Stabilising the moduli of the supersymmetric Standard Model on the $\mathbb{Z}_6'$ orientifold

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Outline

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The supersymmetric Standard Model on the $\mathbb{Z}_6'$ orientifold

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- The $\mathbb{Z}_6'$ orientifold is defined as $T_1^2 \otimes T_2^2 \otimes T_3^2/\mathbb{Z}_6' \times \mathcal{R}$. Using complex coordinates z_k $(k = 1, 2, 3)$ in each torus T_k^2, the $\mathbb{Z}_6'$ point group generator θ acts as $\theta z_k = e^{2\pi i v_k} z_k$, where $\mathbf{v} = \frac{1}{6}(1, 2, -3)$. θ must act as an automorphism of the lattice, and we use an $SU(3)$ root lattice in the tori $T^2_{1,2}$. $\mathcal{R}$ is the embedding of Ω which acts as $\mathcal{R} z_k = \overline{z}_k$.

- We have four supersymmetric stacks $\kappa = a, b, c, d$ of $\mathbb{Z}_6'$-invariant $D6$-branes wrapping fractional 3-cycles with homology class

$$\kappa = \frac{1}{2}(\Pi_\kappa^{\rm bulk} + \Pi_\kappa^{\rm ex})$$

The bulk 3-cycle $\Pi_\kappa^{\rm bulk}$ wraps a 1-cycle in each T_k^2; the exceptional 3-cycle $\Pi_\kappa^{\rm ex}$ wraps a collapsed 2-cycle, associated with the θ^3-twisted sector fixed points in $T_1^2 \times T_3^2$, times a 1-cycle in T_2^2.

- Open strings beginning and ending on a stack κ with N_κ D6-branes give the (massless) gauge bosons of $U(N_\kappa) = U(1)_\kappa \times SU(N_\kappa)$. At the intersections of any two stacks κ and λ there is chiral matter in the bi-fundamental $(\mathbf{N}_\kappa, \overline{\mathbf{N}}_\lambda)$ representation of $U(N_\kappa) \times U(N_\lambda)$, where $\mathbf{N}_\kappa$ and $\overline{\mathbf{N}}_\lambda$ have charges $Q_\kappa = +1$ and $Q_\lambda = -1$ with respect to $U(1)_\kappa$ and $U(1)_\lambda$.
- Under the action of $\mathcal{R}$ each stack λ has an orientifold image $\lambda' = \mathcal{R}\lambda$, and at the intersections of κ with the orientifold image λ' there is chiral matter in the bi-fundamental $(\mathbf{N}_\kappa, \mathbf{N}_\lambda)$ representation of $U(N_\kappa) \times U(N_\lambda)$,
- a has $N_a = 3$, b has $N_b = 2$, and c, d have $N_c = 1 = N_d$, so in the first instance the gauge group from these four stacks is

$$
\begin{aligned}
G &= U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d \\
&= SU(3)_{\text{colour}} \times SU(2)_L \times U(1)_Y \times U(1)^3
\end{aligned}
$$

- The weak hypercharge Y is a linear combination

$$Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{2}Q_d$$

of the charges Q_κ. In our model the number of intersections of a with b, b' is

$$(a \cap b, a \cap b') = (2, 1) \longrightarrow 3Q_L$$

and these give three quark doublets $3Q_L$ having $Y = \frac{1}{6}$, two with $(Q_a, Q_b) = (1, -1)$ and one with $(Q_a, Q_b) = (1, 1)$.

- The remaining intersections are

$$\begin{aligned}
(a \cap c, a \cap c') &= (0, 0) \\
(a \cap d, a \cap d') &= (-3, -3) \longrightarrow (3d_L^c, 3u_L^c) \\
(b \cap c, b \cap c') &= (-2, 1) \longrightarrow 3L \\
(b \cap d, b \cap d') &= (1, 1) \longrightarrow (H_d, H_u) \\
(c \cap d, c \cap d') &= (-3, -3) \longrightarrow (3\ell_L^c, 3\nu_L^c)
\end{aligned}$$

- In orientifolds there is also chiral matter on a stack κ in the $\mathbf{N}_\kappa \times \mathbf{N}_\kappa$ representation of $SU(N_\kappa)$ on the branes (as well as gauge particles). For the $SU(3)$ stack a this gives $\mathbf{3} \times \mathbf{3} = \mathbf{6} + \bar{\mathbf{3}}$, and for the $SU(2)$ stack b $\mathbf{2} \times \mathbf{2} = \mathbf{3} + \mathbf{1}$. In our model, both symmetric and antisymmetric representations are absent on all stacks. $\#(\mathbf{S}_\kappa) = 0 = \#(\mathbf{A}_\kappa)$.
- Overall, we have the spectrum of the supersymmetric Standard Model plus three right-chiral neutrino singlets $3\nu_L^c$.
- Conservation of the $U(1)$ charges Q_κ allows Yukawa couplings of the Higgs $H_{u,d}$ for one (or two) generations, but *not* three.
- Any anomalous $U(1)$ acquires a string scale mass, $\mathrm{O}(10^{17})\ \mathrm{GeV}$ in a supersymmetric theory, and survives only as a global symmetry. In our model $U(1)_Y$ is massless, as required.

HOWEVER,

- So too is $U(1)_{B-L}$, where

$$B - L = \frac{1}{3}Q_a - Q_c$$

The other two $U(1)$s survive as global symmetries.

- Also, for this to be a consistent string theory realisation there must be overall cancellation of RR tadpoles, which requires that the overall homology class of the D6-branes and the O6-plane must vanish, whereas in our model

$$\sum_{\kappa=a,b,c,d} N_\kappa(\kappa + \kappa') - 4\pi_{\mathrm{O6}} \neq 0$$

Fluxes

- The massless fields in the RR sector of type IIA include a 1-form C_1, with field strength $F_2 = dC_1$ dual to the 8-form field strength $F_8 = dC_7 =^* F_2$ associated with the 7-form potential C_7 to which the the D6-branes $\kappa = a, b, c, d$ are (electrically) coupled.

- The type IIA action then includes terms

$$S_{IIA} \supset -\frac{1}{2\kappa_{10}^2} \int F_2 \wedge^* F_2 + \sqrt{2}\mu_6 \sum_{\kappa} N_{\kappa} \int_{\mathcal{M}_4 \times \kappa} C_7$$

The sum in the C_7 tadpole term includes contributions from κ' and the $\mathrm{O6}$-plane. In the massive version, $F_2 = dC_1 + m_0 B_2 + \bar{F}_2$, where $\bar{F}_2$ is the background flux, and

$$\begin{aligned} F_2 \wedge^* F_2 &= F_2 \wedge (dC_7 + ...) \\ &= d(F_2 \wedge C_7) + C_7 \wedge (-m_0 \bar{H}_3 + ...) \end{aligned}$$

with $H_3 = dB_2 + \bar{H}_3$ and $\bar{H}_3$ also background flux. Hence background fluxes $m_0\bar{H}_3$ also contributes to the RR tadpole cancellation conditions

$$\frac{1}{2\kappa_{10}^2}\pi_{m_0\bar{H}_3} + \sqrt{2}\mu_6\left(\sum_\kappa N_\kappa(\kappa + \kappa') - 4\pi_{O6}\right) = 0$$

where $\pi_{m_0\bar{H}_3}$ is the Poincaré dual 3-cycle of the 3-form $m_0\bar{H}_3$. In our model tadpole cancellation *requires* that the flux $m_0\bar{H}_3 \neq 0$.

- There is also a 4-form field strength F_4 associated with C_3, which in the massive theory is given by $F_4 = dC_3 + \bar{F}_4 - C_1 \wedge H_3 - \frac{m_0}{2}B_2 \wedge B_2$. Here $\bar{F}_4$ is the background flux used to stabilise the Kähler moduli

$$\bar{F}_4 = \sum_a e_a \tilde{w}_a$$

with $\tilde{w}_a$ the $h_+^{2,2} = h_-^{1,1}$ untwisted and twisted $(2,2)$-forms having $\mathcal{R} = +1$.

Stabilising moduli

DeWolfe *et al.* JHEP07 (2005) 066

Ihl &Wrase JHEP07 (2006) 027

- Axions enter S_{IIA} via the Chern-Simons term $C_3 \wedge \bar{H}_3 \wedge dC_3$. Since C_3 and $\bar{H}_3$ live only on the compact space Y, this is non-zero only when $dC_3 = f d^4x \equiv \mathcal{F}_0$, which enters the action as a Lagrange multiplier

$$S_{IIA} \supset -\frac{1}{2\kappa_{10}^2} \int (\mathcal{F}_0 \wedge^* \mathcal{F}_0 + 2\mathcal{F}_0 \wedge X)$$

where $X = \bar{F}_6 + B_2 \wedge \bar{F}_4 + C_3 \wedge \bar{H}_3 - \frac{m_0}{6} B_2 \wedge B_2 \wedge B_2$. Eliminating $\mathcal{F}_0$ gives $\int X = 0$, which fixes a single linear combination of the axions in C_3. On the $\mathbb{Z}_6'$ orbifold $h^{2,1} = 5$, and there are four untwisted and eight twisted 3-forms, six with $\mathcal{R} = 1$ and six with $\mathcal{R} = -1$. C_3 has $\mathcal{R} = 1$, so five axions are unfixed by $\bar{H}_3$. Their stabilisation will need non-perturbative world-sheet instanton effects that are allowed by the $\mathcal{R}$-projection and the non-zero fluxes.

- Grimm & Louis NPB 718 (2005)153 Alternatively, the effective four-dimensional $\mathcal{N} = 1$ supergravity is obtained by performing the orientifold projection $\mathcal{R}$ on the $\mathcal{N} = 2$ theory for the vector mutiplets (Kähler moduli t_a) and hypermultiplets (complex structure moduli and dilaton N_k). In a supersymmetric theory, for any field Φ_i,

$$F_i \equiv D_i W = \partial_i W + W \partial_i K = 0, \quad W = W^K(t_a) + W^Q(N_k)$$

$$W^K = \int \bar{F}_6 + \int J_c \wedge \bar{F}_4 - \frac{1}{2} \int J_c \wedge J_c \wedge \bar{F}_2 - \frac{m_0}{6} \int J_c \wedge J_c \wedge J_c$$

$$W^Q = \int \Omega_c \wedge \bar{H}_3$$

and $K = K^K + K^Q$. Here $J_c = \sum_a t_a w_a$ is the complexified Kähler form, and $\Omega_c = \sum_k 2N_k \alpha_k$ the complexified 3-form. Since the derivatives $\partial_{N_k} W^Q$ are real, the equations $\mathrm{Im} F_{N_k} = 0$ yield *one* constraint $\mathrm{Re} W = 0$ which, as before, stabilises *one* linear combination of the (six) axions.

- The equations $\mathrm{Re}F_{N_k} = 0$ fix the $h^{2,1}$complex-structure moduli and the dilaton.

- Since $\mathrm{Re}W = 0$, the equations $\mathrm{Im}F_{t_a} = 0$ for the Kähler moduli $t_a \equiv b_a + iv_a$ require that $\mathrm{Im}(\partial_{t_a} W^K) = 0$. Taking $\bar{F}_2 = 0$, for example, these are most easily satisfied by taking $b_a = 0 \; \forall a$. Thus the (eleven) Kähler moduli axions are *all* fixed.

- With these solutions, the equations $\mathrm{Re}F_{t_a} = 0$ give $h^{1,1}_-$ equations for the volumes v_a. On the $\mathbb{Z}_6'$ orientifold, the θ^3-twisted sector $(1,1)$-forms are associated with fixed points in $T^2_1 \times T^2_3$, so there are trilinear couplings in $W^K(t_a)$, deriving from the $J_c \wedge J_c \wedge J_c$ term, involving the moduli associated with two twisted $(1,1)$-forms and the untwisted $(1,1)$-form associated with T^2_2. The equations therefore couple the blow-up volumes V_j to the untwisted volume v_2 of T^2_2. For example, $v_2 v_3 \sim e_1/m_0, \; v_2 V_j \sim E_j/m_0$, where e_1, E_j are the $\bar{F}_4$ fluxes on the untwisted, twisted $(2,2)$-forms $\tilde{w}_1, \tilde{W}_j$.

- The solutions of these equations are valid provided that both the untwisted volumes v_k and the blow-up volumes V_j are sufficiently large that $\mathrm{O}(\alpha')$ corrections to S_{IIA} are negligible. We also require that $v_k \gg V_j,$, so that we remain within the Kähler cone: $v_k \gg V_j, \gg 1$.

- This requires that the fluxes on the untwisted forms are much larger than those on the twisted forms, and that both are large enough to justify the neglect of the $\mathrm{O}(\alpha')$ corrections: $e_k \gg E_j \gg m_0$. The background flux $\bar{F}_4$ does not contribute to RR tadpole cancellation, so we are free to choose it so that such a geometric solution is justified. Because of the coupling of v_2 to the blow-up volumes V_j, satisfying the constraints requires us to choose $e_1 \sim e_3 \gg e_2 \gg E_j \gg m_0$. Further, in this large-volume limit the coupling constant is small, quantum corrections are suppressed, so the supersymmetric vacuum is stable against corrections.

Summary & conclusions

We have constructed 4-stack models with *just* the spectrum of the (supersymmetric) standard model, plus three neutrino singlets $3\nu_L^c$. The Yukawas couple H_u, H_d to one (or two) matter generations.

However, there remains a single unwanted massless $U(1)_{B-L}$, and all models have uncancelled RR tadpoles.

The RR tadpoles require the introduction of non-zero background fluxes $m_0, \bar{H}_3$ to cancel them. They determine which one of the complex-structure axions is stabilised.

The Kähler moduli and dilaton can be stabilised using other background fluxes $\bar{F}_{2,4}$. Choosing $\bar{F}_4$ large enough ensures the validity of the supergravity approximation.

It remains to consider coupling constant unification, determination of the Yukawas, and stabilisation of the complex-structure axions.