Analog Quantum Computer.

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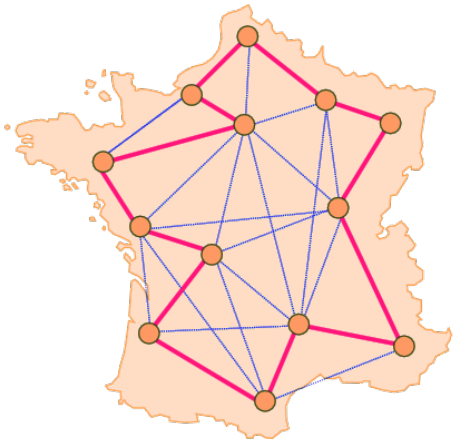
Problems to solve

typically combinatorial optimization problems

- graph colouring, SAT logical problems
- traveling salesman problem, knapsack problem

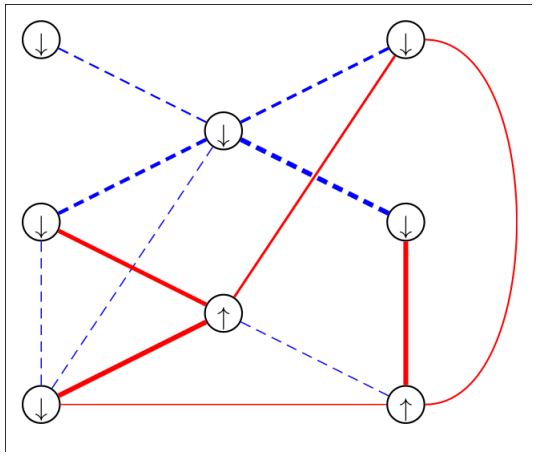


complexity classes:
examples of **NP**-complete problems – particularly hard to solve

Energy landscape of a Spin Glass

- optimization problem related to physics of magnetism
- N spins, which can take only values ± 1 (or $\uparrow, \downarrow$)
- energy of the system is our cost function
  (increases for a pair of misplaced objects)

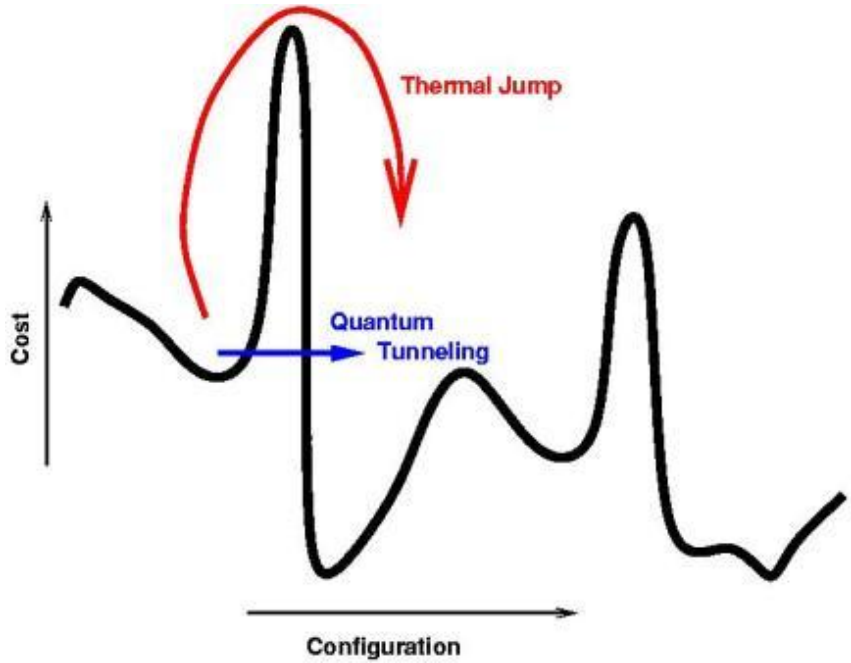
$$H = E(s_1, \ldots, s_N) = -\sum_{ij} J_{ij} s_i s_j - \sum_i h_i s_i$$



- competing links connecting spins: F or AF causes frustration

Annealing in Spin Glass problem

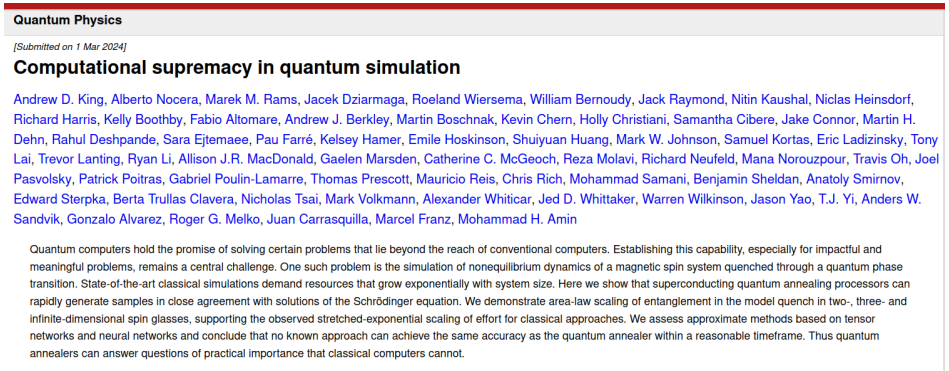
Classical vs. Quantum:
Classical simulated annealing relies on thermal fluctuations to hop *over* high energy barriers. Quantum annealing utilizes **quantum tunneling** to pass *through* narrow, high barriers.



This is the foundational idea behind **Analog Quantum Computers** (Quantum Annealers) built by companies like D-Wave.

The Great Debate: quantum vs. classical computation

Here we show that superconducting quantum annealing processors can rapidly generate samples in close agreement with solutions of the Schrodinger equation – performed on D-wave hardware



Quantum Physics

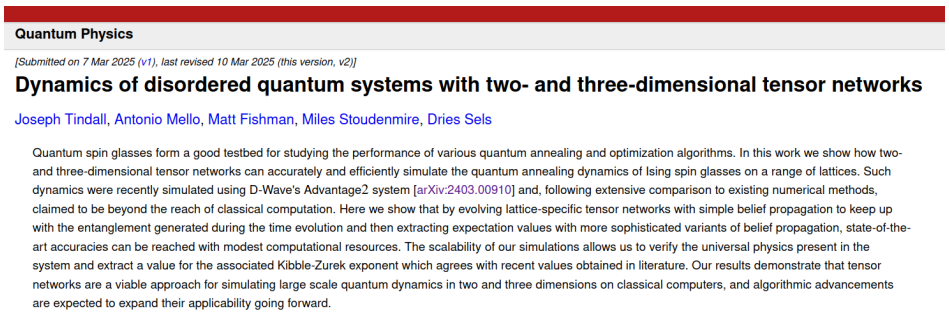
[Submitted on 1 Mar 2024]

Computational supremacy in quantum simulation

Andrew D. King, Alberto Nocera, Marek M. Rams, Jacek Dziarmaga, Roeland Wiersema, William Bernoudy, Jack Raymond, Nitin Kaushal, Niclas Heinsdorf, Richard Harris, Kelly Boothby, Fabio Altomare, Andrew J. Berkley, Martin Boschnak, Kevin Chern, Holly Christiani, Samantha Cibere, Jake Connor, Martin H. Dehn, Rahul Deshpande, Sara Ejtemaee, Pau Farré, Kelsey Hamer, Emile Hoskinson, Shuiyuan Huang, Mark W. Johnson, Samuel Kortas, Eric Ladizinsky, Tony Lai, Trevor Lanting, Ryan Li, Allison J.R. MacDonald, Gaelen Marsden, Catherine C. McGeoch, Reza Molavi, Richard Neufeld, Mana Norouzpour, Travis Oh, Joel Pasvolsky, Patrick Poitras, Gabriel Poulin-Lamarre, Thomas Prescott, Mauricio Reis, Chris Rich, Mohammad Samani, Benjamin Sheldan, Anatoly Smirnov, Edward Sterpka, Berta Trullas Clavera, Nicholas Tsai, Mark Volkmann, Alexander Whiticar, Jed D. Whittaker, Warren Wilkinson, Jason Yao, T.J. Yi, Anders W. Sandvik, Gonzalo Alvarez, Roger G. Melko, Juan Carrasquilla, Marcel Franz, Mohammad H. Amin

Quantum computers hold the promise of solving certain problems that lie beyond the reach of conventional computers. Establishing this capability, especially for impactful and meaningful problems, remains a central challenge. One such problem is the simulation of nonequilibrium dynamics of a magnetic spin system quenched through a quantum phase transition. State-of-the-art classical simulations demand resources that grow exponentially with system size. Here we show that superconducting quantum annealing processors can rapidly generate samples in close agreement with solutions of the Schrödinger equation. We demonstrate area-law scaling of entanglement in the model quench in two-, three- and infinite-dimensional spin glasses, supporting the observed stretched-exponential scaling of effort for classical approaches. We assess approximate methods based on tensor networks and neural networks and conclude that no known approach can achieve the same accuracy as the quantum annealer within a reasonable timeframe. Thus quantum annealers can answer questions of practical importance that classical computers cannot.

Here we show that by evolving lattice-specific tensor networks (...) state-of-the-art accuracies can be reached with modest computational resources.



Quantum Physics

[Submitted on 7 Mar 2025 (v1), last revised 10 Mar 2025 (this version, v2)]

Dynamics of disordered quantum systems with two- and three-dimensional tensor networks

Joseph Tindall, Antonio Mello, Matt Fishman, Miles Stoudenmire, Dries Sels

Quantum spin glasses form a good testbed for studying the performance of various quantum annealing and optimization algorithms. In this work we show how two- and three-dimensional tensor networks can accurately and efficiently simulate the quantum annealing dynamics of Ising spin glasses on a range of lattices. Such dynamics were recently simulated using D-Wave's Advantage2 system [arXiv:2403.00910] and, following extensive comparison to existing numerical methods, claimed to be beyond the reach of classical computation. Here we show that by evolving lattice-specific tensor networks with simple belief propagation to keep up with the entanglement generated during the time evolution and then extracting expectation values with more sophisticated variants of belief propagation, state-of-the-art accuracies can be reached with modest computational resources. The scalability of our simulations allows us to verify the universal physics present in the system and extract a value for the associated Kibble-Zurek exponent which agrees with recent values obtained in literature. Our results demonstrate that tensor networks are a viable approach for simulating large scale quantum dynamics in two and three dimensions on classical computers, and algorithmic advancements are expected to expand their applicability going forward.

Physical reality and NP-completeness (*)

Scott Aaronson's Conjecture
In his paper *"NP-complete Problems and Physical Reality"*, Aaronson postulates a fundamental impossibility:

> *"There is no physical means to solve NP-complete problems in polynomial time."*

If this is true, nature fundamenally forbids it. Not even quantum mechanics (or potentially quantum gravity) can provide a shortcut.

The Consequence:
If we try to solve 3-SAT on a quantum annealer, the physics *must* bottleneck the computation. Let's see exactly where this physical bottleneck occurs.

Plan

Spin as qubit – elementary building block

- states of the form: $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ (spin computational basis)
- important state: $|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$
  random results along z, oriented along x

Spin operators:

along z-axis: $s_z|\uparrow\rangle = |\uparrow\rangle$, $s_z|\downarrow\rangle = -|\downarrow\rangle$
along x-axis: $s_x|\uparrow\rangle = |\downarrow\rangle$, $s_x|\downarrow\rangle = |\uparrow\rangle$
simple check: what is $s_x|\rightarrow\rangle =$?

Quantum spins engineering

TARGET: quantum form of the spin-glass problem

$$H_1 = -\sum_{ij} J_{ij} s_i^z s_j^z - \sum_i h_i s_i^z$$

the corresponding ground state is $|\mathrm{GS}\rangle_1 = |s_1, s_2, \ldots\rangle$ where $\{s_i\}$ minimizes the target function $E(s_1, s_2, \ldots, s_N)$.

FLUCTUATIONS: most democratic state as the groundstate of

$$H_0 = -\sum_i s_i^x$$

with $|\mathrm{GS}\rangle_0 = |\rightarrow_1, \rightarrow_2, \rightarrow_3\rangle$
$= \frac{1}{\sqrt{2}^3} \left(|\uparrow,\uparrow,\uparrow\rangle + |\uparrow,\uparrow,\downarrow\rangle + |\uparrow,\downarrow,\uparrow\rangle + |\uparrow,\downarrow,\downarrow\rangle + \ldots\right)$
for $N = 3$ as an example.

Plan

The adiabatic Hamiltonian

We define a time-dependent (parameter-dependent) Hamiltonian:

$$H(\lambda) = (1 - \lambda)H_0 + \lambda H_1$$

where we control the "experimental knob" $\lambda \in [0, 1]$.

Initial state ($\lambda = 0$):
H_0 is chosen so its ground state is a trivial superposition (easy to prepare):

$$H_0 = -\sum_i \sigma_i^x$$

Final state ($\lambda = 1$):
H_1 is our target optimization problem (the spin glass):

$$H_1 = -\sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z - \sum_i h_i s_i^z$$

The Adiabatic Theorem and the gap

The Adiabatic Theorem:
If a quantum system begins in the ground state of H_0 and the Hamiltonian evolves *slowly enough*, the system will remain in the instantaneous ground state of $H(\lambda)$.

The Bottleneck:
We must avoid exciting the system. The probability of jumping to a higher energy state depends heavily on the energy gap between the ground state E_0 and the first excited state E_1:

$$\Delta E(\lambda) = E_1(\lambda) - E_0(\lambda)$$

The point where ΔE is smallest (**avoided crossing**) is the most dangerous part of the computation.

The speed limit of computation

How slowly must we change λ? The speed limit is governed by the **Landau-Zener formula**:

$$\frac{d\lambda}{dt} \propto (\Delta E(\lambda))^2$$

We define the total running time of the Adiabatic Quantum Computer (T_{AQC}) as:

$$T_{AQC} = \int_0^1 \frac{d\lambda}{v(\lambda)} = \int_0^1 \frac{d\lambda}{[\Delta E(\lambda)]^2}$$

Note: The derivation and simulation of the Landau-Zener transition probability is an excellent topic for a presentation!

(check: https://arxiv.org/abs/2306.11633 "Landau Zener problem in the classroom")

Final remarks

- one wants to study the scaling of T_{AQC} with N
- few analytical results exist eg. database (unstructured) search $T_{\mathrm{AQC}} \propto \sqrt{2^N}$ (as in Grover algorithm for circuit QC)
- efficient solution of the optimization problem for $T_{\mathrm{AQC}} \leq P(N)$ (**BQP** class)

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- trotterization gives the mapping:
  adiabatic evolution $\rightarrow$ circuit model
- adiabatic QC idea is useful as a sub-routine e.g. in QC studies of molecules (VQE, QAOA approaches)
- amazing link with quantum phase transitions (QPT):
  $\Delta E \rightarrow 0$ for $N \rightarrow \infty$ at QPT

Our optional reading material provides some more explanations:
J. Rodriguez-Laguna and S. N. Santalla, Am. J. Phys. **86**, 360 (2018).