Lab III
Simulating an Adiabatic Quantum Computer

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Task 1: The spectrum and adiabatic time

3 points We define a parameter-dependent Hamiltonian for a quantum spin system:

$$H(\lambda) = (1 - \lambda)H_0 + \lambda H_1$$

where $H_0 = -\sum_i S_i^x$ and $H_1 = -\sum_{\langle i,j \rangle} J_{ij} S_i^z S_j^z - \sum_i h_i S_i^z$.

Consider a triangle of 3 spins (labeled 0, 1, 2) with the following parameters:

- Couplings: $J_{01} = -0.4$, $J_{12} = -1.6$, $J_{02} = -1.0$
- Local fields: $h_0 = -0.5$, $h_1 = 0.5$, $h_2 = -0.1$

Construct the 8×8 matrix $H(\lambda)$. Scan over $\lambda \in [0, 1]$ and calculate the eigenvalues.

1. **Plot:** Plot *only* the ground state energy $E_0(\lambda)$ and the first excited state energy $E_1(\lambda)$ as a function of λ.
2. **Calculate & Print:** Calculate the energy gap $\Delta E(\lambda) = E_1(\lambda) - E_0(\lambda)$. Use it to estimate and print the optimal adiabatic evolution time using a discrete sum:

$$T_{AQC} \approx \sum \frac{d\lambda}{(\Delta E(\lambda))^2}$$

Task 2: State evolution and classical solution

2 points As λ sweeps from 0 to 1, the system transitions from a quantum superposition into the classical ground state of the target spin-glass Hamiltonian.

Using the ground state eigenvectors $|\psi_0(\lambda)\rangle$ obtained in Task 1, calculate the expectation value of the magnetization for each of the three spins:

$$\langle S_i^z \rangle = \langle \psi_0(\lambda) | S_i^z | \psi_0(\lambda) \rangle$$

Plot: Create a single plot showing the three curves $\langle S_0^z \rangle$, $\langle S_1^z \rangle$, and $\langle S_2^z \rangle$ as a function of λ. Add a legend to identify each spin.

Observe how the expectation values approach $+1$ or -1 as $\lambda \to 1$, revealing the classical solution to the optimization problem.

Hints for implementation

Spin matrices:
We use the standard Pauli matrices:

$$S^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad S^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Hilbert space construction:
Construct the full system operators $S_i^{x,z}$ using the Kronecker product. For example, the Z operator for spin 1 in a 3-spin system is:

$$S_1^z = \mathbb{I} \otimes S^z \otimes \mathbb{I}$$

You can compute this using `numpy.kron()`.

Data representation:
Store the fields in a simple list `h = [-0.5, 0.5, -0.1]` and the couplings in a dictionary mapping tuples to values:
`J = {(0,1): -0.4, (1,2): -1.6, (0,2): -1.0}`.

Extra credit or presentation topics

1D Antiferromagnetic Chain

Write a recursive function to generate the $2^N \times 2^N$ Hamiltonian for a 1D chain of N spins with uniform antiferromagnetic interactions ($J_{i,i+1} = -1.0, h_i = 0$). Scale N from 3 to 10. How does the minimum energy gap $\Delta E_{\min}$ shrink? (*Hint: Use* `scipy.sparse.linalg.eigsh`).

Real-time simulation

Task 1 assumes the system stays perfectly in the ground state. In a real device, the state evolves dynamically: $i\partial_t|\psi(t)\rangle = H(\lambda(t))|\psi(t)\rangle$. For your presentation, simulate this time sweep iteratively by discretizing the total time T into small steps dt. At each step, update the state using the evolution operator $|\psi(t+dt)\rangle = e^{-iH(\lambda(t))dt}|\psi(t)\rangle$ via `scipy.linalg.expm`. Plot the final fidelity $|\langle\psi_{\text{exact}}|\psi(T)\rangle|^2$ versus T to demonstrate the Landau-Zener transition!

Entanglement entropy

Calculate and plot the von Neumann entanglement entropy of a subsystem during the adiabatic sweep to see how entanglement peaks at the phase transition. More details in *"reading material"*.