Lab I
Hello Qubit!

Jakub Tworzydło

Institute of Theoretical Physics (FUW)
Jakub.Tworzydlo@fuw.edu.pl

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Exercise Topic

We consider a two-level quantum system, serving as an idealized model for a physical qubit. The system is driven by an external pulse, whose frequency is precisely tuned to match the system's transition frequency.

The Hamiltonian is given by:

$$H = \frac{\Omega}{2}\sigma_z + f(t)\cos(\Omega t)\sigma_x,$$

where $f(t)$ is the signal envelope and Ω is the resonant frequency of the drive. We work in dimensionless units, and we also set $\hbar = 1$.

Tasks

(A) 1 point Generate a plot of the driving field as a function of time (solid line) and the envelope function (dashed line). For this example, we use $\Omega = 5\pi$. The pulse $f(t)$ should be a Gaussian centered at $t_0 = 5$ with a width $\tau = 1$, normalized to one.

(B) 3 points Take the initial state $\psi = (1, 0)^T$ and evolve it in time according to the Schrödinger equation with the model Hamiltonian. Time is discretized with the time step dt. The one-step evolution is performed by $\psi(t + dt) = e^{-iH(t)dt}\psi(t)$.
Plot the expectation values $\langle \sigma_z \rangle$ and $\langle \sigma_x \rangle$ as a function of time. Obtain the results for a π and $\pi/2$ pulse by adjusting the amplitude A in the drive term $Af(t)\cos(\Omega t)$.

Hint: To calculate the matrix exponential for the time evolution operator $U = e^{-iH(t)dt}$, you can use the `expm` function from the `scipy.linalg` library.

```
from scipy.linalg import expm
U = expm(-1j * H(t) * dt)
```

Task (C): The Rotating Wave Approximation

(C) 1 point In quantum control, we often use the Rotating Wave Approximation (RWA) to simplify dynamics by moving to a rotating frame and neglecting rapidly oscillating terms (at 2Ω).

In the RWA, the effective Hamiltonian simplifies to $H_{RWA} = \frac{f(t)}{2}\sigma_x$. Because this commutes with itself at different times, it yields an exact analytical solution. The expectation value is given by:

$$\langle \sigma_z(t) \rangle_{RWA} = \cos\left(\int_0^t f(t')dt' \right)$$

Your tasks:

- Plot this analytical RWA solution on top of your exact numerical $\langle \sigma_z \rangle$ from Task (B). Notice what happens to the high-frequency micromotion.
- The RWA assumes the drive is weak compared to the resonant frequency ($f(t) \ll \Omega$). Check what happens for a fast pulse (short τ). At what point does the exact numerical solution visibly deviate from the RWA?