Practical Quantum Computing: Introduction

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Plan

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Scheme of our meetings

- lecture: theoretical foundations and a review
- presentation: results from a real device or extension of the previous lab
- 2 or 3 tasks to complete on your own
- practical (PQC): hands-on learning and experimenting, while rehearsing QM notions

Earning credits

- lecture contains introduction to the lab: attendance is required
- one lab is worth 5 points; a week later 80% points
- written exam 30 points (last week, retake during “sesja”)
- one presentation: 5 points (during semester or last week)

Course topics

1. Analog model of QC
   - classical complexity
   - classical and quantum annealers
   - adiabatic QC
2. Circuit model of QC
   - single qubit gates, two qubit gates
   - entanglement, Bell's inequality
   - teleportation
3. Physical foundations
   - superconducting quantum circuit
   - echo experiments, T_1, T_2 times
   - noise models
4. Algorithms
   - Grover search
   - quantum Fourier transform
   - phase estimation

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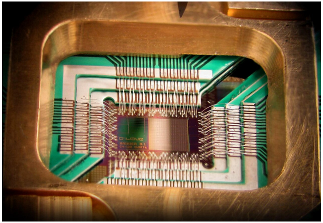
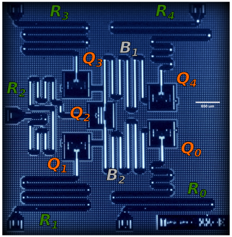
Why quantum computation (QC)?

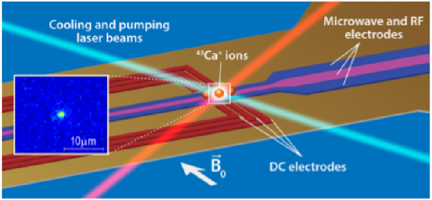
- crossroads of physics and computer science
- circuit model of QC (IBM, IonQ, Rigetti)
- analog model of QC: D-Wave company
- Google, IBM, many startups
  $\rightarrow$ extreme effort to find a practical application



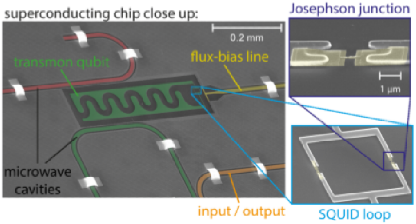


Current platforms

Ions trapped with laser beams



Superconducting circuits: transmon



Other platforms:
neutral atoms, photonic computing, semiconductor spin qubits, NV centers in diamond, nuclear spins, majorana-based qubits.

Current platforms (cont.)

Superconducting qubit (transmon):

- companies: IBM, Google, Rigetti, IQM
- pros: fast gates (~ 10ns), integrated with existing fabrication and electronics techniques
- cons: ultra-low temperatures, limited coherence, crosstalk

Trapped ions:

- companies: IonQ, Quantinuum (formerly Honeywell)
- pros: long coherence times (~ 10s), high-fidelity gates, all-to-all connectivity
- cons: slow gate speeds ($\sim$ms), complex laser-based control, challenges due to ions stability

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Qubit - elementary building block

- ground state $|g\rangle$, excited state $|e\rangle$ of a neutral cold atom, also superconducting qubit
- spin up state $|\uparrow\rangle$, spin down state $|\downarrow\rangle$ of electron in quantum dot, trapped ion, nuclei in NMR
- polarization of a photon $|H\rangle$, $|V\rangle$
- computational basis (physical qubit): $|0\rangle$, $|1\rangle$

We need a general state: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$

Qubit operators in computational basis ('0', '1'):

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \quad \sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \quad \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

are called Pauli matrices, also denoted X, Y, Z.

Physics of a qubit evolution

Hamiltonian of the two-level system, perturbed (driven) by a constant frequency signal:

$$H = \frac{\Omega}{2}\hat{\sigma}_z + \omega_R \cos(\omega t + \phi)\hat{\sigma}_x,$$

where Ω – splitting between the levels, ω – driving frequency, ω_R – coupling (Rabi frequency).

Canonical transformation to the rotating frame $U = e^{i\frac{\Omega}{2}t\sigma_z}$; change of basis $\tilde{\psi} = U\psi$. Schrödinger equation in the new frame is:

$$i\partial_t\tilde{\psi} = (UHU^\dagger + i(\partial_t U)U^\dagger)\tilde{\psi}. \quad (1)$$

Rabi oscillations

Idea: rotating frame cancels explicit time dependence. Result is based on the rotating wave approximation (RWA), which amounts to neglecting quickly oscillating terms. We further simplify to the resonance condition $\Omega = \omega$.

Evolution of the system in the rotating frame after time t:

$$\tilde{\psi}(t) = e^{-i\frac{\omega_R t}{2}(\cos\phi\sigma_x + \sin\phi\sigma_y)}\tilde{\psi}(0),$$

or in the original basis:

$$\psi(t) = e^{-i\frac{\Omega}{2}t\sigma_z}e^{-i\frac{\omega_R t}{2}(\cos\phi\sigma_x + \sin\phi\sigma_y)}\psi(0).$$

Example: for $\phi = 0$, $\frac{\omega_R t}{2} = \pi/2$: $e^{-i\frac{\pi}{2}\sigma_x} = -i\sigma_x \equiv X$ gate up to a phase (we call it a π-pulse).

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Driven Qubit Setup

Two-level system (qubit) with splitting Ω.

Hamiltonian (set $\hbar = 1$):

$$H(t) = \frac{\Omega}{2}\sigma_z + \omega_R \cos(\omega t + \phi)\sigma_x$$

- Ω — qubit transition frequency
- ω — drive frequency
- ω_R — drive amplitude (bare Rabi frequency)
- ϕ — drive phase

Goal: understand qubit evolution under resonant driving.

Rotating Frame Transformation

Move to a frame rotating at the drive frequency ω:

$$U(t) = e^{i\frac{\omega t}{2}\sigma_z} \qquad \tilde{\psi} = U\psi$$

The Schrödinger equation becomes:

$$i\partial_t\tilde{\psi} = \tilde{H}\tilde{\psi}, \quad \tilde{H} = UHU^\dagger + i(\partial_t U)U^\dagger$$

Evaluating the transformation $U\sigma_x U^\dagger = \sigma_x \cos(\omega t) - \sigma_y \sin(\omega t)$:

$$\tilde{H} = \frac{\Delta}{2}\sigma_z + \omega_R \cos(\omega t + \phi)\left(\sigma_x \cos\omega t - \sigma_y \sin\omega t\right)$$

where $\Delta = \Omega - \omega$ is the detuning.

Rotating Wave Approximation (RWA)

Using trigonometric identities for the drive terms:

$$\cos(\omega t + \phi)\cos(\omega t) = \frac{1}{2}\left[\cos(\phi) + \cos(2\omega t + \phi)\right]$$
$$-\cos(\omega t + \phi)\sin(\omega t) = \frac{1}{2}\left[\sin(\phi) - \sin(2\omega t + \phi)\right]$$

In $\tilde{H}$ this produces:

- slowly varying terms (dependent only on ϕ)
- fast terms oscillating at 2ω

If $\omega \gg \omega_R$, the fast-oscillating terms average to zero over the timescale of the qubit evolution.

Effective Hamiltonian (RWA):

$$H_{\text{RWA}} = \frac{\Delta}{2}\sigma_z + \frac{\omega_R}{2}\left(\cos\phi\,\sigma_x + \sin\phi\,\sigma_y\right)$$

The Hamiltonian is now time-independent.

Solution and Rabi Oscillations

Time evolution in the rotating frame:

$$\tilde{\psi}(t) = e^{-iH_{\mathrm{RWA}}t}\tilde{\psi}(0)$$

On resonance ($\Delta = 0$):

$$H_{\mathrm{RWA}} = \frac{\omega_R}{2}\left(\cos\phi\,\sigma_x + \sin\phi\,\sigma_y\right)$$

Using the matrix exponential identity:

$$e^{-i\theta \boldsymbol{n}\cdot\boldsymbol{\sigma}} = \cos\theta\, I - i\sin\theta\, \boldsymbol{n}\cdot\boldsymbol{\sigma}$$

we obtain coherent oscillations (Rabi oscillations).

π-Pulse and Quantum Gates

Take $\phi = 0$ (drive along x axis):

$$H_{\text{RWA}} = \frac{\omega_R}{2}\sigma_x$$

Evolution operator:

$$U(t) = e^{-i\frac{\omega_R t}{2}\sigma_x}$$

A π-pulse satisfies the condition where the argument of the exponential is $\frac{\pi}{2}$:

$$\frac{\omega_R t_\pi}{2} = \frac{\pi}{2} \implies t_\pi = \frac{\pi}{\omega_R}$$

This yields:

$$U(t_\pi) = e^{-i\frac{\pi}{2}\sigma_x} = -i\sigma_x$$