Lasers

lecture 3

Czesław Radzewicz

laser amplifier

from lecture 2 we have

1. photon flux $F \equiv \frac{I}{\hbar\omega}$ $\left[\frac{1}{\mathrm{cm}^2\mathrm{s}}\right]$ evolves according to:

$$\frac{\partial}{\partial z}F = \sigma_{12}(\omega)\cdot\Delta N\cdot F$$

2. the populations inversion ΔN $\left[\frac{1}{\mathrm{cm}^3}\right]$ evolution is governed by:

$$\frac{\partial}{\partial t}\Delta N = -\frac{\Delta N}{T_1} - 2\sigma_{12}\cdot\Delta N\cdot F$$

note1: in more realistic models we will go beyond the two-level model and the second equation will be modified accordingly. In systems with a short lifetime of the lower level (the most common case) the factor 2 is missing.

note2: we switch from 0 to 12 when indexing the cross-section. From now on we will use σ_{12} to signify a typical energy level system with 1 and 2 being the lower and upper level of laser transition.

2 variables and 2 first order differential equations. The problem is that the equations are nonlinear – there are no general analytic solutions.

options:

- numerical integration
- approximate solutions

simple dynamics of the laser amplifier

$$\frac{\partial}{\partial z}F(t,z) = \sigma_{12}\Delta N(t,z)F(t,z)$$

$$\frac{\partial}{\partial t}\Delta N(t,z) = -\frac{\Delta N(t,z)}{T_1} - 2\sigma_{12}\Delta N(t,z)F(t,z)$$

Note that, for the amplifier to work, we need some initial population inversion ΔN_0. This modifies the second equation which now reads (time and space dependence are dropped for clarity):

$$\frac{\partial}{\partial t}\Delta N = -\frac{\Delta N - \Delta N_0}{T_1} - 2\sigma_{12}\Delta N F$$

We formally transform the two equations as follows: From the first one we calculate $\Delta N = \frac{\partial F}{\partial z}/(\sigma_{01}F)$ and insert it into the second equation

$$\frac{\partial^2}{\partial t \partial z}\ln F + \sigma_{12}\frac{\partial}{\partial z}F + \frac{1}{T_1}\left(\frac{\partial}{\partial z}\ln F - \sigma_{12}\Delta N_0\right) = 0$$

There are 2 characteristic time scales involved in this problem: (1) the population decay time T_1 and (2) light pulse duration (t_p). Wee will attempt to solve those equations in two limiting cases

„short" pulse laser amplifier

„short" pulses ($t_p \ll T_1$). They actually cannot be too short – we have previously neglected the transverse relaxation time so we need $t_p \gg T_2$

$$\frac{\partial^2}{\partial t \partial z} \ln F + 2\sigma_{12} \frac{\partial}{\partial z} F + \underbrace{\frac{1}{T_1}\left(\frac{\partial}{\partial z} \ln F - \sigma_{01} \Delta N_0\right)}_{\text{this term is small; we will drop it}} = 0$$

solutions

$$F(z,t) = \frac{F(0,t)}{1 - e^{-S(t)}\left(1 - e^{-G(z)}\right)}$$

$$\Delta N(z,t) = \frac{\Delta N(z,0) e^{-G(z)}}{e^{S(t)} + e^{-G(z)} - 1}$$

$$S(t) = 2\sigma_{12} \int_{-\infty}^{t} F(0,t') dt'$$

$$G(z) = \sigma_{12} \int_{0}^{z} \Delta N(z',0) dz'$$

L.M. Frantz and J.S. Nodvik, *Theory of pulse propagation in a laser amplifier*, J. Appl. Phys. 34, 2346 (1963)

„short" pulse laser amplifier, 2

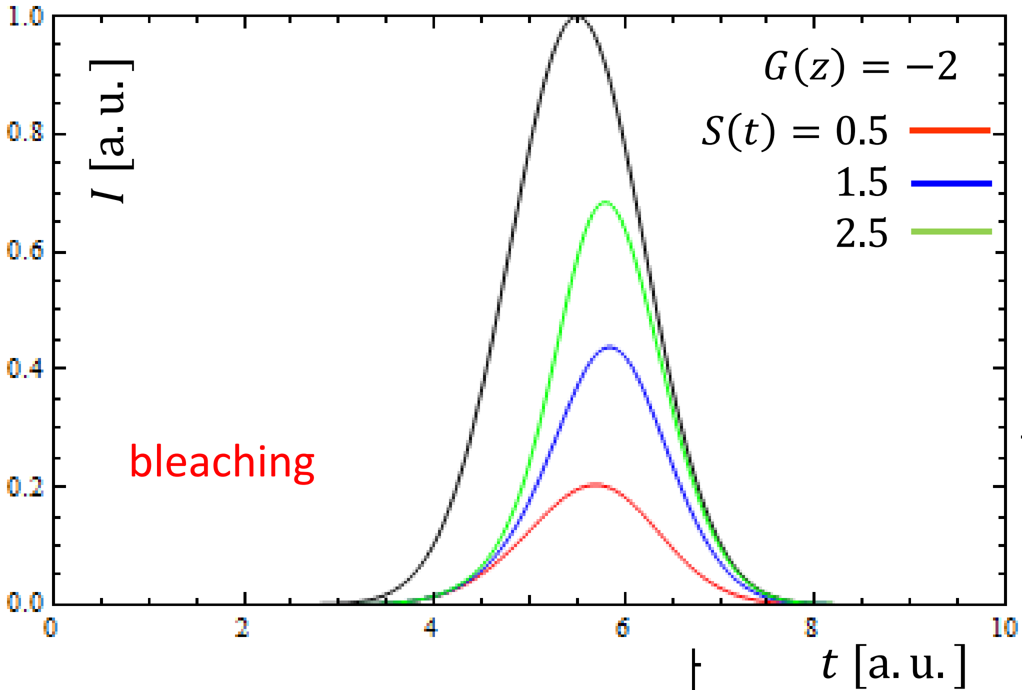
gain saturation leads to pulse shaping.

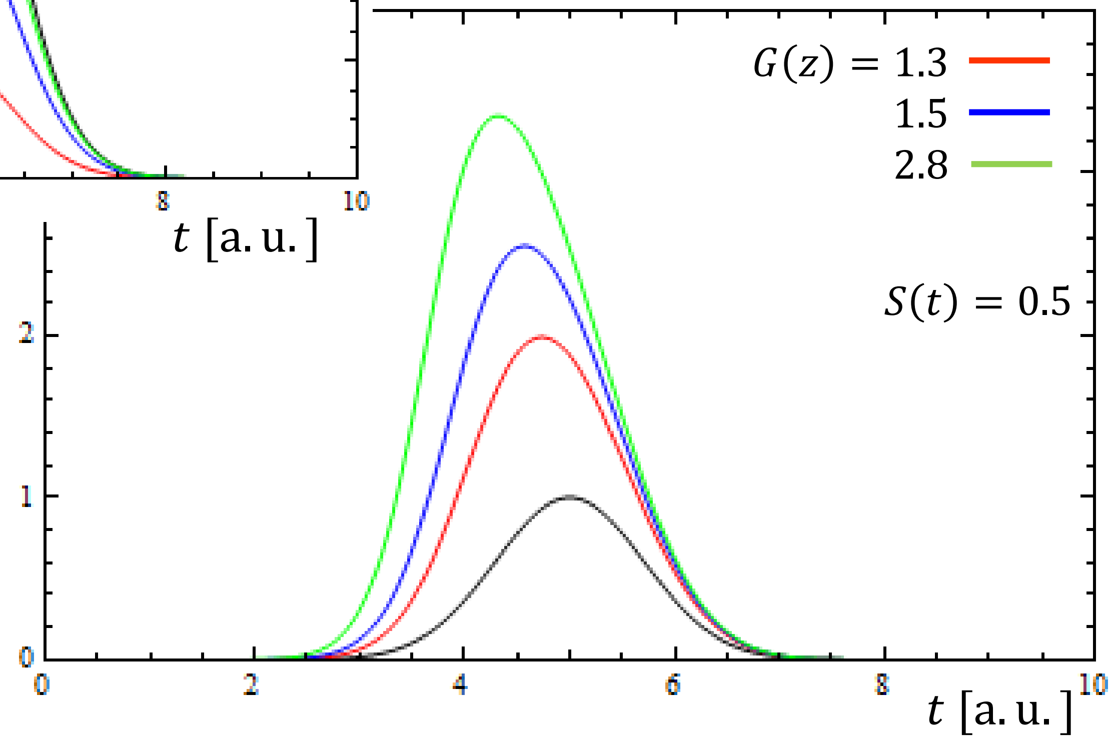
$$F(z,t) = \frac{F(0,t)}{1 - e^{-S(t)}\left(1 - e^{-G(z)}\right)}$$

$$\Delta N(z,t) = \frac{\Delta N(z,0)e^{-G(z)}}{e^{S(t)} + e^{-G(z)} - 1}$$

$$S(t) = 2\sigma_{12}\int_{-\infty}^{t} F(0,t')dt'$$

$$G(z) = \sigma_{12}\int_{0}^{z} \Delta N(z',0)dz'$$





„long” amplified pulses ($\tau_p \gg T_1 , T_2$).

„long” pulse laser amplifier

$$\frac{\partial^2}{\partial t \partial z} \ln F + \sigma_{12} \frac{\partial}{\partial z} F + \frac{1}{T_1} \left(\frac{\partial}{\partial z} \ln F - \sigma_{12} \Delta N_0 \right) = 0$$

small –we drop it

Note: no factor 2 in this eq. - explanation will be given later.

$$\frac{\partial}{\partial z} F = \frac{\gamma_0 F}{1 + F/F_s}$$

$F_s = (\sigma_{12} T_1)^{-1}$

the formal solution is

$$\ln \frac{F(z,t)}{F(0,t)} + \frac{F(z,t) - F(0,t)}{F_s} = \gamma_0 z$$

two limits:

$I \ll I_s \quad \Longrightarrow \quad F(z,t) = e^{\gamma_0 z} F(0,t)$ unsaturated laser amplifier

$I \gg I_s \quad \Longrightarrow \quad F(z,t) = F(0,t)$ completely saturated laser amplifier

$\gamma_0 = \sigma_{12} \Delta N_0$ - unsaturated gain coefficient

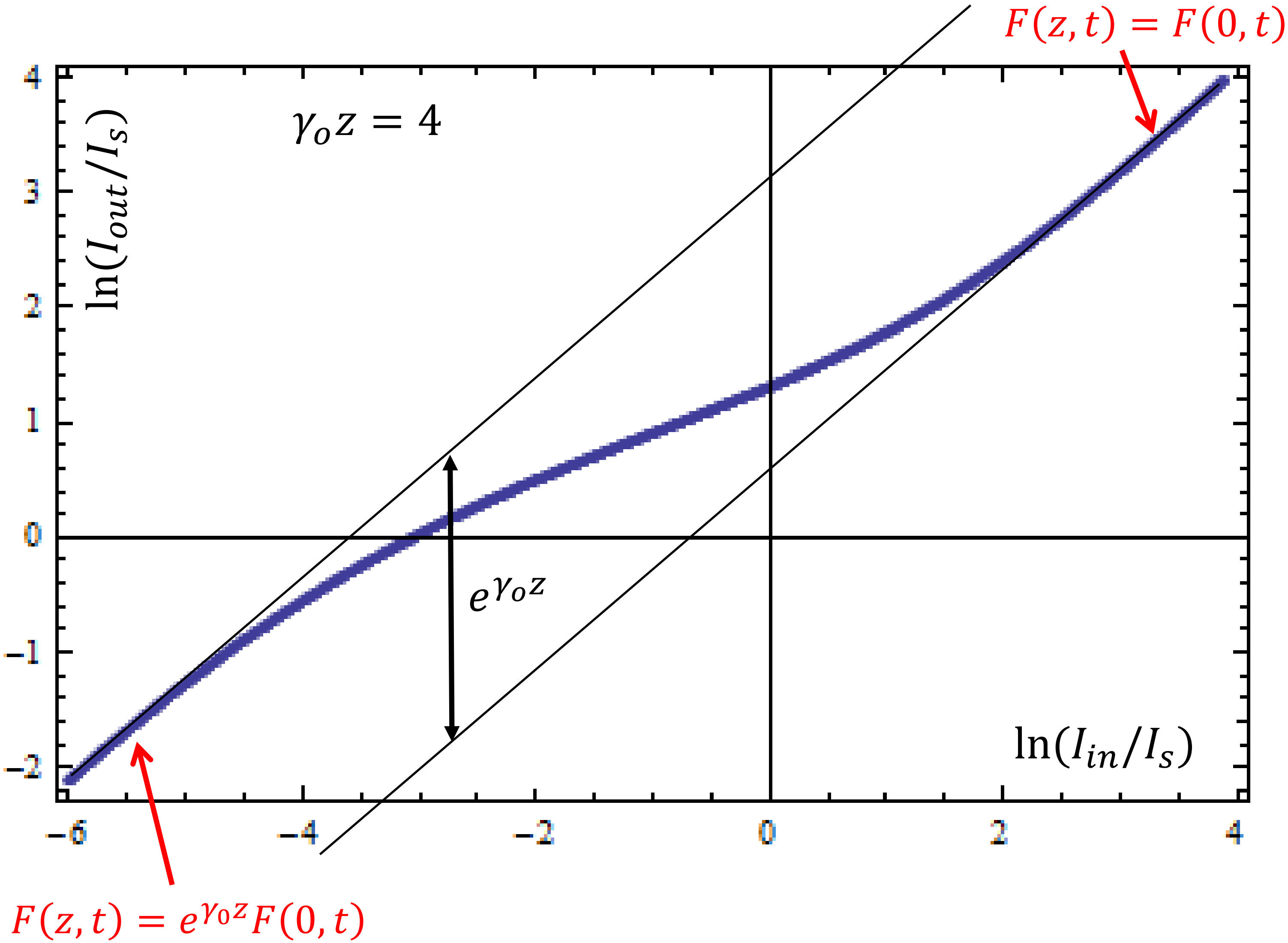
$F_s \equiv \frac{1}{T_1 \sigma_{12}}$ - saturating photon flux $\left[\frac{1}{\mathrm{s \cdot cm^2}}\right]$

$I_s = \hbar \omega F_s$ - saturating intensity $\left[\frac{\mathrm{W}}{\mathrm{cm^2}}\right]$

„long” pulse laser amplifier, 2

$$\ln\frac{F(z,t)}{F(0,t)}+\frac{F(z,t)-F(0,t)}{F_s}=\gamma_0 z$$

$$F_s=(\sigma_{01}T_1)^{-1}$$



„intermediate” pulse laser amplifier

- „intermediate” pulses ($t_p \cong T_1$). The equation has to be integrated numerically in its full splendor

$$\frac{\partial^2}{\partial t \partial z}\ln F + 2\sigma_{01}\frac{\partial}{\partial z}F + \frac{1}{T_1}\left(\frac{\partial}{\partial z}\ln F - \sigma_{01}\Delta N_0\right) = 0$$

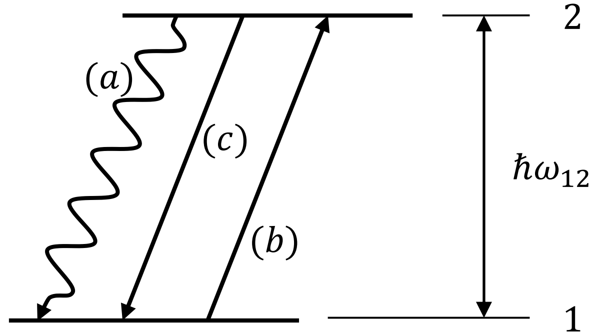
a simpler picture for light-matter interaction; Einstein coefficients

a two-level atom/ion. There are 3 radiative transitions:

a. spontaneous emission
b. absorption
c. stimulated emission

populations: $N_1 + N_2 = N$

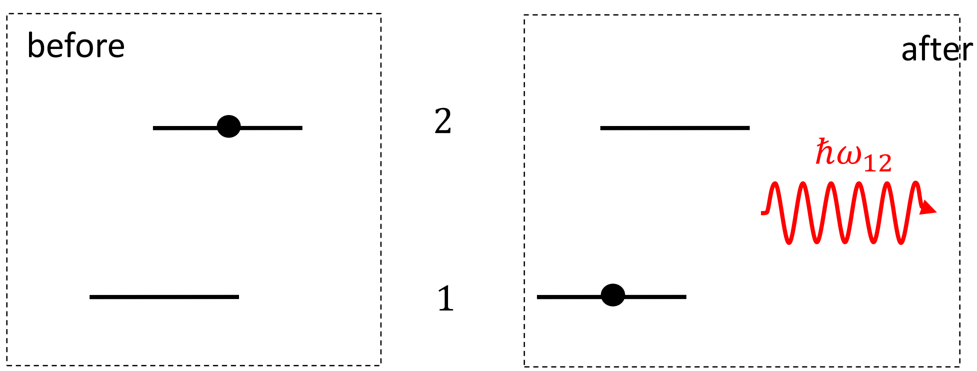
density of atoms/ions



Simple properties of the radiative transitions:

- spontaneous emission

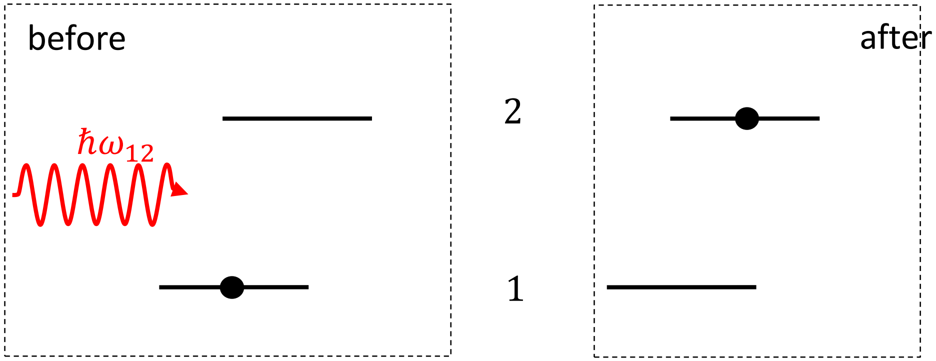
$\frac{dN_2}{dt} = -A_{21}N_2$, A_{21} is a constant (coefficient)



another picture ....

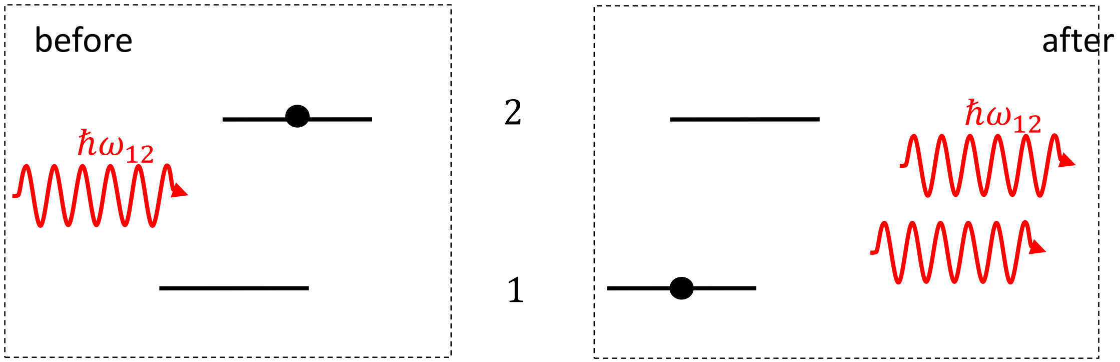
- absorption

$$\frac{dN_2}{dt} = -\frac{dN_1}{dt} = B_{12}\ \varrho(\omega_{12})N_1 \qquad B_{12}\ -\text{coefficient}, \varrho(\omega_{12})\ \text{power density of em filed}$$

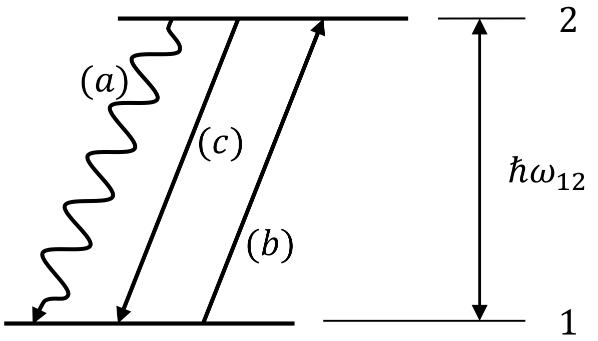


- stimulated emission

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt} = B_{21}\ \varrho(\omega_{12})N_2$$



an alternative picture ….



relations between Einstein coefficients

$$B_{21} = B_{12}$$

$$A_{21} = \frac{\hbar {\omega_{21}}^3}{\pi^3 c^3} B_{21}$$

population evolution:

$$\frac{dN_2}{dt} = -A_{21}N_2 + B_{21}\varrho(\omega_{21})(N_1 - N_2)$$

$$N_1 + N_2 = N$$

consequences:

- the same speed of stimulated transitions
- stimulated transitions dominate at low frequencies
- at high frequencies the spontaneous emission dominates

ϱ_{cr} $\left[\frac{\mathrm{J}}{\mathrm{m^3 Hz}}\right]$ - critical spectral density and critical intensity of the em field:

$$A_{21} = B_{21}\varrho_{cr}(\omega) = \frac{\hbar {\omega_{21}}^3}{\pi^3 c^3} B_{21} \Rightarrow \varrho_{cr}(\omega) = \frac{\hbar {\omega_{21}}^3}{\pi^3 c^3}$$

$$I(\omega) = \mathrm{c}\ \varrho_{\omega}(\omega) d\omega \Rightarrow I_{cr}(\omega) = \frac{\hbar {\omega_{21}}^3}{\pi^3 c^2} d\omega$$

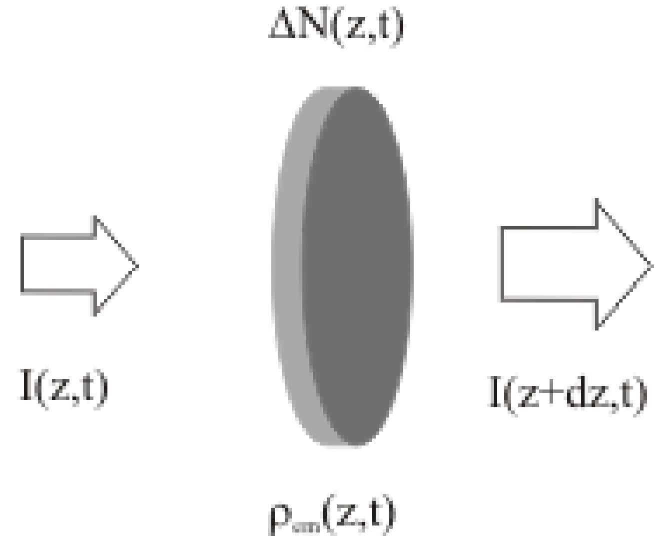
in vacuum

energy transport equation

$$I(z,t) = \varrho_{em}(z,t)d\omega \cdot v_g$$

spectral density of em field $\left[\frac{\mathrm{J}}{\mathrm{m^3 Hz}}\right]$

group velocity $v_g \equiv d\omega/dk$



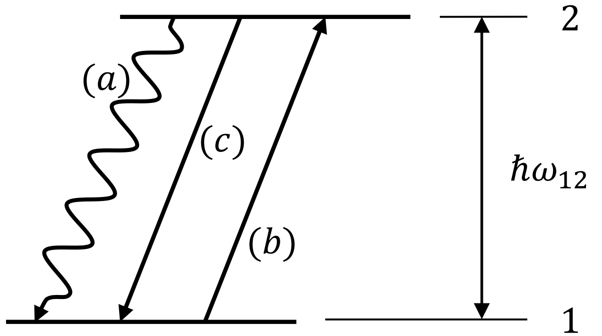
em field propagates in the z direction. consider a slice with area S and thickness dz. em field energy change within the slice:

$$\underbrace{\frac{d\varrho}{dt}}_{\frac{dI}{dt}\frac{1}{v_g}} d\omega S dz = [I(z,t) - I(z+dz,t)]S + \underbrace{\hbar\omega B_{21}\varrho d\omega}_{\sigma I \;\; (\sigma = \hbar\omega B_{21}/c)} \Delta N(z,t) S dz \qquad /(Sdz)$$

$$\frac{\partial I}{\partial z} + \frac{1}{v_g}\frac{\partial I}{\partial t} = \sigma \cdot \Delta N \cdot I$$

$$\frac{\partial F}{\partial z} + \frac{1}{v_g}\frac{\partial F}{\partial t} = \sigma \cdot \Delta N \cdot F$$ - the same as in lecture 2

populations



from Einstein's eqs.:

$$\frac{dN_2}{dt} = -A_{21}N_2 + B_{21}\varrho(\omega_{21})(N_1 - N_2)$$

$$N_1 + N_2 = N$$

which gives

$$\Delta N = N_2 - N_1 = 2N_2 - N$$

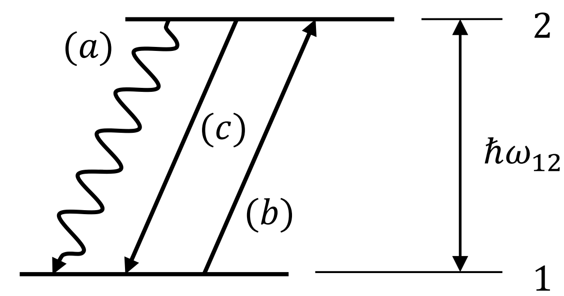
$$\frac{d}{dt}\Delta N = 2\frac{dN_2}{dt} = \underbrace{-A_{21}(\Delta N + N\,)}_{\text{spontaneous emission}} - \underbrace{2B_{21}\varrho(\omega_{21})}_{\sigma F}\Delta N$$

something is wrong; we know from lecture 2 that

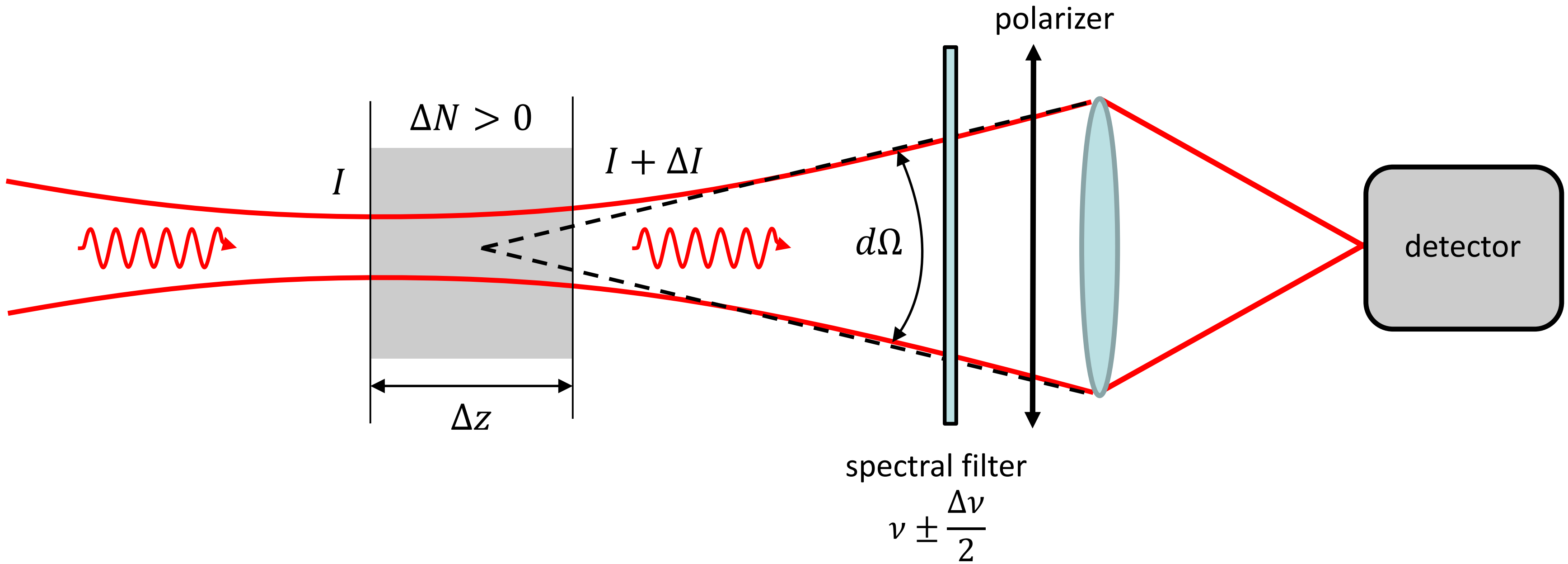
$$\frac{d}{dt}\Delta N = -\frac{1}{T_1}(\Delta N - \Delta N_0) - 2\sigma\Delta NF$$

- the rate of stimulated emission is OK
- In lecture 2 we have ignored sponatneous emission!

spontaneous emission



Again, consider a thin slice of the amplifying medium. The thing we measure is intensity and the detector cannot distinguish between photons from stimulated and spontaneous emission.



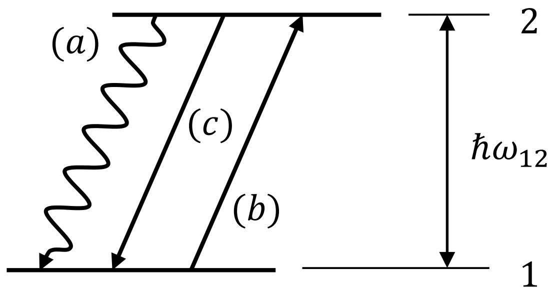
$$\Delta I(\nu) = \sigma(\nu)\Delta N I(\nu) + h\nu \times A_{21} \times g(\nu)\Delta\nu \times \frac{1}{2} \times \frac{d\Omega}{4\pi} \times N_2 dz$$

- $\sigma(\nu)\Delta N I(\nu)$: coherent light - amplification
- $h\nu$: energy of a photon
- A_{21}: transition rate
- $g(\nu)$: spectral factor
- $\frac{1}{2}$: polarization
- $\frac{d\Omega}{4\pi}$: solid angle
- $N_2 dz$: number of atoms

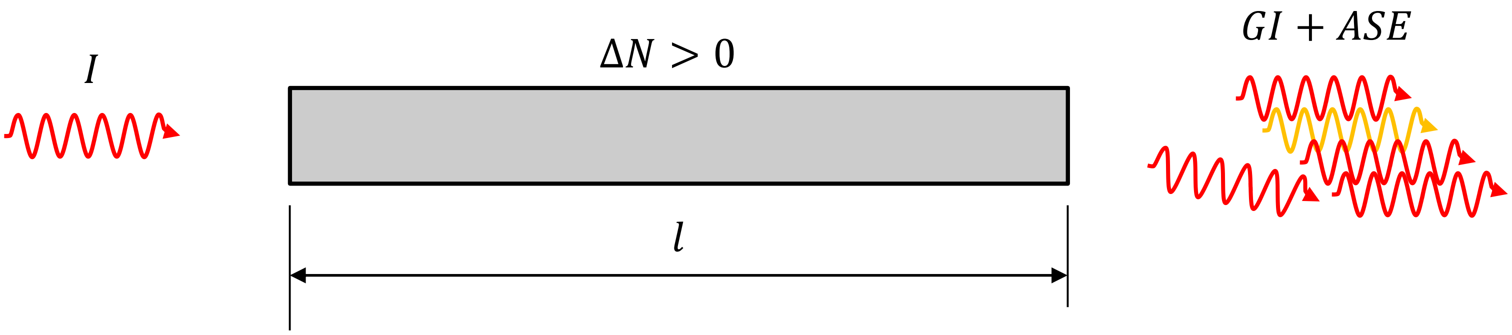
energy of a photon … number of atoms: incoherent light - spontaneous emission

in the laser amplifier:
spontaneous emission = noise

Amplified Spontaneous Emission (ASE)



when the medium is long and/or amplification coefficient large $\gamma_0 l \gg 1$ the spontaneous emission can be amplified to macroscopic intensities



No simple and convenient formulas for accounting for ASE. The spontaneous emission rate is given by $\frac{1}{2}h\nu$ for every spatio-temporal mode of the amplifier.

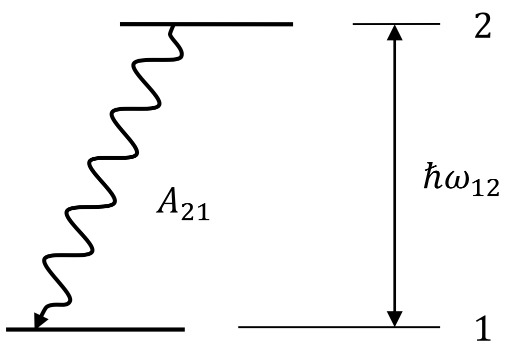
Consequences of ASE:

- ❑ Noise at the output of the amplifier
- ❑ In extreme cases ASE can saturate amplifier

Simple rule: to avoid problems with ASE the input intensity has to be much larger than the spontaneous emission intensity

homogenous line-broadening (we cannot address atoms by spectral methods)

- Natural broadening (always present). At least one of the two energy levels involved in light amplification corresponds to an excited state which has a finite life-time because atoms spontaneously drop to lower energy levels while emitting photons. In addition, in condensed phase, the life-time can shortened by non-radiative transition which increase the total transition rate.



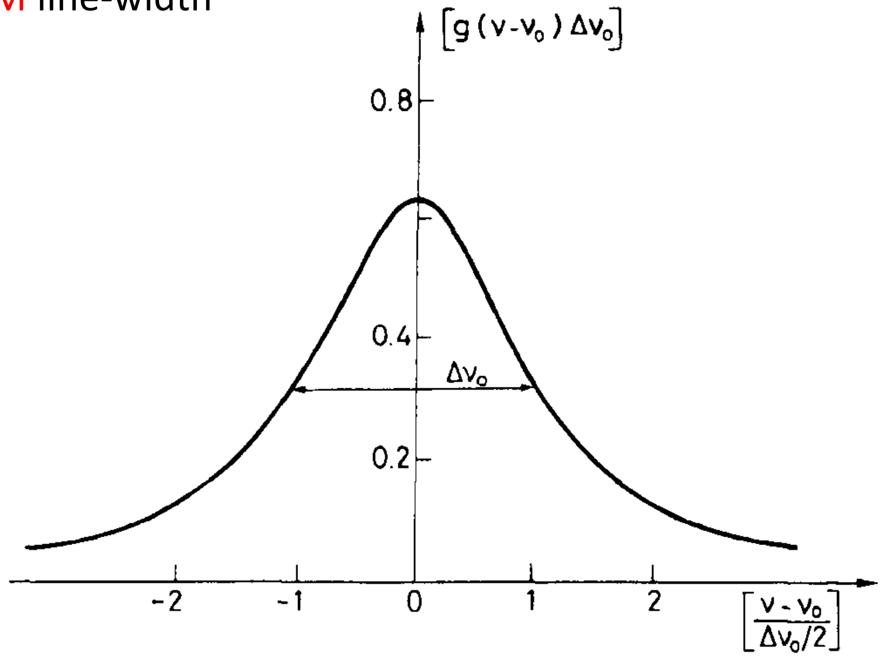
spontaneous emission leads to the Lorentzian line – shape

$$g(\nu) = \frac{\Delta\nu}{2\pi[(\nu-\nu_0)^2+(\Delta\nu/2)^2]}, \quad g(\omega) = 2\pi g(\upsilon)$$

with FWHM line-width

$\Delta\nu = \frac{A_{21}}{2\pi}$:

FWHM – Full Width at Half-Maximum



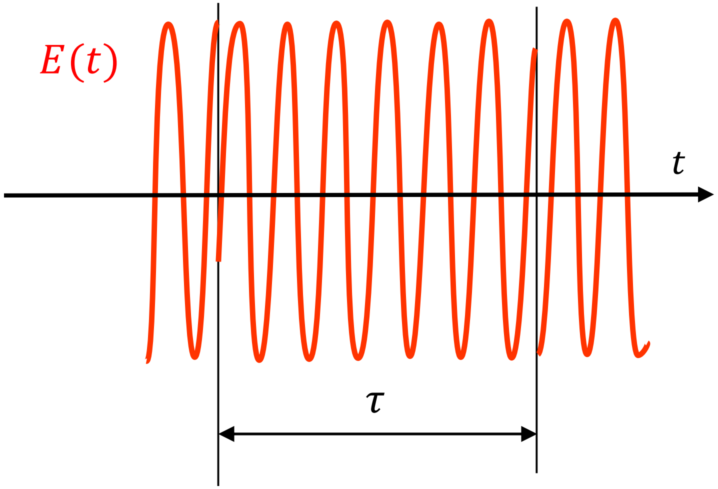
homogenous line-broadening, 2

- Pressure broadening

probability density for atomic collisions in gas phase

$$p(\tau) = \frac{e^{-\tau/\tau_c}}{\tau_c}$$

$p(\tau)d\tau$ – the probability for that the atom to undergo a collision in the time interval $\tau, \tau + d\tau$



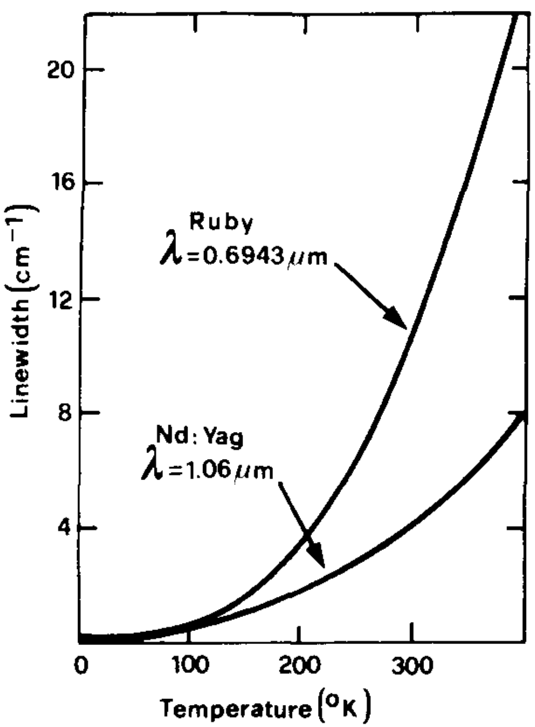
calculations.....

$$g(\nu) = \frac{\Delta\nu}{2\pi[(\nu-\nu_0)^2+(\Delta\nu/2)^2]}$$

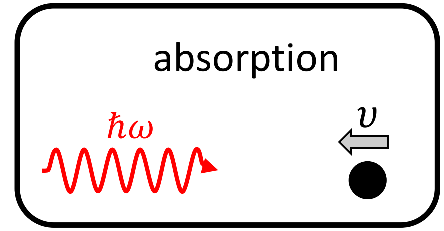
with $\Delta\nu = \pi/\tau_c$.

in glasses and crystals the interrupting events are phonons

for gases $\cong$ MHz/mbar

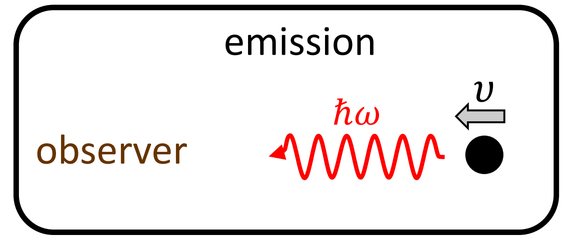


inhomogenous line-broadening (we can address atoms by spectra methods)



❑ Doppler broadening

Doppler shift; if the atoms moves slowly compared to the speed of light in vacuum ($\upsilon \ll c$) the largest shift comes form linear Doppler effect, which depends on the velocity component parallel to the direction of observation (we assume υ_z):



$$\nu' = \left(1 + \frac{\upsilon_z}{c}\right)\nu \;\Rightarrow \upsilon_z = \frac{c}{\upsilon_z}(\nu' - \nu)$$

the convention is that $\upsilon_z > 0$ for atom moving towards the light source (absorption) or the observer (emission).

For gas at the temperature T the velocity distribution is given by Maxwell function

$$p(\upsilon) = \left(\frac{M}{2\pi kT}\right)^{1/2} \exp[-(M\upsilon_z^2/T)]$$

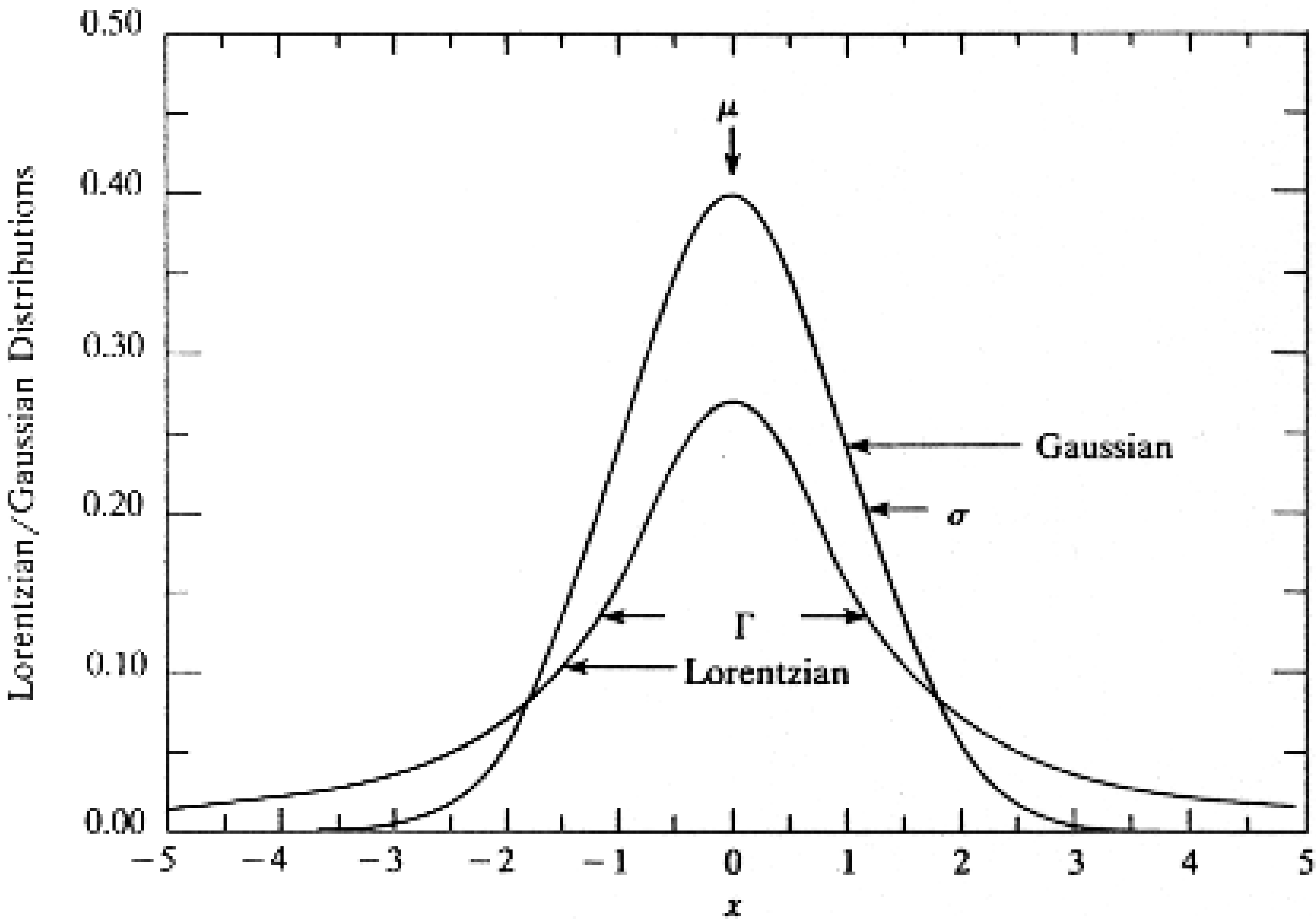
Let's mark the resonant frequency in atom is by ν_0 and let's assume that homogenous line-broadening is small. Then the line-shape function id given by Gaussian function:

$$g(\nu) = \frac{1}{\nu_0}\left(\frac{M}{2\pi kT}\right)^{1/2} \exp\left[-\frac{Mc^2}{2kT}\,\frac{(\nu-\nu_0)^2}{{\nu_0}^2}\right]$$

with FWHM:

$$\Delta\nu = 2\nu_0\left(\frac{2kT\ln 2}{Mc^2}\right)^{1/2}$$

Gauss vs Lorentz



mixed line-broadening

example: Doppler broadening + collisional broadening

Atoms with a given value of υ_z are characterized by homogenously broadened

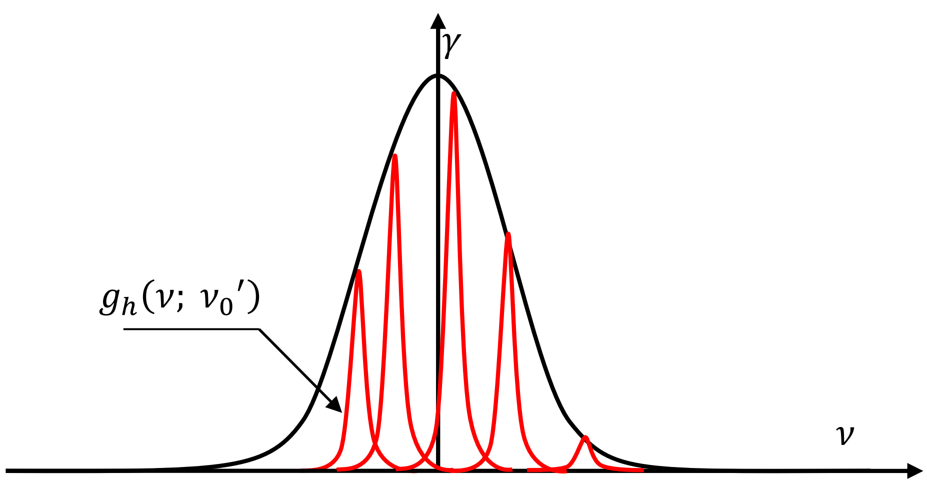
$$g_h(\nu') = \frac{\Delta\nu}{2\pi[(\nu' - \nu_0{}')^2 + (\Delta\nu/2)^2]}$$

Index h signifies homogenous (in this example collisional broadening), $\Delta\nu$ is the linewidth of homogenous broadening, and $\nu_0{}' = \left(1 + \frac{\upsilon_z}{c}\right)\nu_0$ the Doppler shifted resonance frequency .

The probability of atom having a given value of υ_z is given by Maxwell's distribution , we integrate over the possible values of $\nu_0{}'$

$$g_V(\nu) = \left(\frac{M}{2\pi kT}\right)^{1/2} \int_{-\infty}^{\infty} d\nu_0{}' \frac{\Delta\nu}{2\pi[(\nu - \nu_0{}')^2 + (\Delta\nu/2)^2]} \exp\left[-\frac{Mc^2}{2kT}\frac{(\nu - \nu_0{}')^2}{\nu_0{}^2}\right]$$

$g_V(\nu)$ is Voigt's profile.

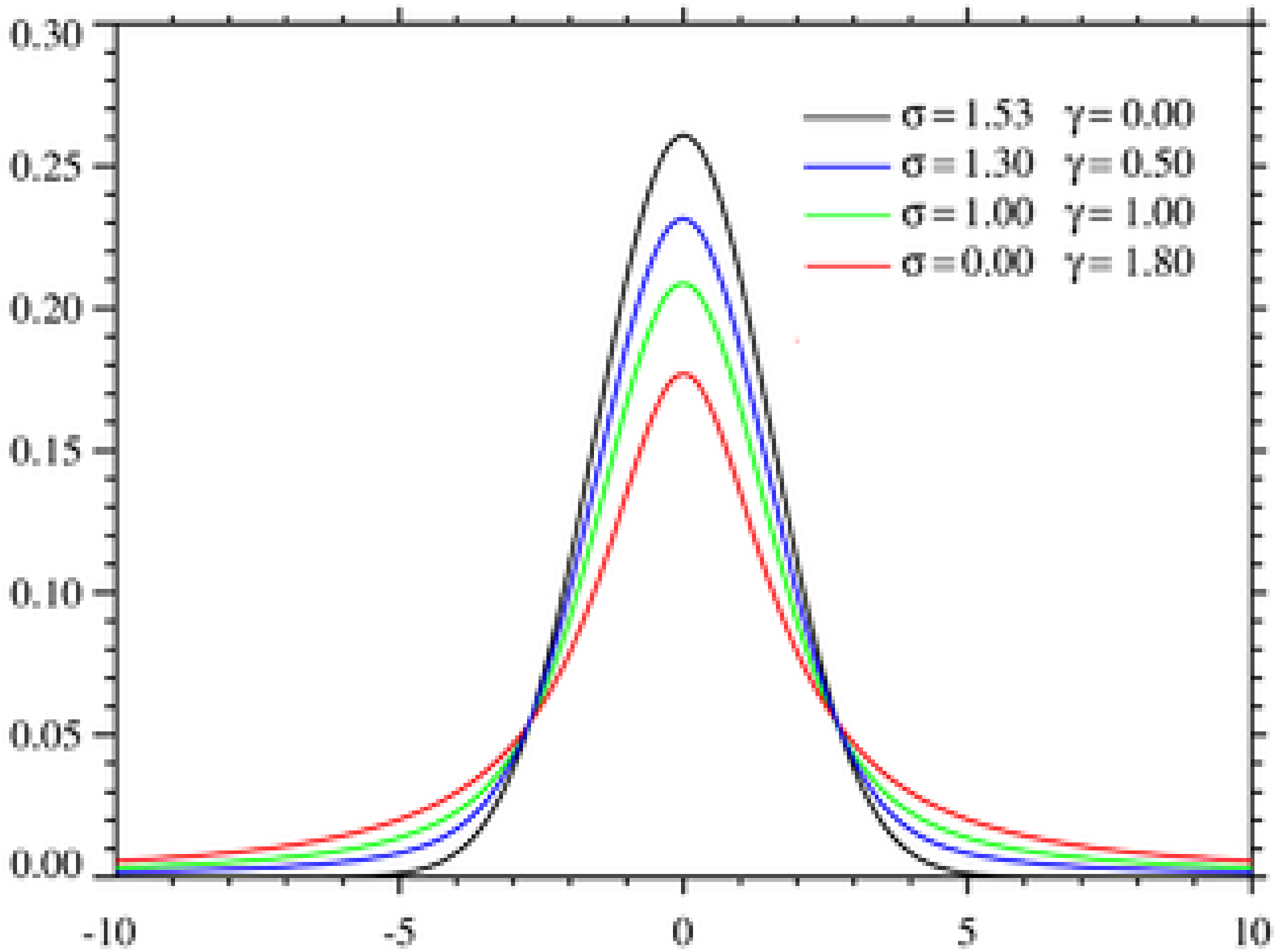


mixed line-broadening, 1

Voigt's profile is a convolution of Lorentz and Gauss functions

$$g_V(x) = \int_{-\infty}^{\infty} dx' G(x';\sigma) L(x-x';\gamma)$$

$$G(x;\sigma) \equiv \frac{e^{-\frac{x^2}{(2\sigma^2)}}}{\sigma\sqrt{\pi}}, \quad L(x;\gamma) \equiv \frac{\gamma}{\pi(x^2+\gamma^2)}$$



mixed line-broadening, 2

absorption coefficient α $\alpha \propto p \times {}^{1}/_{\Delta\nu}$

Doppler broadening: $\alpha \propto p$
pressure broadening: $\alpha \propto p \times {}^{1}/_{p} = \text{const}$

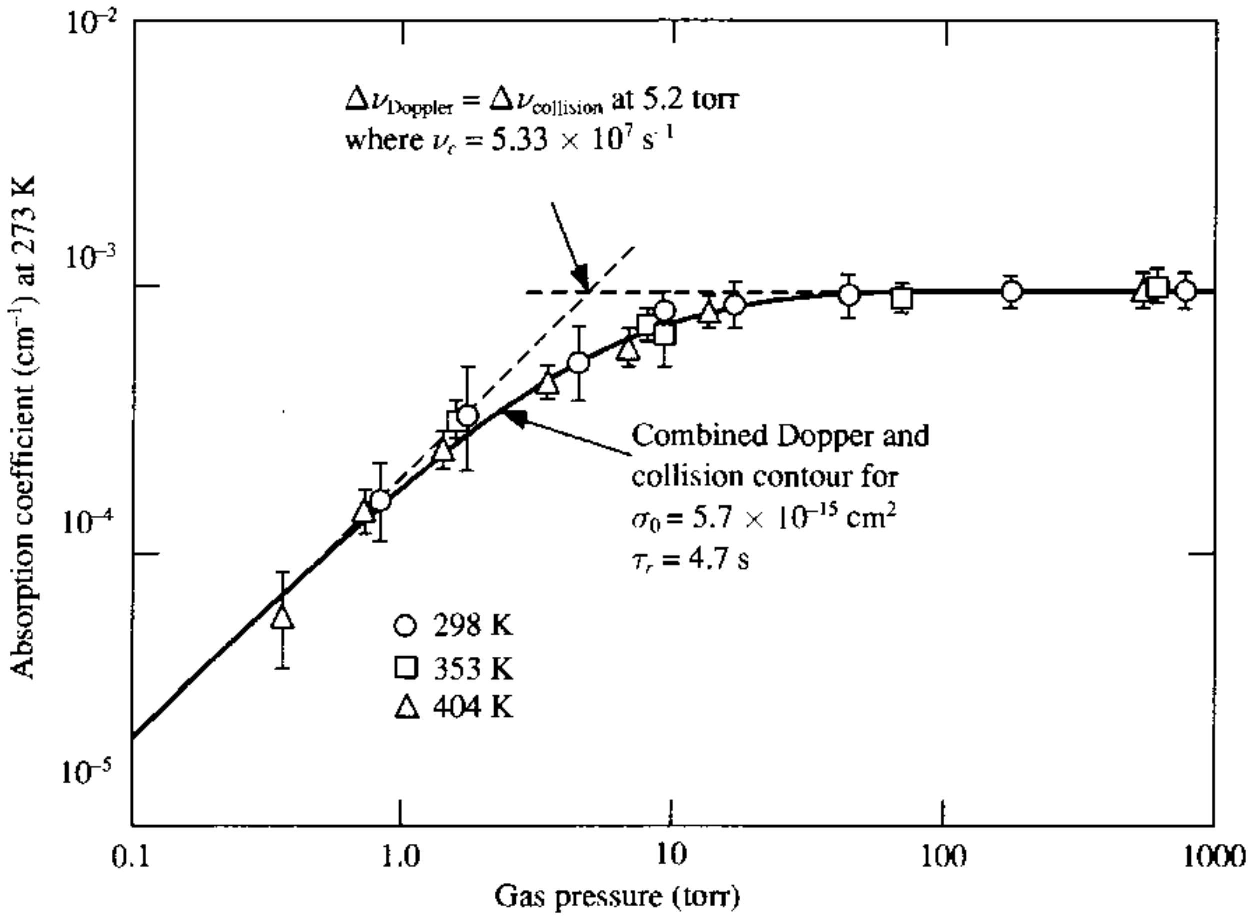


FIGURE 7.9. Absorption coefficient in CO_2 at 10.6 μm as a function of CO_2 pressure. (After E. T. Gerry and D. A. Leonard, *Appl. Phys. Lett.* 8, 227, 1966.)

”typical” linewidths

	effect	gas	liquid	condensed matter
homogenous	natural	0.001Hz-10MHz	n *	n
	atomic collision	5-10MHz/mbar	≈ 300 cm^{-1}	----
	phonons	---	---	≈ 10 cm^{-1}
inhomogeneous	Doppler	50MHz-1GHz	n	---
	Local fields	---	≈ 500 cm^{-1}	1-500 cm^{-1}

*n - negligible

cm^{-1} units are often used in spectroscopy

$$\tilde{\nu} \equiv \frac{1}{\lambda[\mathrm{cm}]}$$

$$\tilde{\nu} \equiv \frac{1}{\lambda}[\mathrm{cm}^{-1}] = \frac{\nu}{c\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]} = 10^{-2}\frac{\nu}{c}$$

numbers: $\lambda = 1\ \mu\mathrm{m} \Leftrightarrow 10\ 000\ \mathrm{cm}^{-1}$

for $\lambda = 1\ \mu\mathrm{m}$: $1\ \mathrm{cm}^{-1} = 30\mathrm{GHz}$

gain saturation in media with different line broadening

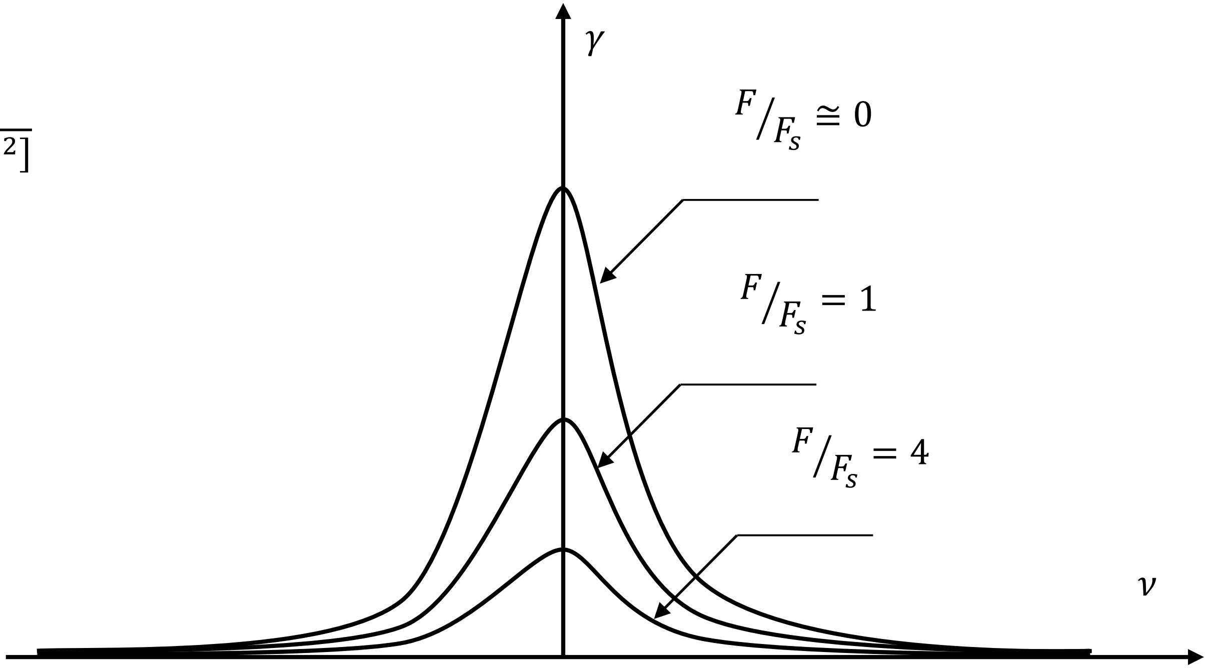
We will concentrate on the case $\tau_p \gg T_1$. Similar reasoning can be extende to other cases as well.

$$\gamma(F) = \frac{\gamma_0}{1 + F/F_s}$$

- Homogenous broadening dominates. As the population inversion decreases the gain drops for all frequencies because all atoms interact with the em wave in the same way. – Saturation requires higher intensities for frequencies far away for the resonance.

$$\gamma_0(\nu) \propto \frac{\Delta\nu}{2\pi[(\nu - \nu_0)^2 + (\Delta\nu/2)^2]}$$

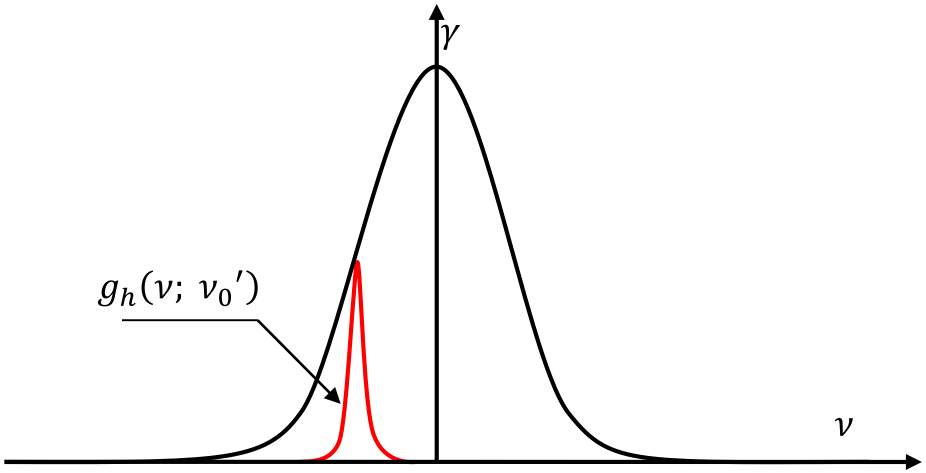
$$\gamma(F,\nu) = \frac{\gamma_0(\nu)}{1 + F/F_s}$$



gain saturation in media with different line broadening, 2

- inhomogeneous broadening dominates. A monochromatic em wave of frequency ν inetracts only with atoms that have their resonant frequencies close to ν (closer than homogenous linewidth). The saturation affects only this selected group of atoms – other groups „do not see” the em field.

$$\gamma_0(\nu) \propto \int_{-\infty}^{\infty} d\nu_0{}' \underbrace{\frac{\Delta\nu}{2\pi[(\nu-\nu_0{}')^2+(\Delta\nu/2)^2]}}_{g_h(\nu,\nu_0{}')} g(\nu_0{}')$$



$g_j(\nu,\nu_0{}')$

homogenously broaden line centered at $\nu_0{}'$.

Saturation „burns a hole” in the gain profile. Its width corresponds to the homogenous linewidth. The depth of the hole scales with saturation (em field intensity).