Overview of quark fragmentation functions

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Outline

1. Introduction
   - Phenomenology
   - QCD framework

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2. Processes to study FF
   - Anihilation $e^+e^- \longrightarrow h + X$
   - SIDIS: $lN \longrightarrow l'h + X$
   - $pp \longrightarrow h + X$
   - Experimental data

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3. Analysis technique
   - Fitting groups
   - Parametrizations
   - Assumptions

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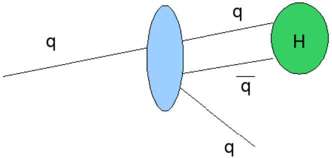
4. Results

Fenomenology

Initial and final state (hadrons) are colorless!

$q \longrightarrow h + X$:



$D_j^h(z, Q^2)$ – probability density that parton j fragments into hadron h where

Q^2 – energy scale of the particular process

z – fraction of energy of intermidiate boson carried by final hadron

Evolution of FF in QCD framework

DGLAP

$$\frac{d}{d\ln Q^2}\vec{D}^h(z,Q^2) = [\vec{P}^{(T)} \otimes \vec{D}^h](z,Q^2)$$

where

$$\vec{D}^h = \begin{pmatrix} D^h_\Sigma \\ D^h_g \end{pmatrix}, \qquad D^h_\Sigma = \sum(D^h_q + D^h_{\bar{q}})$$

$$\vec{P} = \begin{pmatrix} P_{qq} & 2n_f P_{gq} \\ \frac{P_{qg}}{2n_f} & P_{gg} \end{pmatrix}$$

split functions P_{ij} have perturbative expansion of the form:

$$P_{ij}(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}P^{(0)}_{ij}(z) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 P^{(1)}_{ij}(z) + \ldots$$

$\otimes$ denoting convolution

Limitaton

Range of applicability for fragmentation function as defined previously is limited to medium-to-large values of z.

- In NLO $P_{gq}(z) \approx ln^2 z/z$.
  $z \ll 1 \Rightarrow D_i^h < 0$ through Q^2 evolution and $d\sigma < 0$.
- Massless approximation:
  finite mass correction $\propto M_h/(sz)^2 \gg 0$ at low z.

Energy conservation

$$\sum_h \int_0^1 dzzD_i^h(z, Q^2) = 1$$

but since small z are problematic only truncated moments are meaningful $\int_{z_{min}}^1 dzzD_i^h(z, Q^2)$ so the energy conservation cannot be used as a constraint.

$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow h + X$

Cross-section in NLO accuracy

$$\frac{d\sigma^h}{dz^h} = \sigma_{tot} \frac{\sigma_0}{\sum_q e_q^2} \left[2(F_1^h(z, Q^2)) + F_L^h(z, Q^2) \right]$$

where

$$\sigma_{tot} = \sum_q e_q^2 \sigma_0 \left[1 + \frac{\alpha_s(Q^2)}{\pi} \right], \qquad \sigma_0 = \frac{4\pi\alpha^2(Q^2)}{s},$$

$$z = \frac{2E_h}{\sqrt{s}}, \qquad \sqrt{s} = Q,$$

Q – momentum of the intermediate boson (γ, Z)

E_h – energy of observed hadron

$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow h + X$

structure functions

$$F_1^h(z, Q^2) = \frac{1}{2} \sum_q e_q^2 \Big\{ \left[D_q^h(z, Q^2) + D_{\bar{q}}^h(z, Q^2) \right] + $$

$$+ \frac{\alpha_s(Q^2)}{2\pi} \left[C_q^1 \otimes \left(D_q^h + D_{\bar{q}}^h \right) + C_g^1 \otimes D_g^h \right] (z, Q^2) \Big\}$$

$$F_L^h(z, Q^2) = \frac{1}{2} \frac{\alpha_s(Q^2)}{2\pi} \sum_q e_q^2 \left[C_q^L \otimes \left(D_q^h + D_{\bar{q}}^h \right) + C_g^L \otimes D_g^h \right] (z, Q^2)$$

where

$D_{q,g}$ – quark (gluon) fragmentation functions

$C_{q,g}^{1,L}$ – coefficient functions calculated in NLO

advantages

- General: no cross section dependence on parton density function
- Practical: very high statistics recorded in experiments (CERN: LEP; SLAC: TPC, SLD)

disadvantages

- no q-$\bar{q}$ separation
- at scale of M_Z electroweak couplings roughly the same $\Rightarrow$ only flavor siglet combination can be determined
- gluon FF available only at NLO
- lack of accurate data at low scales and at large hadron energy fraction

$IN \longrightarrow l'h+X$

Cross-section

$$\frac{d\sigma^h}{dxdQ^2dz^h} = \frac{2\pi\alpha^2}{Q^2}\left[\frac{(1+(1-y)^2)}{y}2F_1^h(x,Q^2,z^h) + \frac{2(1-y)}{y}F_L^h(x,Q^2,z^h)\right]$$

where

$$x = \frac{Q^2}{2p_N\cdot q}, \qquad Q^2 = -q^2 = -(k-k')^2, \qquad Q^2 = sxy$$

$$y = \frac{p_N\cdot q}{p_N\cdot k}, \qquad z_h = \frac{E_h}{E_l - E_{l'}}$$

k, k', p_N – four-momentum of incoming lepton, outcoming lepton and nucleon

$lN \to l'h + X$

Assuming factorization:

Structure functions

$$F_1^h(z,x,Q^2) = \frac{1}{2}\sum_{q,q'} e_q^2 \Big\{ q(x,Q^2) D_q^h(z,Q^2) + \\ + \frac{\alpha_s(Q^2)}{2\pi}\Big[q \otimes C_{qq}^1 \otimes D_q^h + q \otimes C_{gq}^1 \otimes D_g^h + q \otimes C_{qg}^1 \otimes D_q^h \Big](x,Q^2,z)\Big\}$$

$$F_L^h(z,x,Q^2) = \frac{1}{2}\frac{\alpha_s(Q^2)}{2\pi}\sum_{q,q'} e_q^2 \Big[q \otimes C_{qq}^1 \otimes D_q^h + q \otimes C_{gq}^1 \otimes D_g^h + q \otimes C_{qg}^1 \otimes D_q^h \Big](x,Q^2,z)$$

where
$q(x,Q^2)$ – probability that the quarks of paricular flavor carry a fraction x of proton momentum (PDF)

$lN \longrightarrow l'h + X$

advantages

- practical: experiments probe fragmentation in energy regime complementary to e^+e^-
- general: sensitivity to FF of individual quark and anti-quark flavors

disadvantages

- one has to assume x vs. z factorization
- non-trivial dependence of cross sections on PDF of the nucleon
- different parametrizations of PDF brings addtional uncertainties

$pp \longrightarrow h + X$

Cross section

$$E_h \frac{d^2\sigma}{d^3 p_h} = \sum_{a,b,c} \left(q_a \otimes q_b \otimes d\sigma_{ab\to c} \otimes D_c^h \right)(s, p_T, z)$$

- the sum is over all contributing partonic channels $a + b \to c + X$ with $d\sigma_{ab\to c}$ the assoctiated partonic cross section
- $d\sigma_{ab\to c}$ can be expanded as a power series in the strong coupling of α_s (NLO corrections are available)

$pp \longrightarrow h + X$

advantages

- quarks and gluons come at the same order
- sensitive to gluon FF through dominance of gg→gX processes at low p_T (gluons are on average softer than quarks)
- sensitive to fragmentation at high z

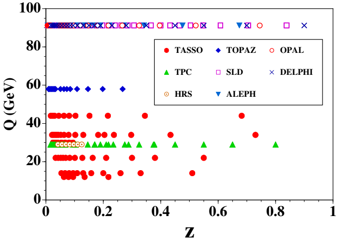
disadvantages

- large uncertainties (2 PDFs and 3 convolutions)
- NLO corrections are very important because of large contributions from elementary subprocesses involving gluons

Data sets: e^+e^-

Fully inclusive charged pion and kaon production in e^+e^-:

Pion data



SLD(SLAC),ALEPH,DELPHI,OPAL(CERN);

TOPAZ (KEK)

TASSO (DESY),

TPC, HRS (SLAC)

- Access to flavor singlet at $Q = M_Z$
- small z region cut at $z_{min} = 0.05(0.1)$ for all pion (kaon) data sets.

Data sets: e^+e^-

- "flavor tagged" data from ALEPH, DELPHI and TPC:
  Light quarks separated from heavy quarks
  Not measured directly nor clearly calculable in QCD (MC dependence)
- fully flavor separated data from OPAL:
  probabilities $\eta_i^h(z_p)$ for a quark flavor $i = q + \bar{q}$ to produce a jet containing the hadron h with z larger than z_p. At LO $\eta_i^h(z_p) = \int_{z_p}^1 dzzD_i^h(z)$; problems for pQCD at NLO.
- Other e^+e^- data:
  -three jet events: $q\bar{q}g$ at LO - gluon FF, but not clear at NLO
  -unidentified hadrons: dominant by π, K

Data sets: $lN \longrightarrow l'h+X$, $pp \longrightarrow h+X$

- SIDIS (HERMES)
  - charged pions and kaon multiplicities - flavor separation
  - measurement at scales $\mu \approx Q = 2GeV \ll M_Z$
- hadronic collision (RHIC)
  - neutral pions at central ($|\eta| < 0.35$) and forward ($\langle\eta\rangle \approx 3.5$) rapidities (PHENIX and STAR)
  - charged pions and kaons at forward rapidities ($\langle\eta\rangle \approx 3$, BRAHMS)
  - K_S^0 production at $|\eta| < 0.5$ (STAR)

Fits to inclusive e^+e^-

- [KKP] *Kniehl, Kramer, Potter* (NPB 582, 2000)
- [KRE] *Kretzer* (PRD 62, 2000)
- [AKK] *Albino, Kniehl, Kramer* (NPB 725, 2005)
- [HKNS] *Hirai, Kumano, Nagai, Sudoh* (PRD 75, 2007)

Fits to inclusive e^+e^- + other processes

- [KLC] *Kretzer, Leader, Christova* (EPJ C 22, 2001)
  - LO analysis of charged pion in SIDIS (fair agreement with previous Kretzer set)
- [DSS] *De Florian, Sassot, Stratmann* (PRD 75, 2007)
  - LO and NLO anlysis. SIDIS and *pp* data included

Parametrizations

History...

$$D_i^h(z,\mu_0) = N_i z^{\alpha_i}(1-z)^{\beta_i}$$

$$D_i^h(z,\mu_0) = N_i z^{\alpha_i}(1-z)^{\beta_i}\left[1+\gamma_i(1-z)^{\delta_i}\right]$$

$$D_i^h(z,\mu_0) = \frac{N_i z^{\alpha_i}(1-z)^{\beta_i}\left[1+\gamma_i(1-z)^{\delta_i}\right]}{B(2+\alpha_i,\beta_i+1)+\gamma_i B(2+\alpha_i,\beta_i+\delta_i+1)}$$

where

- $B()$ is a Euler *beta* function: $B(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}dt$
- The normalization N, and the parameters α, β, γ, δ in general depend on the energy scale μ_0.

Pions

Charged hadron fragmentation function

$$D_i^{h^+} = D_i^{\pi+} + D_i^{K+} + D_i^{p} + D_i^{res^+}$$

Symmetry assumption for π^+ FF:

- $D_{\bar{u}}^{\pi^+} = D_{d}^{\pi^+}$ (izospin symmetry in sea)
- $D_{d+\bar{d}}^{\pi^+} = ND_{u+\bar{u}}^{\pi^+}$ (DSS only, $N = N' = 1$ for other groups)
- $D_{s}^{\pi^+} = N' D_{\bar{u}}^{\pi^+}$ (DSS only, $N = N' = 1$ for other groups)
- $D_{c}^{\pi^+} = D_{\bar{c}}^{\pi^+}$ (only e^+e^- contribute)
- $D_{b}^{\pi^+} = D_{\bar{b}}^{\pi^+}$ (only e^+e^- contribute)
- $\gamma = 0$ for heavy quarks

Kaons

Symmetry assumption for K^+ FF:

- $D_{\bar{u}}^{K^+} = D_{d}^{K^+} = D_{\bar{d}}^{K^+} = D_{s}^{K^+}$
- $D_{c}^{K^+} = D_{\bar{c}}^{K^+}$ (only e^+e^- contribute)
- $D_{b}^{K^+} = D_{\bar{b}}^{K^+}$ (only e^+e^- contribute)
- $\gamma = 0$ for gluons
- $\gamma = 0$ for heavy quarks
- $D_{u}^{K^+} \neq D_{\bar{s}}^{K^+}$ (favored fragmentations are not equal in DSS)

Protons

Symmetry assumption for proton FF:

- $D_u^p = ND_d^p$ (izospin symmetry for favored fragmentation)
- $D_{\bar{u}}^p = ND_{\bar{d}}^p$ (izospin symmetry for unfavored fragmentation)
- $2D_{\bar{u}}^p = (1-z)^\beta D_{u+\bar{u}}^p$ with $\beta > 0$
- $2D_{\bar{d}}^p = (1-z)^\beta D_{d+\bar{d}}^p$ with $\beta > 0$
- $D_s^p = D_{\bar{s}}^p = N' D_{\bar{u}}^p$
- $D_c^p = D_{\bar{c}}^p$ (only e^+e^- contribute)
- $D_b^p = D_{\bar{b}}^p$ (only e^+e^- contribute)
- $\gamma = 0$ for gluons
- $\gamma = 0$ for heavy quarks

Rest of hadrons

Symmetry assumption for the rest of hadrons:

- $D_u^{res^+} = D_d^{res^+} = D_s^{res^+}$ (SU(3) flavor symetry for quarks)
- $D_{\bar{u}}^{res^+} = D_{\bar{d}}^{res^+} = D_{\bar{s}}^{res^+}$ (SU(3) flavor symetry for antiquarks)
- $2D_{\bar{u}}^{res^+} = (1-z)^{\beta} D_{u+\bar{u}}^{res^+}$ with $\beta > 0$
- $2D_{\bar{d}}^{res^+} = (1-z)^{\beta} D_{d+\bar{d}}^{res^+}$ with $\beta > 0$
- $\gamma = 0$ for heavy quarks

Other possible assumptions

- $D_i^{\pi^0} = \frac{\left(D_i^{\pi^+} + D_i^{\pi^-}\right)}{2}$, for all flavors
- $D_i^{K^0} = \frac{\left(D_i^{K^+} + D_i^{K^-}\right)}{2}$, with $u \to K^+$ and $d \to K^+$ FF interchanged

Charge conjugation assumed to obtain FF for h^-

$$D_q^h = D_{\bar{q}}^{\bar{h}}$$

Minimization technichque

Definition of $\chi 2$

$$\chi^2 = \sum_{i=1}^{N} \frac{(T_i - E_i)^2}{\delta E_i^2}$$

where

E_i – measured value of a given observable

δE_i – error associeted with this measurement ($E_i = \sqrt{E_{i_{stat}}^2 + E_{i_{syst}}^2}$)

T_i – theoretical estimate for a given set of parameters (α, β, γ, δ)

Mellin transformation

The Mellin moments

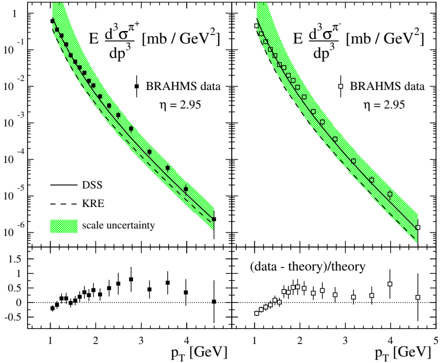
$$D_i^h(n, Q^2) = \int_0^1 z^{n-1} D_i^h(z, Q^2) dz$$

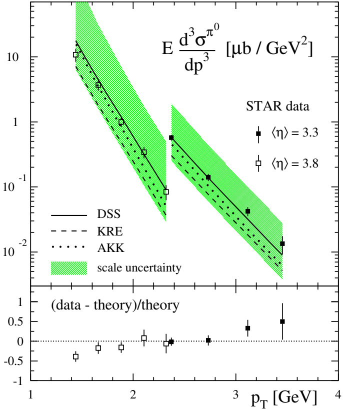
Inverse Mellin transform

$$D_i^h(z, Q^2) = \frac{1}{2\pi i} \int_{C_n} z^{-n} D_i^h(n, Q^2) dn$$

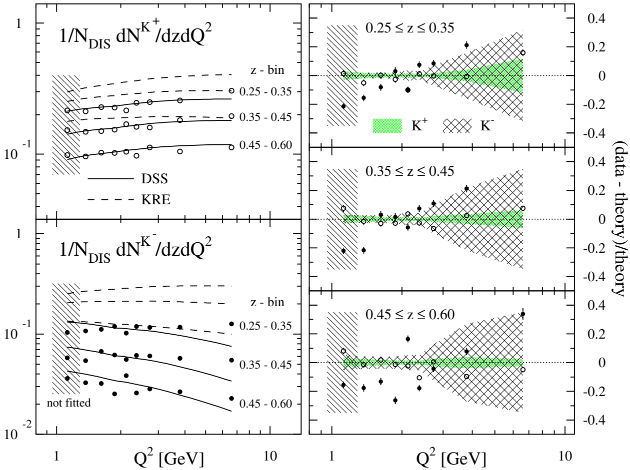
- very fast procedure – 10^3 faster than direct minimization
- about 100 first moments calculated (DSS) to reproduce the cross section to an accuracy of better than 1% for all data points

Comparision of chargd pion production $pp \to \pi^{\pm} X$ from BRAHMS with DSS and KRE parametrization



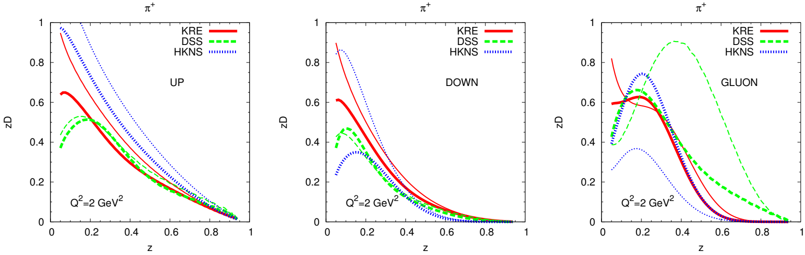


Comparision of charged pion multiplicities from HERMES with DSS and KRE parametrization



Pion FF

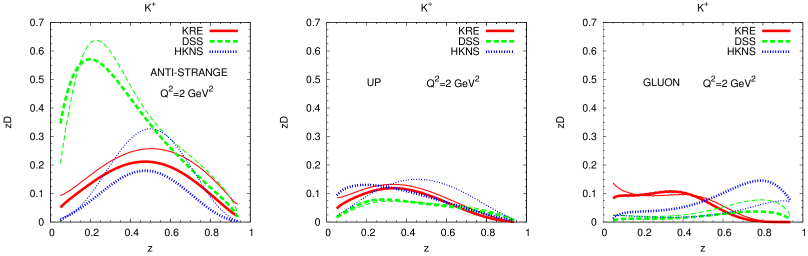
Comparision of pion FF determined by three different groups: DSS, HKNS and KRE



Pion FFs at NLO (thick) and LO (thin) at $Q^2 = 2\ \mathrm{GeV}^2$.

Kaon FF

Comparision of kaon FF determined by three different groups: DSS, HKNS and KRE



Kaon FFs at NLO (thick) and LO (thin) at $Q^2 = 2\ \mathrm{GeV}^2$.

Results - numbers LO

Fitting parameters for fragmentation function $D_i^{\pi^+}(z, \mu)$ at scale $\mu_0 = 1 GeV$ in LO

flavor i	N_i	α_i	β_i	γ_i	δ_i
$u + \overline{u}$	0.367	-0.228	1.20	5.29	4.51
$d + \overline{d}$	0.404	-0.228	1.20	5.29	4.51
$\overline{u} = d$	0.117	0.123	2.19	7.80	6.80
$s + \overline{s}$	0.197	0.123	2.19	7.80	6.80
$c + \overline{c}$	0.256	-0.310	4.89	0.00	0.00
$b + \overline{b}$	0.469	-1.108	6.45	0.00	0.00
g	0.493	1.179	2.83	-1.00	6.76

Results - numbers NLO

Fitting parameters for fragmentation function $D_i^{\pi^+}(z,\mu)$ at scale $\mu_0 = 1 GeV$ in NLO

flavor i	N_i	α_i	β_i	γ_i	δ_i
$u+\overline{u}$	0.345	-0.015	1.20	11.06	4.23
$d+\overline{d}$	0.380	-0.015	1.20	11.06	4.23
$\overline{u}=d$	0.115	0.520	3.27	16.26	8.46
$s+\overline{s}$	0.190	0.520	3.27	16.26	8.46
$c+\overline{c}$	0.271	-0.905	3.23	0.00	0.00
$b+\overline{b}$	0.501	-1.305	5.67	0.00	0.00
g	0.279	0.899	1.57	20.00	4.91

Summary

- FFs determined by several groups.
- Most recent analysis by DSS is the most complete one:
  - all data sets from e^+e^-, pp and SIDIS taken into account
  - analyses done in NLO
  - weaker assumptions on relations between FFs
  - more flexible parametrization of FFs
- Improvements (in terms of agreement with data) found wrt earlier analyses (e.g. Kretzer)
- Uncertainties of FF are at level of 2-5% (non-strange quarks) and above 10% (for strange quarks)
- All analyses determined so far only spin-idenpendent FFs.

Error estimation

Theoretical errors estimated using Lagrange multipliers:

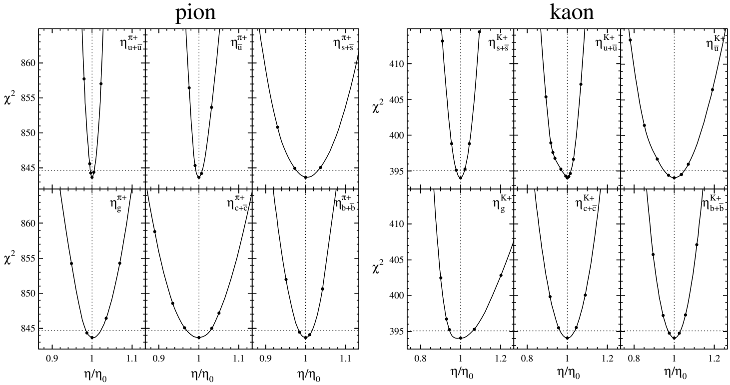
Lagrange technique

$$\Phi(\lambda_i, \{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i O_i(\{a_j\})$$

where
λ_i – Lagrange multipliers related to O_i
O_i – observable depending on $\{a_j\}$
$\{a_j\}$ – set of parameters descibing PDF for fixed value of λ_i

Estimated errors

$\sigma(D_u, D_d) \approx 2 - 5\%$
$\sigma(D_s) \approx 10\%$



Profiles of χ^2 vs. $\eta_i^{h^+} = \int_{x_p}^1 dz z D_i^{h^+}(z)$ at $x_p = 0.2$, $Q = 5$ GeV (DSS).