Playing Quantum Games

How Quantum Mechanics rewrites the rules of game theory...

Jakub Tworzydło

Institute of Theoretical Physicis, FUW
Jakub.Tworzydlo@fuw.edu.pl

April 2026 Pasteura, Warszawa

Plan

1. The Classical Setup

2. The Quantum Generalization

3. The Quantum Advantage

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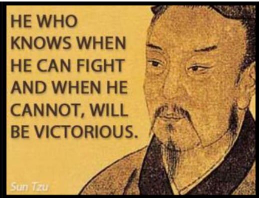
What is Game Theory?

Game theory studies the ways in which **strategic interactions among rational players** produce outcomes with respect to the players' preferences.

- **Players:** rational decision-makers (e.g., Alice and Bob).
- **Strategies:** the choices available to each player.
- **Payoffs:** the reward/penalty a player receives, dependent on the *combined* choices of all players.



Sun Tzu, 544–496 BC

The Prisoner's Dilemma

Two players, Alice and Bob, are arrested. They are interrogated separately and face two choices:

- **Cooperate (C)** with each other (remain silent).
- **Defect (D)** against each other (betray the other).

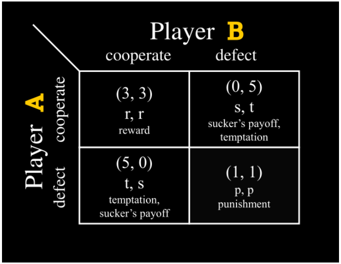
The payoff matrix: (normalized form)

- $r = 3$: reward for mutual cooperation (C, C)
- $t = 5$: temptation to defect (D, C)
- $s = 0$: sucker's payoff (C, D)
- $p = 1$: punishment for mutual defection (D, D)

Expected payoff for A: $\$_A = 3P_{CC} + 5P_{DC} + 0P_{CD} + 1P_{DD}$

Nash equilibrium vs. Pareto efficiency

The trap: no matter what Bob does, Alice's highest payoff comes from Defecting (*D*). Bob faces the exact same logic.



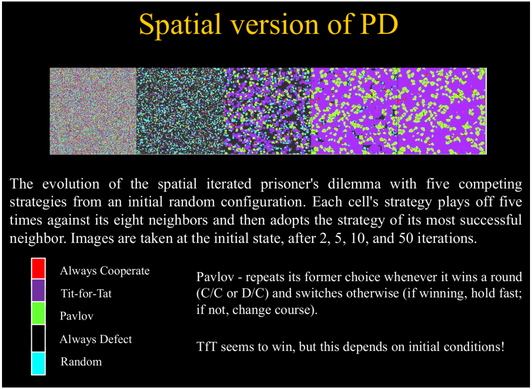
Player **A** \ Player **B**	cooperate	defect
cooperate	(3, 3) r, r reward	(0, 5) s, t sucker's payoff, temptation
defect	(5, 0) t, s temptation, sucker's payoff	(1, 1) p, p punishment

- **Nash equilibrium:** a state where no player can gain by unilaterally changing their strategy. In PD, this is **(*D*, *D*)**, yielding 1 point each.
- **Pareto efficient outcome:** an outcome where no one can be made better off without making someone else worse off. Mutual cooperation **(*C*, *C*)** yields 3 points each, best for both.

Escaping the dilemma classically?

Classically, escaping the sub-optimal (D, D) trap usually requires changing the rules of the game:

- **Iterated games:** playing multiple rounds allows for strategies with memory (e.g., Tit-for-Tat, Pavlov).
- **Spatial games:** evolution of strategies over a grid or network.



Spatial version of PD

The evolution of the spatial iterated prisoner's dilemma with five competing strategies from an initial random configuration. Each cell's strategy plays off five times against its eight neighbors and then adopts the strategy of its most successful neighbor. Images are taken at the initial state, after 2, 5, 10, and 50 iterations.

🟥	Always Cooperate
🟪	Tit-for-Tat
🟩	Pavlov
⬛	Always Defect
🟦	Random

Pavlov - repeats its former choice whenever it wins a round (C/C or D/C) and switches otherwise (if winning, hold fast; if not, change course).

TfT seems to win, but this depends on initial conditions!

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Making the Game Quantum

What if Alice and Bob have access to qubits instead of classical bits?

- **basis states:** $|C\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|D\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- **strategies as unitaries:** A player's choice is no longer a static coin flip, but a unitary transformation $\hat{U}$ applied to their qubit.

The Eisert-Wilkens-Lewenstein (EWL) protocol

To be a *proper* quantum generalization, the protocol must reduce to the classical game if players are restricted to classical moves.

The EWL circuit steps:

1. **Initialization:** Qubits start in state $|CC\rangle$.
2. **Entanglement:** A referee applies an entangling gate $\hat{J}$.
3. **Moves:** Alice applies $\hat{U}_A$ and Bob applies $\hat{U}_B$ independently.
4. **Disentanglement:** The referee applies the inverse gate $\hat{J}^\dagger$.
5. **Measurement:** Qubits are measured in the computational basis with either C or D outcome.

The entangling gate

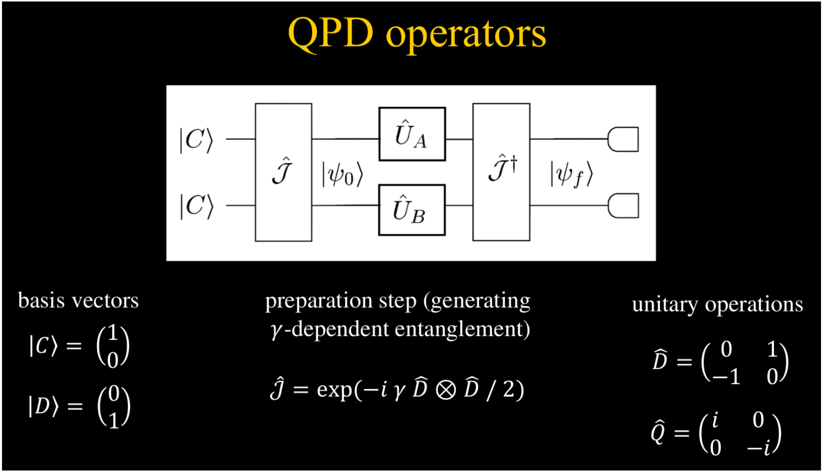
The key to the quantum advantage is the entangling operator $\hat{J}(\gamma)$, which correlates Alice and Bob's qubits before they make their moves.

$$\hat{J}(\gamma) = \exp\left(-i\frac{\gamma}{2}\hat{D} \otimes \hat{D}\right)$$

The parameter $\gamma \in [0, \pi/2]$ acts as a "dial" for the quantumness of the game:

- $\gamma = 0$**:** The game is separable – reduces entirely to the classical Prisoner's Dilemma.
- $\gamma = \pi/2$**:** The game is maximally entangled (... some magic happens).

Summary of quantum PD game



QPD operators

basis vectors

$$|C\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|D\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

preparation step (generating γ-dependent entanglement)

$$\hat{J} = \exp(-i\,\gamma\,\hat{D}\otimes\hat{D}\,/\,2)$$

unitary operations

$$\hat{D} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$$\hat{Q} = \begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix}$$

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Classical vs. Quantum Operators

The classical move: The traditional ”Defect” strategy acts like a bit-flip (with a phase shift), ”Cooperate” is the identity:

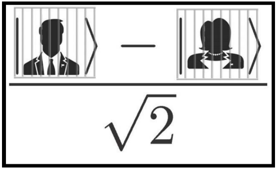
$$\hat{D} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \hat{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The quantum strategy: Quantum players have access to the full Bloch sphere. We define a specific ”Quantum” strategy $\hat{Q}$, which alters the phase without flipping the state:

$$\hat{Q} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

If Alice plays $\hat{Q}$ while Bob plays $\hat{D}$, Alice is using superposition and interference to manipulate the final probability amplitudes to her advantage.

The Quantum Advantage: a new equilibrium



When the game is sufficiently entangled ($\gamma > \gamma_{th2}$):

- The traditional mutual defection strategy $(\hat{D}, \hat{D})$ is no longer the best option.
- If both players adopt the quantum strategy $(\hat{Q}, \hat{Q})$, the final state gracefully disentangles back into $|CC\rangle$.
- **Result:** a new, Pareto-efficient Nash equilibrium emerges. The players escape the dilemma and guarantee the maximum mutual payoff of 3!

We will test some of these findings in the lab...