‘dependent’ to define extensivity: this might include the situation where mass is constant, but none of the references is explicit about that aspect. The situation where  $m$  is commonly the constant of proportionality applies to many mechanical quantities (kinetic energy, energy of gravity, force, momentum) and many thermodynamic quantities (internal energy, heat capacity). Note that constant mass  $m$  does not imply constant number of particles  $N$ , so  $N$  can be and often is a variable of state for closed (reactive) systems.
- (f) For both the true proportionality and conditional proportionality,  $m = 0$  implies  $Z = 0$ . For volume  $V$ , a vacuum is an obvious violation of both PRs, but not of the additivity rule. If  $V$  is not proportional to  $m$  then  $(U + pV) = H$  may not be proportional to  $m$  either, although enthalpy  $H$  is an extensive quantity. For entropy even the extensivity-test seems to fail, even for ideal gases. However, the additivity of entropy is implicit in the calculation of the entropy of mixing. In paragraph 4, the distinction between two forms of additivity is useful in this aspect. At this point it is important to note that entropy does not have to be proportional to the number of particles or mass, if that would be the issue (see Tolman 1938). The free enthalpy  $G$  ( $G = H - TS$ ) will only be proportional to  $m$  when both  $H$  and  $S$  are: mass is a constant of proportionality for  $G$  if it is for both  $H$  and  $S$ .
- (g) There is a group of extensive quantities not related to mass at all. This applies to several forms of energy (displacement, spring, and surface energy) and to all power variables, including heat and work. It also applies to some electrical quantities (current, charge), and

to several chemical quantities (extent of reaction and reaction rate). It includes mass-flow  $\Phi$  (discharge) and surface area  $A$ .

- (h) If extensivity–intensivity is an inherent property of all quantities, then it seems reasonable to assume that whether a quantity is extensive or not, should not depend on the system chosen, not on the relation chosen and not on the conditions chosen. This makes additivity the preferred criterion for ‘extensive’, as ‘proportional to mass’ will fail eventually on all three accounts.

**Table 2.** Classification of extensive quantities, based on their relation with mass.

Symbol	Quantity	Unit	Conditions	Formula	Notes
$(Z/m = )$					
Z1	<i>proportional to mass</i>				
$m$	mass	kg	none <sup>a</sup>	$m/m = 1$	
$F_g^b$	weight	$\text{J m}^{-1}$	g constant	$F_g/m = g$	$g = 9.81 \text{ m s}^{-2}$
Z2	<i>proportional to mass under constant composition (<math>Z/m = k</math>), else <math>Z2 = Z3</math>,<sup>c</sup></i>				
$C_p$	heat capacity	$\text{J K}^{-1}$	$n_i$ constant	$C_p/m = c_p$	$c_p$ = specific heat cap.
$n$	amount of substance	mol	$n_i$ constant	$n/m = 1/M$	$M$ = molar mass
$V_{\text{SL}}$	volume (solid, liquid)	$\text{m}^3$	$n_i$ constant	$V_{\text{SL}}/m = \rho$	$\rho$ = density
$N$	number of particles	–	$n_i$ constant		
Z3	<i>mass is constant of proportionality, or conditional proportional (<math>Z/m = z</math>)</i>				
$E_k$	kinetic energy	J		$E_k/m = \frac{1}{2}v^2$	$v$ = velocity
$E_g$	gravitational energy	J		$E_g/m = gh$	$h$ = height, $g = 9.81 \text{ m/s}^2$
$p$	momentum	$\text{kg} \cdot \text{m s}^{-1}$		$p/m = v$	$v$ = velocity
$F^b$	force	$\text{J m}^{-1}$		$F/m = a$	$a$ = acceleration
$U$	internal energy	J		$U/m = u(T)$	$T$ = temperature
$G$	free enthalpy	J		$G/m = g(T,p)$	$g$ = specific free enthalpy
$H$	enthalpy	J		$H/m = h$	$h$ = specific enthalpy
$S$	entropy	$\text{J K}^{-1}$		$S/m = s$	$s$ = specific entropy
$V_G$	volume (gas)	$\text{m}^3$		$V_G/m = M.v_G(T,p)$	$v_G$ = molar volume
$V$	volume (all phases)	$\text{m}^3$		$V = V_{\text{SL}} + V_G$	
Z4	<i>extensive quantities independent of mass</i>				
$A$	area (interface)	$\text{m}^2$		–	
$E_{\text{el}}$	electric energy	J		–	
$E_{\text{sp}}$	spring energy	J		$E_{\text{sp}} = \frac{1}{2}k.x^2$	$k$ = spring constant
$E_{\text{pv}}$	displacement energy	J		$E_{\text{pv}} = p.V$	$p$ = ambient pressure
$E_{\text{sur}}$	surface energy	J		$E_{\text{sur}} = \gamma.A$	$\gamma$ = surface tension
$i$	current (electric-)	$\text{C s}^{-1}$		–	
$P$	power (most forms)	$\text{J s}^{-1}$		–	–
$Q$	Heat	J		–	–
$q$	charge (electric)	C		–	–
$W$	work	J		–	–
$\xi$	extent of reaction	eq <sup>d</sup>		–	conjugate to affinity
$d\xi/dt$	reaction rate	eq $\text{s}^{-1}$ <sup>d</sup>		–	
$\Phi$	flow (fluid)	$\text{kg s}^{-1}$		–	
total quantities = 29		total			

<sup>a</sup> non-relativistic conditions assumed throughout.

<sup>b</sup> See box #3 for this classification.

<sup>c</sup>  $m, n, N$ , and  $V$  are mentioned in table 1, [D].

<sup>d</sup> the unit ‘equivalent’ (eq) = ‘reaction-mole’.



Now additive quantities can be classified into four classes (Z1/Z4), as indicated in table 2. Of the extensive quantities listed, only five have a true proportionality to mass  $m$ : but none of these unconditionally so. For a large group of the extensive quantities, proportionality to mass is absent. For the common thermodynamic and mechanical systems,  $m$  is the constant of proportionality.

Conclusion: additivity is to be preferred as the distinctive property of extensive quantities, in line with IUPAC recommendations, but taking into account the observations of paragraph 4.

#### 4. Additivity: two of a kind

Two problems can be identified with the concept of additivity: the first problem arises when no proper distinction is made between a formal addition and the physical addition. In the case of formal addition, a system is pronounced to exist of two (arbitrary) systems taken together, where 'formal' is to be understood as a matter of assignment or administration, and not involving any physical process. In the case of physical addition the systems are brought together and may or may not merge in the end: both in the process of approach and merging, the systems might exchange energy and/or mass. The exchange of energy will change variables of state of the two systems, and thus affect their extensive quantities. As long as the systems ( $i$  and  $j$ ) are distinguishable from each other, at any time  $Z = Z_i + Z_j$ , but  $Z_i$  nor  $Z_j$  need to be constant, hence their sum is unequal to the initial value. If two systems have merged it is improbable that the total equals the sum of the initial values of the two systems, except for mass.

*Example.* If we have a droplet with surface area  $A'$  and another droplet with area  $A''$ , then together they have a surface area  $A = A' + A''$ . This is the formal addition. What happens when these droplets merge (physical addition) is a different question: one droplet may be water and the other sulphuric acid: upon mixture surface area  $A$ , volume  $V$  and entropy  $S$  will become different from their initial (formal) sum.

The second problem with additivity is that there are instances in which the AR applies to intensive quantities. For a (sub-)system  $m = \sum m_j$ , and  $n = \sum n_j$ , where the summations are over all compounds constituting the system. If these equation are divided by  $m$  and  $n$  respectively, we get  $1 = \sum x_j$ , where  $x_j$  is the mass- or mol-fraction, which are additive but intensive. A similar derivation applies to concentrations.

Additivity applies also to partial pressures (Dalton's Law:  $p = \sum p_j$ ), reaction enthalpies  $\Delta H$ , and reaction affinities  $A_r$ . In these cases, the summed quantities also apply to partial systems, i.e. part of a system or subsystem that consists of one chemical species, or a few chemical species (e.g. reactants or products), that share a common space (volume). Note that specific thermodynamic quantities (like specific enthalpy, molar entropy) commonly are partial quantities, rather than totals. Most intensive variables not additive. These include temperature, density, surface tension, electric potential and time and velocity (see box 3).

#### 5. Conclusions with recommendations

On the basis of the 40 references gathered here, the following can be concluded.

There is little agreement on the *nomenclature* of 'extensive quantities'; the most commonly used term is 'extensive properties'. 'Extensive' is used adjectivally with property, quantity, variable and parameter, and several other nouns. There is also little agreement on the *definition* of an 'extensive quantity'. A minority defines 'extensive' as 'following the additivity rule'. A

large number define an extensive quantity as proportional to or dependent on some other extensive quantity, most commonly the mass. Nowhere is it obvious that the word ‘proportional’ is to be taken to have a strict, mathematical meaning: sometimes a conditional proportionality is assumed. Some publications state an unspecified dependency on mass, but none of the references indicates that  $m$  could be the constant of proportionality. It is not obvious from the references that *all* quantities can be classified as either intensive or extensive, and the usefulness or importance of distinguishing between intensive and extensive quantities is not elaborated.

Classification of extensive variables, based on their relationship with mass, shows that many extensive quantities are independent of mass, and that for many systems  $m$  is the constant of proportionality. Only a few variables can be considered to be genuinely proportional to mass, but never unconditionally so. For the proper definition of extensive quantities as additive quantities it is necessary to distinguish a formal addition from physical addition. ‘Extensive’ is to be understood in the first sense. At the same time however, additivity of quantities does not imply extensivity, unless the quantity refers to the whole system or subsystem.

For thermodynamics these conclusions suggest the following recommendations.

- (1) Extensivity is best defined by the additivity rule, as in the IUPAC Gold Book. Using the proportionality rule should be discouraged: if used, true proportionality is to be distinguished from conditional proportionality.
- (2) The expression ‘extensive quantity’ is to be preferred over ‘extensive property’.
- (3) Additivity is to be taken as a formal addition, not as a physical merger. There is no need to restrict the additivity rule to identical systems or to systems that are in equilibrium with each other.
- (4) It could be emphasized that extensivity is not only a property of the state variable *energy*, but also of the process variables *heat* and *work*.
- (5) It seems worthwhile to investigate whether force  $F$  can be considered an extensive quantity and displacement  $x$  and velocity  $v$  as intensive quantities, as suggested by the ‘extensivity test’.
- (6) The definition of extensivity by Landsberg (based on homogeneous functions) needs further scrutiny.
- (7) The evidence that all quantities can be classified as either extensive or intensive seems rather scarce in the thermodynamic literature and further attention might be warranted. This also holds for the assumption made that extensivity is an intrinsic property of a quantity.

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