

New Non-Trivial Vacuum Structures in Supersymmetric Field Theories and String Theories

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Tucson, Arizona

- 0806.3364 (PRD 78, 106011)
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The physical properties of most theoretical models of particle physics --- both in field theory and in string theory --- are in large part determined by the structure of their vacua.

- Spontaneous symmetry breaking
 - Gauge symmetries
 - Supersymmetry
- *Non-unique vacuum?*
 - True ground-state vacuum
 - Possible metastable vacua

Indeed, even when the true ground state of a theory preserves the apparent symmetries of a model, these might be broken in a metastable vacuum. In such situations, the resulting phenomenology might be determined by the properties of the metastable vacuum rather than those of the ground state.

Knowing the vacuum structure of a theory is the critical first step towards understanding its resulting phenomenology.

In particular, the possibility of metastable vacua has recently attracted considerable attention, both in field theory...

- Ellis, Lewellen-Smith, Ross
- Dine, Nelson
- Dine, Nelson, Nir, Shirman
- Luty
- Luty, Terning
- Intriligator, Seiberg, Shih
- Kitano, Ooguri, Ookouchi
- Murayama, Nomura
- Csaki, Shirman, Terning
- Abel, Khoze
- Dine, Mason
- Aharony, Seiberg
- Forste
- Abel, Chu, Jaeckel, Khoze
- Craig, Fox, Wacker
- Fischler, Kaplunovsky, Krishnan, Mannelli, Torres
- Gomez-Reino, Scrucca
- Abe, Kobayashi, Omura
- Giveon, Kutasov
- Giveon, Katz, Komargodski
- Dine, Feng, Silverstein
- Antoniadis, Benakli, Delgado, Quiros
- ...

...and in string theory, through a variety of different D-brane constructions.

- Franco, Uranga
- Braun, Buchbinder, Ovrut
- Bena, Gorbатов, Hellerman, Seiberg, Shih
- Aganagic, Beem, Seo, Vafa
- Tatar, Wetenhall
- Dudas, Papineau, Pokorski
- Lebedev, Lowen, Mambrini, Nilles, Ratz
- Giveon, Kutasov
- Angelantonj, Dudas
- Ooguri, Ookouchi, Park
- Dudas, Mourad, Nitti
- Serone, Westphal,
- Aganagic, Beem, Freivogel
- de Wolfe, Kachru, Mulligan
- ...

Many different setups are available.

The reasons for this intense interest in metastable vacua are clear.

- Such vacua can have very long lifetimes --- we might be living in one!
- They have different phenomenologies than the true ground states --- hence, theories with non-viable true ground states may actually be viable after all!
- They can easily arise as the low-energy limits of string theories, where they can significantly affect the phenomenologies of string models and alter vacuum-counting statistical studies of the string landscape.
- They may be populated (perhaps even preferentially) in thermal theories, in the early universe.
- Transitions into/out of metastable vacua can give rise to domain walls, cosmic strings, other important cosmological effects... even the cosmological constant.



Theories with multiple vacua can be expected to have rich phenomenologies that are unseen in theories with single vacua.

Such ideas provide ample motivation to investigate whether there might exist relatively simply field theories which give rise to additional, hitherto-unexplored vacuum structures.

If so, such structures could potentially provide new ways of addressing a variety of unsolved questions about the universe we inhabit, both in field theory and in string theory.

Unfortunately, most models with metastable vacua are hard to analyze...

- Tools which exist for understanding properties of global ground states (e.g., Witten index, ADS criterion, etc.) are unavailable for metastable states. Thus, metastable states are generally difficult to locate, enumerate, or analyze in a given model.
- Full stability of true ground state and/or metastable state (*including elimination of all flat directions*) typically requires assumptions about unknown strong-coupling behavior and introduction of high-scale non-perturbative physics.
- Even if a metastable vacuum state can be identified, it is usually separated from the true ground state by an infinite distance in field space.



As a result, it is **difficult to obtain a low-energy effective description of the full theory**, *including both the true vacuum and the metastable vacuum simultaneously*. Hard to perform reliable calculations of quantities which depend on the *global* features of the theory --- e.g., calculations of metastable lifetimes.

In this talk, I will present two examples of new, non-trivial vacuum structures which can occur in supersymmetric field theories, along with explicit models in which they arise.

Moreover, the models realizing these new vacuum structures will have the properties that

- All vacua --- both true as well as metastable --- are present *classically* (i.e., at tree level) and are expected to survive quantum corrections.
- All vacua are separated by only *finite distances* in field space.
- *All flat directions are eliminated*, and no non-perturbative physics is required for vacuum stability.
- These models can naturally arise as the low-energy limits of string models with multiple U(1) gauge groups.

Two new vacuum structures

1.

Vacuum towers: A vacuum structure consisting of *large (and even infinite) towers of metastable vacua with higher and higher energies*. As the number of vacua grows towards infinity, the energy of the highest vacuum remains fixed while the energy of the true ground state tends towards zero. We will study the instanton-induced tunneling dynamics associated with vacuum towers, and find that many distinct decay patterns along the towers are possible: “collapses” and “cascades”, as well as forbidden regions into which tunneling cannot occur.

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2.

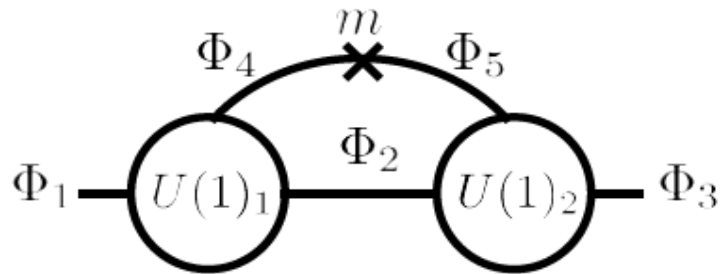
Bloch waves: A large, potentially infinite set of *degenerate vacua*, with a shift symmetry relating one vacuum to the next. Thus the true ground states of such theories are *Bloch waves*, with energy eigenvalues approximating a continuum and giving rise to a vacuum “band” structure.

But first, a quick warm-up...

A model with both a true SUSY vacuum and a single long-lived non-SUSY metastable vacuum *at tree level*, with no flat directions whatsoever and with only a finite separation in field space.

- KRD and B. Thomas, 0806.3364 (PRD)

“A Moose and a Mass”



Field	$U(1)_1$	$U(1)_2$	R
Φ_1	-1	0	$2/3$
Φ_2	+1	-1	$2/3$
Φ_3	0	+1	$2/3$
Φ_4	+1	+1	1
Φ_5	-1	-1	1

$$W = \lambda \Phi_1 \Phi_2 \Phi_3 + m \Phi_4 \Phi_5$$

Renormalizable

- Two $U(1)$ gauge groups and five chiral superfields.
- Φ_4 and Φ_5 form a vector-like pair with supersymmetric mass m and are charged under both gauge groups.
- A Fayet-Iliopoulos term for each gauge group: ξ_1 and ξ_2 .
- Gauge coupling constants are taken to be equal: $g_1 = g_2 \equiv g$
- With this R-charge configuration F-term breaking implies R-symmetry breaking.

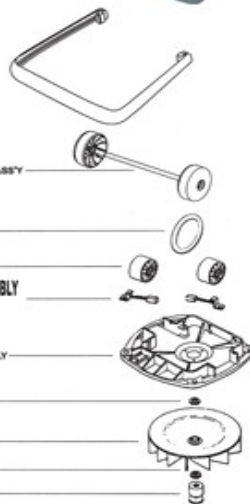
Despite initial appearances...

PARTS TO FIT PERFECT VAC, SANITAIRE MODELS, PERFECT P101 AND PERFECT P102, POWER FLIGHT, KENT

- 1 HOOD
 - 2 STRAIN RELIEF BUSHING
 - 3 SHOULDER SCREW
 - 4 ADJUSTMENT KNOB
 - 6 AIR INLET DUCT(R,H.)
 - 7 MOTOR ASS'Y
 - 8 FOOT PEDAL SPRING
 - 9 FOOT PEDAL
 - 10 HOOD SPRING
 - 11 BASE ASS'Y (INCLUDE MAGNET)
 - 11A. BASE ASSEMBLY 10" VAG
 - 12 WHEEL COVER
 - 13 PUSH SWITCH
 - 14 MOTOR MTG.GASKET
 - 15 AIR INLET DUCT ASS'Y(L,H)
 - 16 ADAPTER RETAINER SPRING
 - 17 FAN CHAMBER COVER
-
- 54151-2 PLASTIC STEP ON SWITCH
 - 52282-3 VG2 BRUSH STRIP SET
 - 52246-1 VG2 BRUSH STRIP SET
 - 53270 STEEL 12" DISTURBULATOR VG2
 - 14660-1 684 SWITCH BUTTON



19 FURNITURE GUARD



- 20 REAR WHEEL & AXLE ASS'Y
- 21 PUSH NUT
- 22 REAR WHEEL
- 23 BELT
- 24 FRONT WHEEL
- 25 MOTOR BRUSH ASSEMBLY
- 26 MOTOR BASE ASSEMBLY
- 27 FAN WASHER
- 28 FAN
- 27 FAN WASHER
- 29 PULLEY

Model No. P105

*** TRUE ERGONOMIC HANDLE - Finger Tip Control for back & forth movement - No longer do you have to grip with your whole hand. ANTI-SLIP HANDLE**

Re-Engineered Xtra Heavy, Xtra Large Reinforced Quick Release Cord Clip (For easy cord release)

50 ft. heavy duty 3-wire cord
Insulated 50 ft. Plyable Cord (Best in the industry)
ETL Listed - USA & Canada

TEFLEX FILTER BAG

Easy Lock Clips

See-Thru Large Capacity Dirt Cup

Heat overload protection system

Extra strength metal head

870 watt motor - 1,000+ hours motor life with carbon brush changes

12 Inch

Easy to turn 6-position carpet adjustment knob

NEW NAME PLATE COLORS Black & Grey
Easier to read Height Adjustment Settings

Extra-wide non-marking see-through transparent bumper So your customers can view the patent pending built-in magnet

Built In Super-Charged Perfect Super Magnet (Patent Pending) NOW 3 TIMES MORE CHARGE

Hardened Steel Handle Bail

Heavy Duty Metal Stop-On Switch

Hardened Steel, Large Area Metal Foot Release Pedal (Built to last)

Large easy-to-roll wide back wheels

"V" SHAPED MOTOR VENTS (Greater Air Flow)

12" Cleaning Path

LARGEST CAPACITY DIRT CUP AVAILABLE TODAY. Compare to All competitors

COMPETITORS PERFECT

...analyzing the vacuum structure of a given model is relatively easy.

Consider the scalar potential

$$V = \frac{1}{2} \sum_a g_a^2 D_a^2 + \sum_i |F_i|^2$$

where

$$D_a = \xi_a + \sum_i q_i^{(a)} |\phi_i|^2, \quad F_i = -\frac{\partial W^*}{\partial \phi_i^*}$$

Extrema are located
at locations in field
space where

$$\frac{\partial V}{\partial \phi_i} = 0$$

U(1)'s are broken if charged fields
get a vev.

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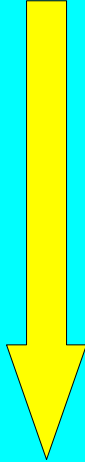
Extrema are located at locations in field space where

$$\frac{\partial V}{\partial \phi_i} = 0$$

U(1)'s are broken if charged fields get a vev.

To understand *stability* properties of extrema, calculate (mass)² matrix

$$\mathcal{M}^2 \equiv \begin{pmatrix} \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j} & \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j^*} \\ \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} & \frac{\partial^2 V}{\partial \phi_i \partial \phi_j^*} \end{pmatrix}$$



Diagonalize. At extremum, will have one zero eigenvalue (Nambu-Goldstone boson) for each broken U(1). Remaining eigenvalues describe extremum...

- **Stable vacuum** if all other $m^2 > 0$
- **Unstable** if at least one $m^2 < 0$
- **Flat direction** if at least one additional $m^2 = 0$.

Metastability occurs if two or more stable vacua emerge; true ground state has minimum V.

Let us now make a particular parameter choice in our model:

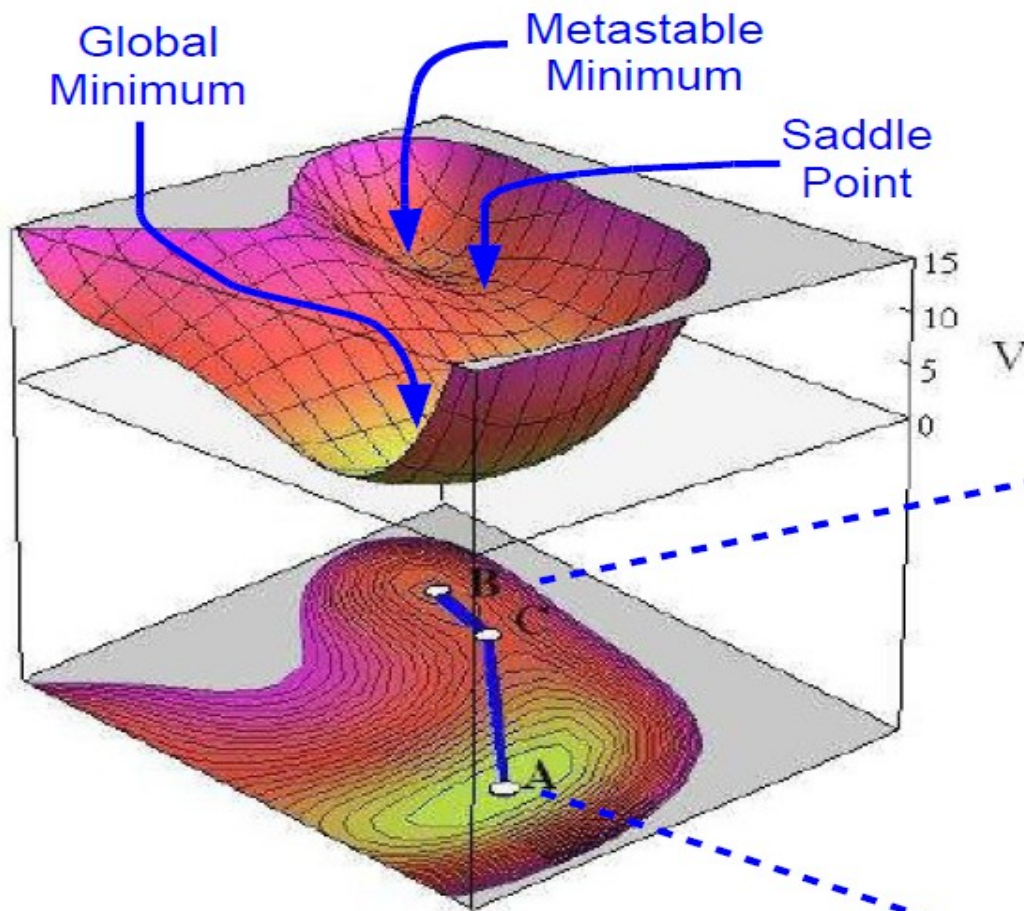
$$(\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0) \text{ and } g = 1$$

We then find that our model has three extrema:

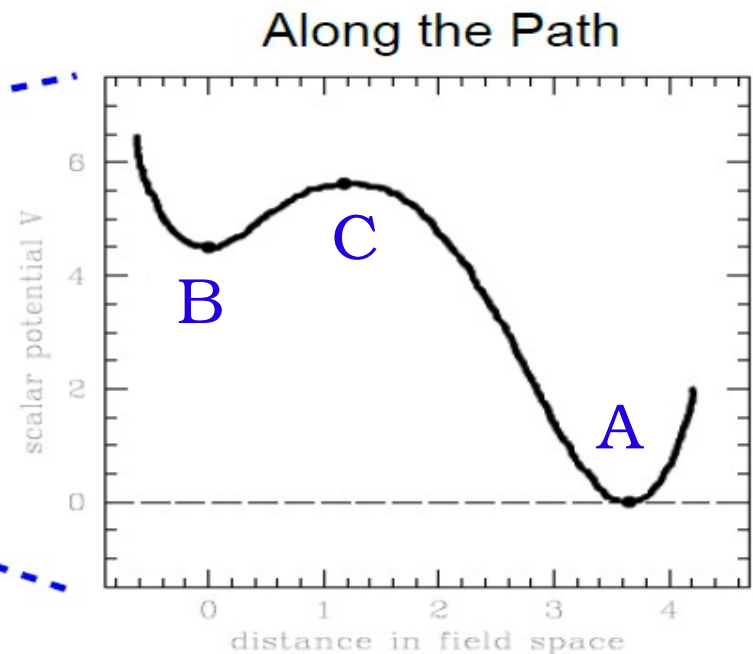
- **Extremum A:** a true ground state in which both SUSY and R-symmetry are preserved;
- **Extremum B:** a metastable vacuum state in which both SUSY and R-symmetry are broken, and whose gauge symmetry also differs from that of the true vacuum; and
- **Extremum C:** a saddle point associated with a vacuum-energy barrier between the two that results in a long lifetime (potentially exceeding the age of the universe) for the metastable vacuum.

All three are located at finite distances in field space...

Label	(v_1, v_3, v_5)	V	Stability	SUSY	R -symmetry	Gauge Group
A	$(\sqrt{5}, 0, 0)$	0	Stable	Yes	Yes	$U(1)_b$
B	$(0, 2, 2)$	9/2	Metastable	No	No	None
C	$(\sqrt{3}/2, \sqrt{7}/2, \sqrt{5}/2)$	45/8	Unstable	No	No	None



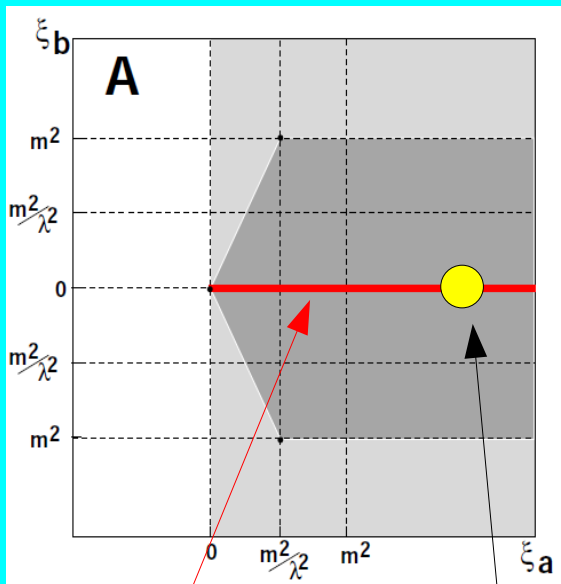
- Solution C represents the saddle point in field space between solutions A and B through which the classical path runs.



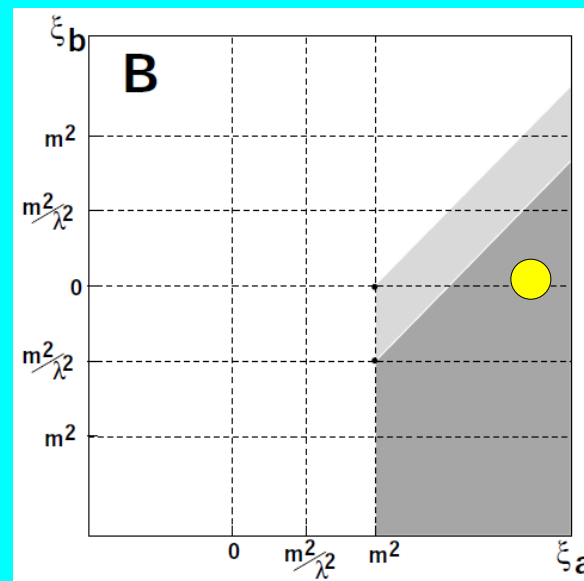
Thus we see that our metastable vacuum structure arises *classically*, with all flat directions lifted, and involves only finite distances in field space.

Moreover, our original model parameters did not need to be fine-tuned; similar behavior exists over large regions of (ξ_1, ξ_2) parameter space...

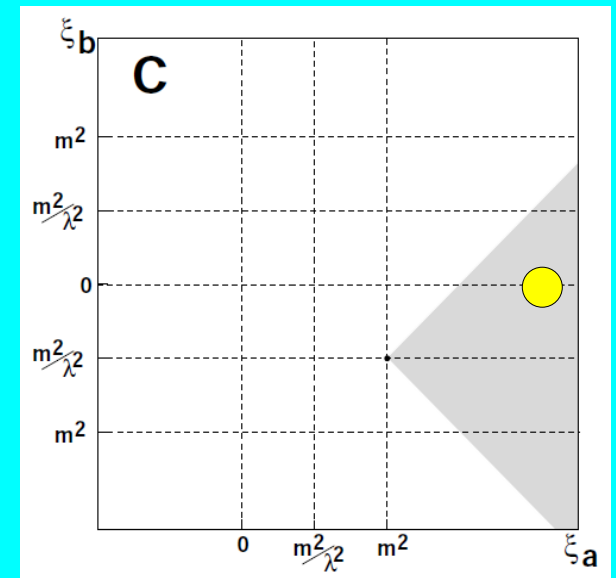
True vacuum



Metastable vacuum



Saddle point



Unbroken SUSY

Original parameter choice

- Light gray = extremum exists
- Dark gray = extremum stable

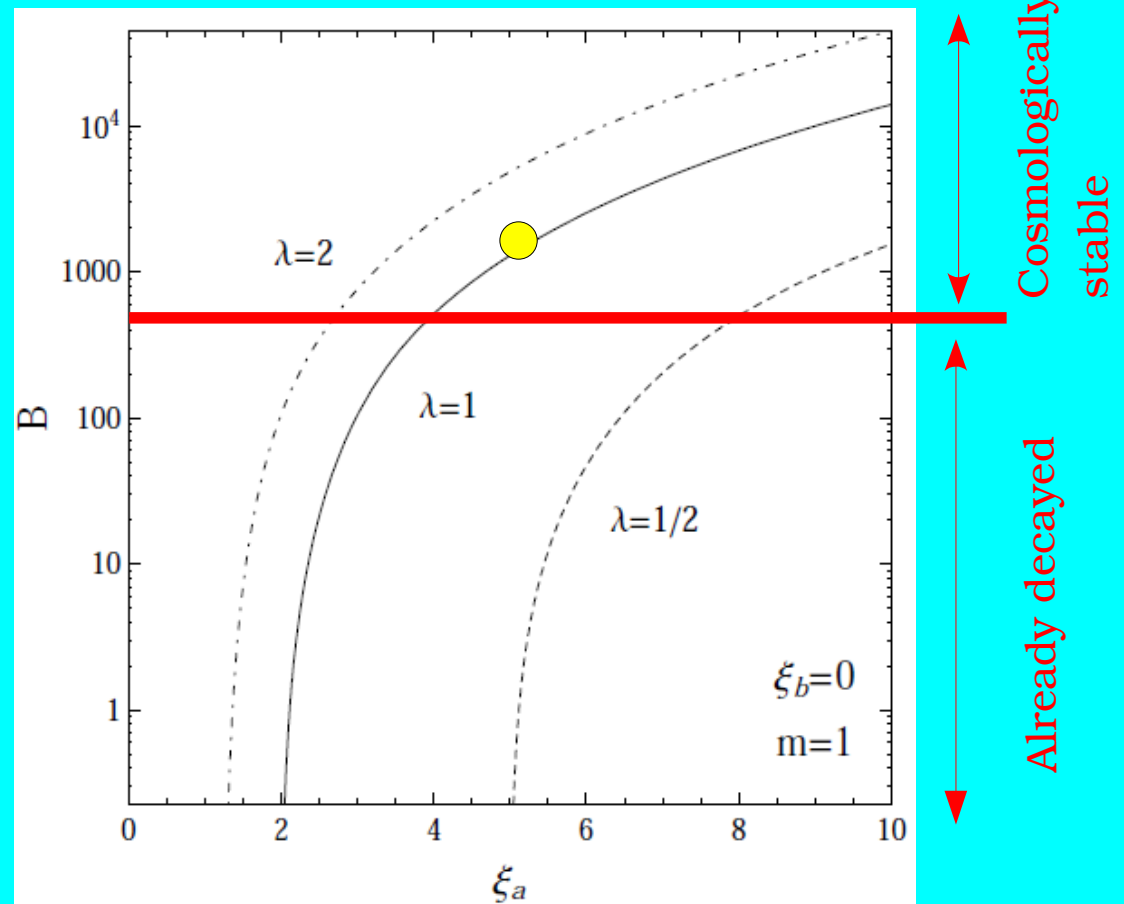
Moreover, it is straightforward to demonstrate that the metastable vacuum has a lifetime which easily exceeds the age of the universe.

Recall that instanton-induced tunneling to the true ground state leads to a metastable decay rate which is governed by a “bounce action” B :

$$\frac{\Gamma_{\text{inst}}}{\text{Vol}} = A e^{-B}$$

B can be calculated through standard techniques, given the depths of the two wells, their relative distance in field space, and the height of the potential barrier between them.
For our model, we find...

- Coleman
- Coleman, de Luccia
- Duncan, Jensen



Clearly, this is not a complete phenomenological model of metastable SUSY-breaking.

However, this might be a simple, field-theoretic “kernel” which could serve as a SUSY-breaking sector in a fully developed model, and be connected to the SM through a suitable messenger sector.

More importantly for our purposes, however, **this model shows that it is possible to have a remarkably simple vacuum structure in which there exists a SUSY ground state and a SUSY-breaking metastable state, with both arising classically in a theory with no flat directions or infinite distances in field space.**

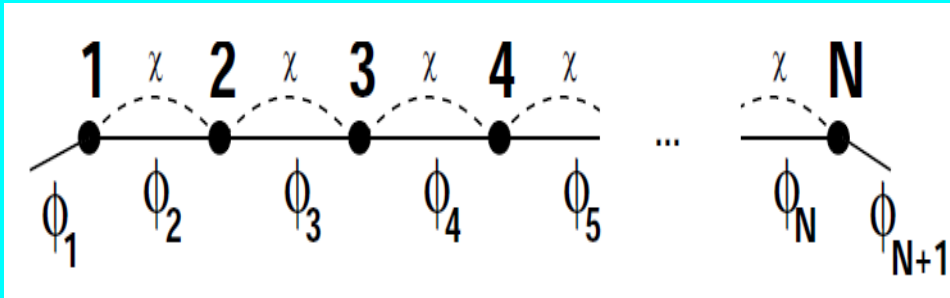
The key feature that makes this possible is that SUSY-breaking in this model arises through F-term breaking which is ultimately sourced by D-term breaking (FI terms).

Given this “warm-up” exercise, we can now proceed to construct a model with *infinite towers* of metastable vacua, all realized classically without flat directions or infinite distances in field space.

- KRD and B. Thomas, 0811.3335 (PRD).

The Model

Generalizing the previous model, we now consider an N -site “orbifold moose” with N different $U(1)$ gauge groups and $N+1$ scalar “link” fields.



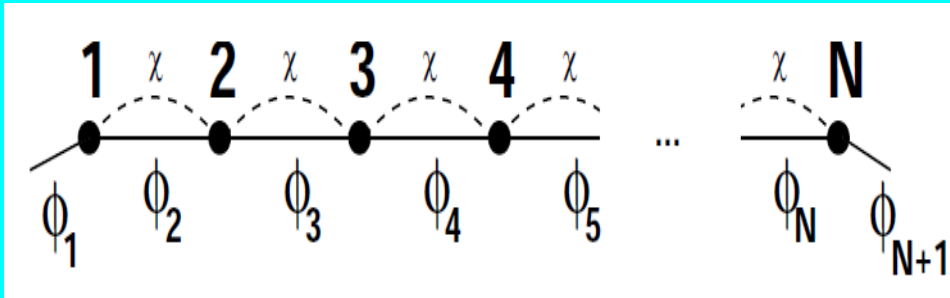
	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$...	$U(1)_{N-1}$	$U(1)_N$
Φ_1	-1	0	0	0	...	0	0
Φ_2	+1	-1	0	0	...	0	0
Φ_3	0	+1	-1	0	...	0	0
Φ_4	0	0	+1	-1	...	0	0
\vdots	\vdots	\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
Φ_{N-1}	0	0	0	0	...	-1	0
Φ_N	0	0	0	0	...	+1	-1
Φ_{N+1}	0	0	0	0	...	0	+1

where $g_1 = g_2 = g_3 = \dots = g_N = g$.

To this core model, we then add
three critical ingredients:

The Model

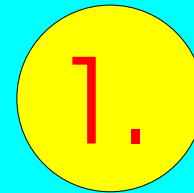
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Φ_2	+1	-1	0	0	...	0	0
Φ_3	0	+1	-1	0	...	0	0
Φ_4	0	0	+1	-1	...	0	0
\vdots	\vdots	\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
Φ_{N-1}	0	0	0	0	...	-1	0
Φ_N	0	0	0	0	...	+1	-1
Φ_{N+1}	0	0	0	0	...	0	+1

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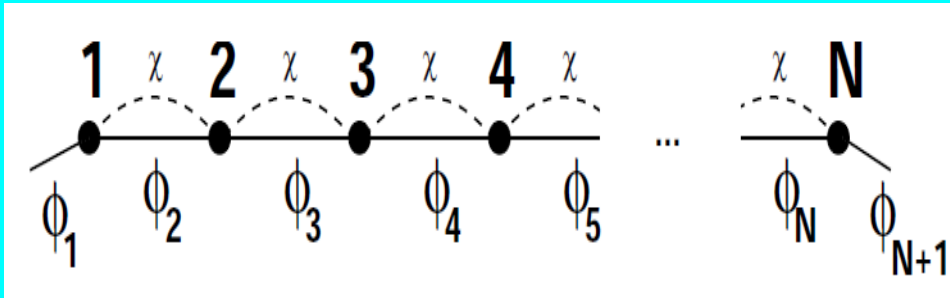
Wilson line operator

$$W = \lambda \prod_{i=1}^{N+1} \Phi_i$$

- Most general superpotential contribution that can be written.
- Renormalizable for N=2 only, but term exists for all N.

The Model

Generalizing the previous model, we now consider an N-site “orbifold moose” with N different U(1) gauge groups and N+1 scalar “link” fields.



2.

	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$...	$U(1)_{N-1}$	$U(1)_N$
Φ_1	-1	0	0	0	...	0	0
Φ_2	+1	-1	0	0	...	0	0
Φ_3	0	+1	-1	0	...	0	0
Φ_4	0	0	+1	-1	...	0	0
\vdots	\vdots	\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
Φ_{N-1}	0	0	0	0	...	-1	0
Φ_N	0	0	0	0	...	+1	-1
Φ_{N+1}	0	0	0	0	...	0	+1

Fayet-Iliopoulos terms
 ξ_1, ξ_N for “endpoint”
gauge groups only.

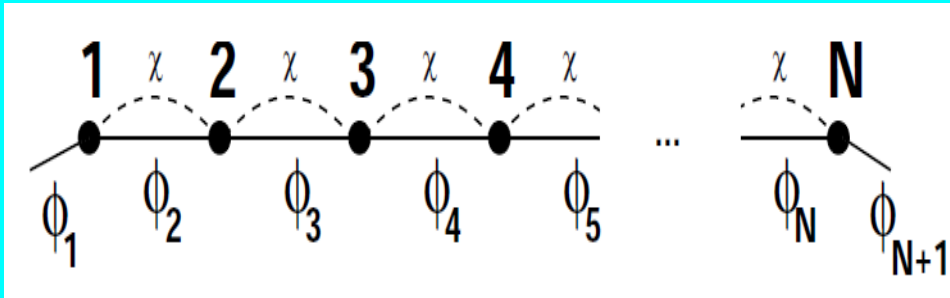
Take $\xi_1 = \xi_N = \xi$
for simplicity.

where $g_1 = g_2 = g_3 = \dots = g_N = g$.

To this core model, we then add
three critical ingredients:

The Model

Generalizing the previous model, we now consider an N-site “orbifold moose” with N different U(1) gauge groups and N+1 scalar “link” fields.



3.

Kinetic mixing!

$$\mathcal{L} \ni \frac{1}{32} \int d^2\theta \ W_{a\alpha} X_{ab} W_b^\alpha$$

For simplicity, restrict mixing to “nearest neighbors” and take all kinetic mixing parameters = χ .

Note:

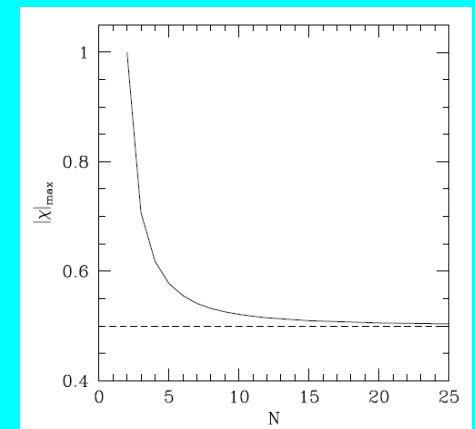
$$0 < \chi < 1/2$$

	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$...	$U(1)_{N-1}$	$U(1)_N$
Φ_1	-1	0	0	0	...	0	0
Φ_2	+1	-1	0	0	...	0	0
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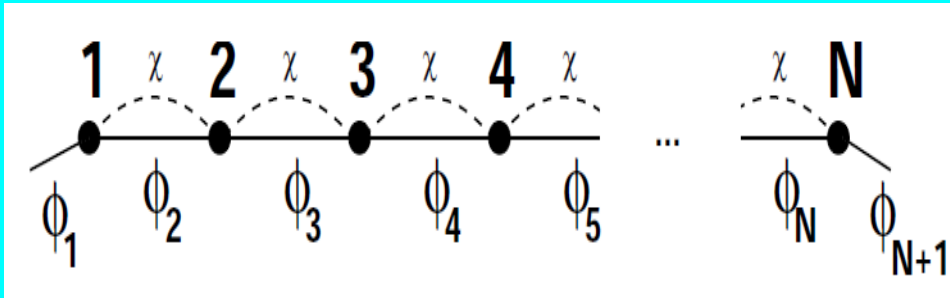
To this core model, we then add three critical ingredients:

Maximum χ allowed for non-singular mixing matrix with positive eigenvalues, versus N:



The Model

Generalizing the previous model, we now consider an N-site “orbifold moose” with N different U(1) gauge groups and N+1 scalar “link” fields.



3.

Kinetic mixing!

$$\mathcal{L} \ni \frac{1}{32} \int d^2\theta \ W_{a\alpha} X_{ab} W_b^\alpha$$

Note: Kinetic mixing can be “undone” through re-diagonalization:

$$\hat{W}_a \equiv M_{ab} W_b$$

But this then redistributes the U(1) charges and FI terms in a non-trivial way across the entire moose:

$$\begin{aligned} \hat{Q}_{ai} &= [(M^{-1})^T]_{ab} Q_{bi} \\ \hat{\xi}_a &= [(M^{-1})^T]_{ab} \xi_b . \end{aligned}$$

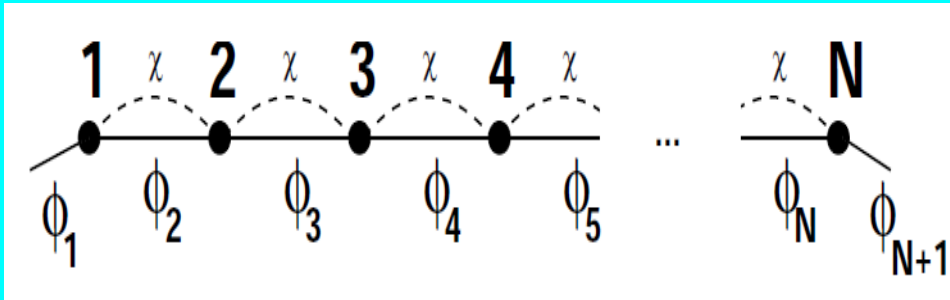
where $g_1 = g_2 = g_3 = \dots = g_N = g$.

To this core model, we then add three critical ingredients:

	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$...	$U(1)_{N-1}$	$U(1)_N$
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Φ_2	+1	-1	0	0	...	0	0
Φ_3	0	+1	-1	0	...	0	0
Φ_4	0	0	+1	-1	...	0	0
\vdots	\vdots	\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
Φ_{N-1}	0	0	0	0	...	-1	0
Φ_N	0	0	0	0	...	+1	-1
Φ_{N+1}	0	0	0	0	...	0	+1

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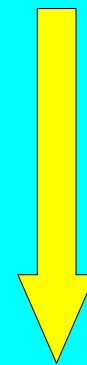
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where $g_1 = g_2 = g_3 = \dots = g_N = g$.

To this core model, we then add three critical ingredients:

Thus, our model is defined by five parameters: N, g, λ, ξ, χ .

For simplicity, we can form dimensionless quantities by dividing by ξ and scaling out g , then dropping the primes:



$$\lambda'' \equiv \xi^{N/2-1} \lambda / g$$

$$\Phi''_i \equiv \Phi_i / \sqrt{\xi}$$

$$V'' \equiv V / (g\xi)^2$$

Thus, model now depends on only N, λ (dimensionless), and χ .

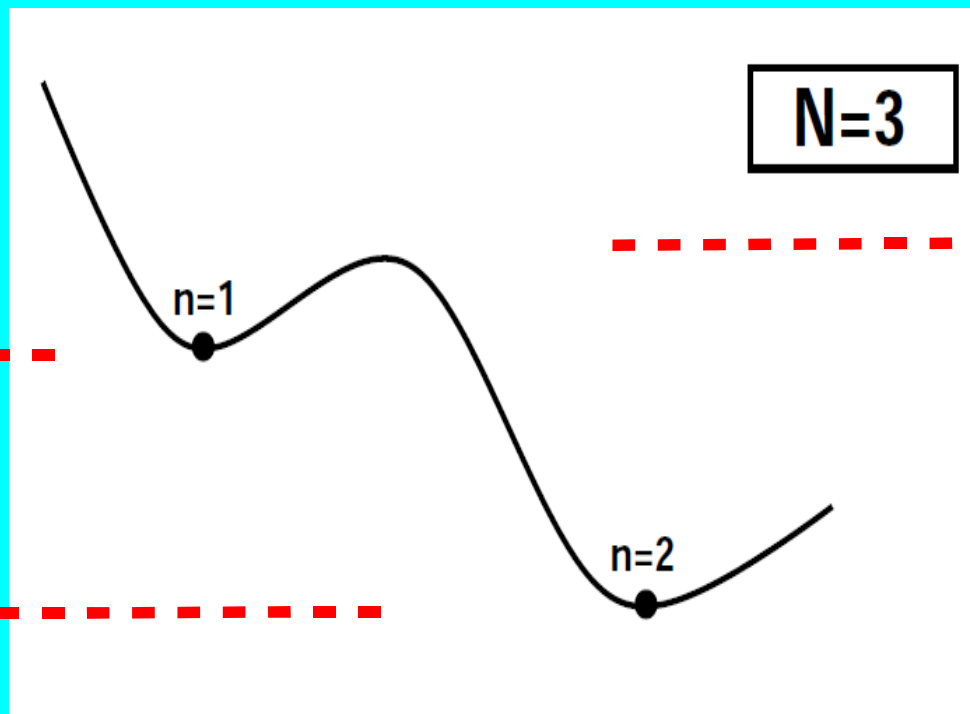
So what do we find?

Consider $N=3$ special case: 3 U(1)'s, 4 chiral superfields --- 4D field space.

→ Two vacua and a single potential barrier between them.

Stable vacua

Barrier



$$V_1 = 1/2$$

$$V_{12} = \frac{1}{2}(1 - \chi^2)^{-1}$$

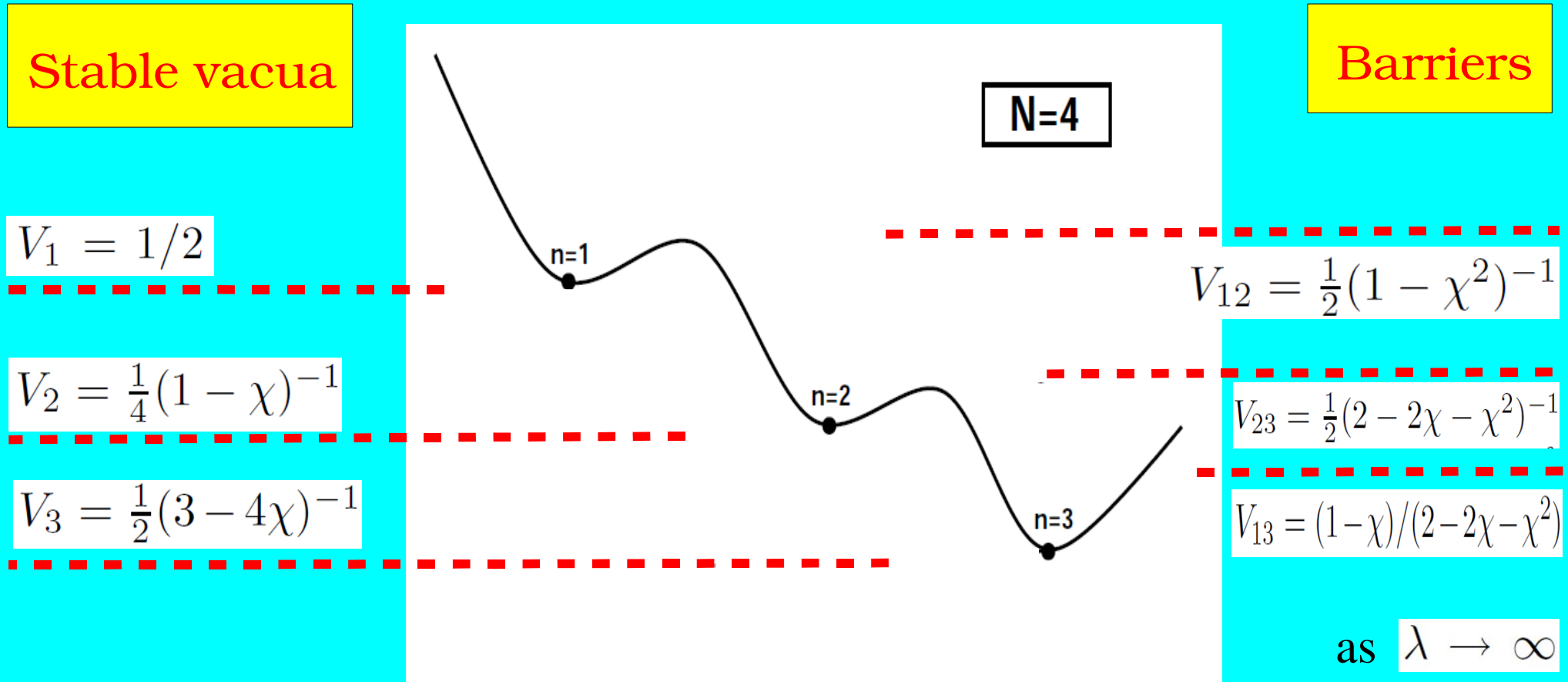
as $\lambda \rightarrow \infty$

$$V_2 = \frac{1}{4}(1 - \chi)^{-1}$$

- Energies (and field configurations) of vacua are λ -independent.
- Height of barrier depends on λ , takes simple form in $\lambda \rightarrow \infty$ limit.
- Both vacua in tower are stable if $\lambda > \lambda_3^*$ where $\lambda_3^* \equiv 1/\sqrt{\chi(1 + \chi)}$

Now consider $N=4$ case: 4 U(1)'s, 5 chiral superfields --- 5D field space.

→ **Three** vacua and **three** potential barriers (one between each pair).



- Energies of vacua are λ -independent, but barrier heights depend on λ .
- All vacua in tower are stable if $\lambda > \lambda_4^*$ where $\lambda_4^* \equiv \frac{1}{\chi\sqrt{1+\chi}}$
- Energies of n=1,2 vacua and barriers same as N=3 case!
- New n=3 vacuum with even lower energy “slides in” under the previous two!

For general N : N $U(1)$'s, $N+1$ chiral superfields --- $(N+1)$ -dim'l field space.

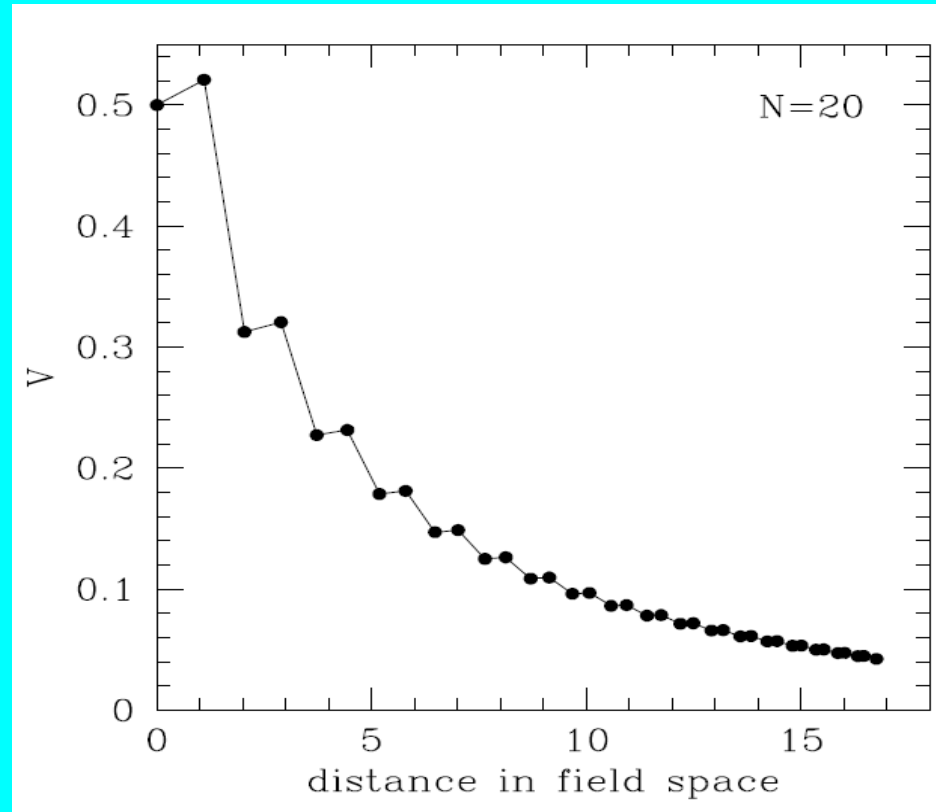
→ $N-1$ vacua and $(N-1)(N-2)/2$ potential barriers between them.

Stable vacua

$$V_n = \frac{1}{2} \left(\frac{1}{\chi R_n} \right)$$

where

$$R_n \equiv \left(\frac{1}{\chi} - 2 \right) n + 2$$



Barriers

$$V_{nn'} = \frac{1}{2\chi} \left(\frac{R_{n'-n}}{R_n R_{n'-n} - 1} \right)$$

as $\lambda \rightarrow \infty$

- As N increases, new vacua keep “sliding in” under the previous tower, with energies approaching zero.

- All vacua in tower are stable if $\lambda > \lambda_N^*$ where $\lambda_N^* \equiv \max_{1 \leq n \leq N-1} \lambda_{N,n}^*$

with

$$\lambda_{N,n}^{*2} = y^n \frac{\Gamma(y)}{\Gamma(n+y)} \frac{R_n^{N-2}}{\chi(1+R_n)} \quad \text{and} \quad y \equiv \chi/(1-2\chi)$$

There is also a rich set of instanton-induced tunneling decay patterns along the length of the tower.

Tunneling rate for any single transition $n_i \longrightarrow n_f$

- Coleman
- Coleman, de Luccia

is governed by a bounce action $B(n_i, n_f)$.


$$\frac{\Gamma_{\text{inst}}}{\text{Vol}} = A e^{-B}$$

But unlike the traditional case of only two vacua separated by a single barrier, we now have *an infinite spectrum of vacua along with a unique saddle-point barrier between any pair of vacua.*

Therefore, the full quantum-mechanical decay process can be rather complex:

- Many possible final states for a given initial vacuum n_i .
- For any decay $n_i \longrightarrow n_f$, now can have possibility of QM interference terms from
 - $n_i \longrightarrow n' \longrightarrow n_f$,
 - $n_i \longrightarrow n' \longrightarrow n'' \longrightarrow n_f$, *etc.*

Simplifying approximations

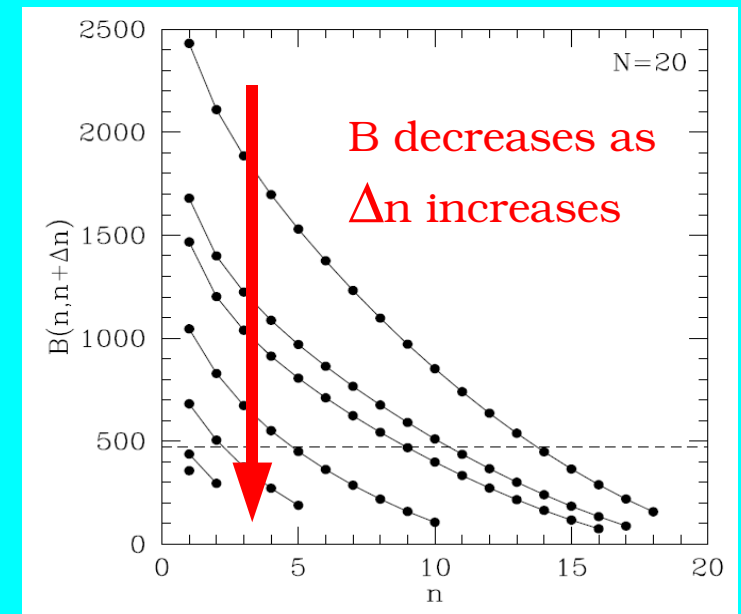
- Neglect multi-transition interference terms, since $B(n_i, n_f) < B(n_i, n') + B(n', n_f)$  such terms are exponentially suppressed.
- A given vacuum n_i prefers to decay to that vacuum n_f for which $B(n_i, n_f)$ is minimized. This decay is exponentially preferred, so neglect all others.



What emerges, then, are semi-classical decays from single vacuum to single vacuum, with each hop governed by its own bounce action.

Thus, problem boils down to understanding the behavior of $B(n_i, n_f)$ as function of n_i , n_f , N , and χ . We find general trends...

- $B(n_i, n_f)$ vanishes as $\chi \rightarrow 0$ and diverges as $\chi \rightarrow 1/2$.
 - Vacua become unstable in first limit.
 - There are no CdL instanton transitions in second limit (to be discussed later).
- $B(n_i, n_f)$ increases with N and with n_i .
 - Vacua become more stable as N increases, since field space distances increase with N .
 - Vacua become more unstable in lower portions of tower, more stable near the top.
- Generally, $B(n_i, n_f)$ decreases as $\Delta n = n_f - n_i$ increases.
 - Generally, preferred decay mode has maximum possible value of Δn . Thus, a given metastable vacuum generally prefers to decay directly to the ground state.



Taken together, what emerges is

“Collapse” behavior

Imagine each vacuum state initially populated

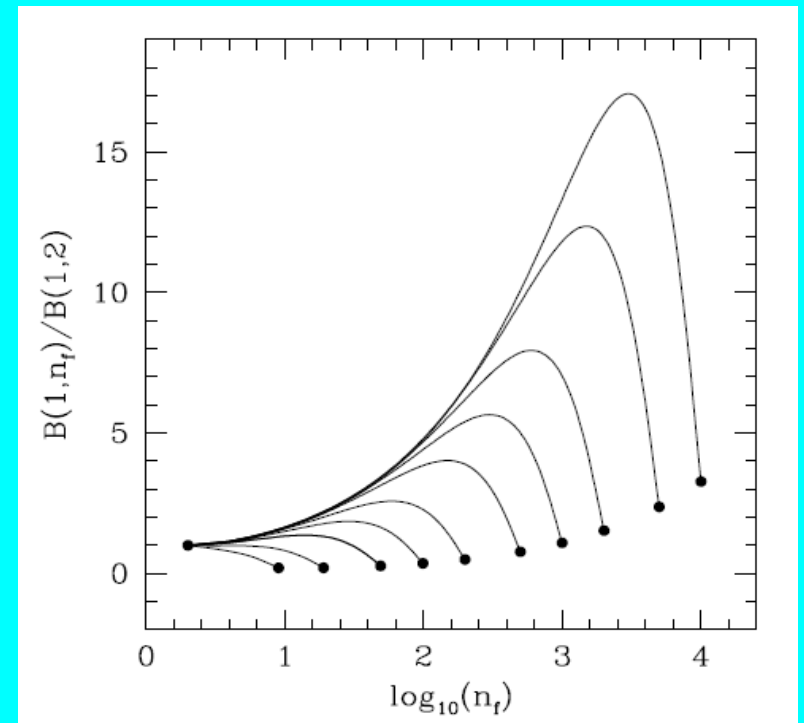
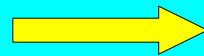
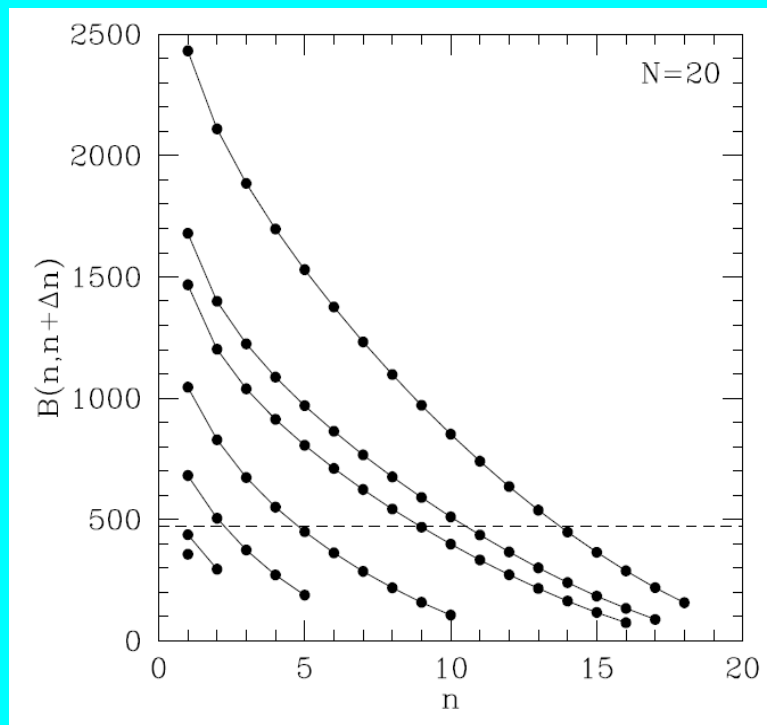
(e.g., eternal inflation, multiverse, or even one universe with multiple vacua)...

- Each vacuum state individually collapses directly to the ground state.
- Lower vacua collapse first, then higher and higher vacua begin to collapse as the collapse behavior creeps up the tower.
- These sequential collapses continue until they reach a maximum height (if any) at which collapse has not yet occurred. This is the onset of the region of cosmological stability at the top of the tower.

Collapse behavior is fairly generic.

But we naturally expect kinetic mixing χ to be small, and we are particularly interested in the case of large N .

It turns out that in the double limit $\begin{cases} \chi \rightarrow 0 \\ N \rightarrow \infty \end{cases}$ (with χN fixed), this behavior can change significantly...



Smallest B when Δn is **maximized**.
Favors *large* hops directly to ground state.

Smallest B when Δn is **minimized**.
Favors *small* steps down the tower.

In this case, what emerges is

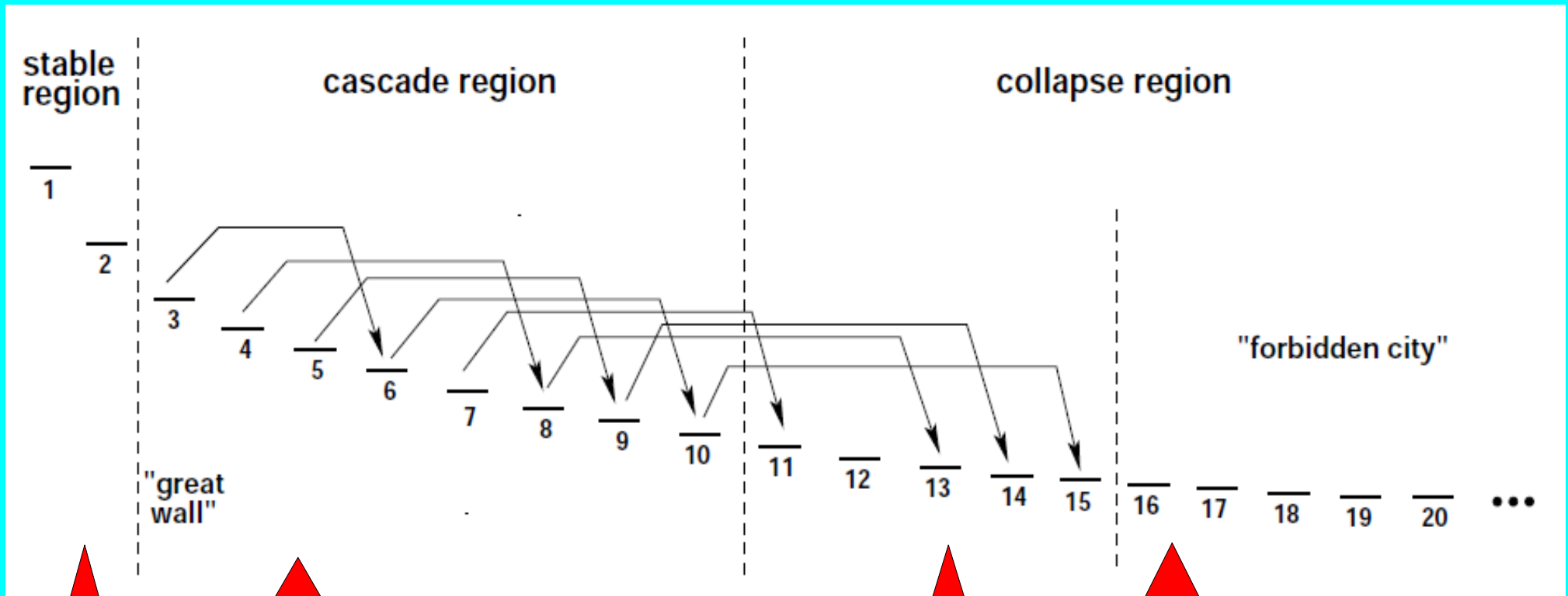
A Metastable Vacuum Cascade

Imagine the top vacuum state(s) initially populated...

- In general, a given metastable vacuum state decays to another (lower) metastable vacuum state.
- This process repeats, forming a **metastable vacuum cascade**.
- Many independent cascade trajectories are possible, “leapfrogging” over each other.
- The size of each hop is generally governed by the product χ^N .
- These cascades continue down the tower until collapse behavior reemerges. All subsequent decays go directly to the ground state.
- The uppermost portion of any tower may still be cosmologically stable.

Explicit example:

- $N = 5000$
- $\chi = 2.8 \times 10^{-4}$



Cascade region:
Four independent trajectories:

- $3 \rightarrow 6 \rightarrow 10 \rightarrow 15 \rightarrow \text{GS}$
- $4 \rightarrow 8 \rightarrow 13 \rightarrow \text{GS}$
- $5 \rightarrow 9 \rightarrow 14 \rightarrow \text{GS}$
- $7 \rightarrow 11 \rightarrow \text{GS}$

Vacua are stable on cosmological timescales.

Upon arriving here, all subsequent decays are directly to the ground state.

Can never be reached from outside. To populate this region, universe must be "born" here.

The particle spectrum of the model: A Quick Glance

In each n -vacuum, only one $U(1)$ remains unbroken:

$$B^\mu = \frac{1}{\sqrt{n}} \sum_{a=N-n+1}^N A_a^\mu$$

This gauge boson couples to the components of only two chiral superfields: Φ_{N-n+1} and Φ_{N+1} .

The remaining fields in the theory couple only to the massive Z' fields, and are thus essentially unobservable at low energies.

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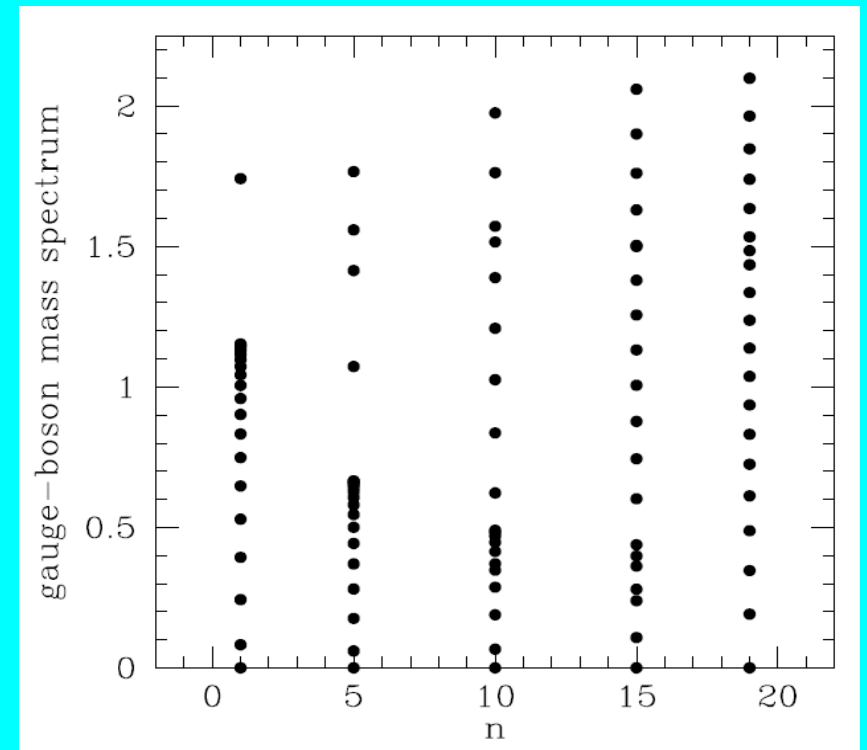
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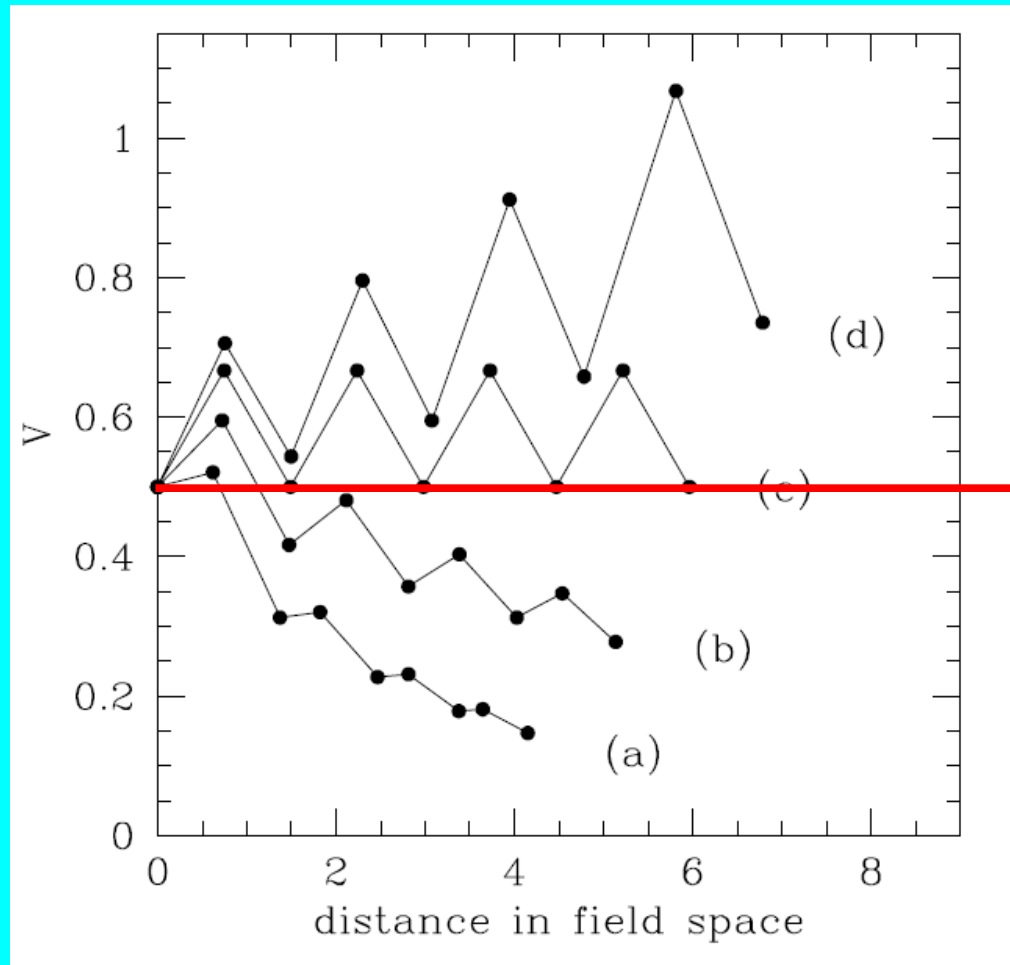
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How does the spectrum of states evolve as we cascade down the tower?

Of course, the spectrum can vary significantly if the underlying model parameters are changed... plenty of room for realistic model-building exists.



Finally, how do these metastable vacuum towers depend on χ ?



$$\chi = 0.54$$

$$\chi = 0.5$$

$$\chi = 0.4$$

$$\chi = 0.2$$

For $\chi = 1/2$, all vacua in the tower become degenerate and an exact shift symmetry emerges relating one vacuum state to the next. This shift symmetry is a reflection of the underlying translational symmetry of the original moose.

As a result of this symmetry,

- The true ground state of the theory is no longer any of the individual n -vacua by itself. Instead, what results is a **Bloch wave** across the entire set of degenerate vacua.
- Moreover, the vacuum energies associated with such Bloch states fill out a **continuous band**. As a result, the vacuum energy of the true ground state of the theory will be smaller than the energy of any individual vacuum.

This, then, is the first known explicit example of a supersymmetric theory with a Bloch-wave vacuum structure!

- KRD and B. Thomas, 0811.3335
- KRD and B. Thomas, to appear.

Moreover, our Bloch-wave vacuum states are linear combinations of the individual \mathbf{n} -vacua. Thus, any process that causes our system to initially populate a given \mathbf{n} -vacuum (such as a coupling of a single U(1) to the SM via a messenger sector) will then induce non-trivial time-dependent *vacuum oscillations* across the set of \mathbf{n} -vacua as a whole.

Such vacuum oscillations are analogous to multi-flavor neutrino oscillations, and can potentially have dramatic effects on the phenomenology resulting from such theories.

- KRD and B. Thomas, in progress.

We caution, however, that not merely *any* moose will give rise to this sort of Bloch-wave band structure.

Translational symmetry is not enough: we also require an infinite set of fully stable vacua, with all flat directions lifted and with equal non-zero transition probabilities between them.

This in turn requires not only a specific pattern of FI terms, but also the specific kinetic-mixing pattern discussed above.

Thus, the emergence of a Bloch-wave vacuum structure is highly non-trivial.

Conclusions

In this talk, we have presented explicit examples of two new highly non-trivial vacuum structures that can arise in supersymmetric field theories and string theories:

- Infinite towers of metastable vacua
- Bloch-wave vacuum structure and a corresponding vacuum-energy “band”.

In each case, these vacuum structures are realized classically (*i.e.*, at tree level) without flat directions and without recourse to infinite field-space distances or non-perturbative physics.

Nevertheless, numerous theoretical questions/extensions remain...

- How can we populate the vacuum towers *thermally*? Which states in the tower (top? middle? bottom?) are preferred *thermally*?
- **For the metastable vacuum tower**, we have thus far restricted our attention to instanton-induced vacuum transitions. What about thermal transitions (sphalerons)?
- **For the Bloch wave**, CdL instantons are infinitely massive. Must consider Hawking-Moss instantons instead and perform rigorous analysis of band structure.
- Is there a 5D geometric interpretation of our moose structure, as in deconstructed models? What is the 5D geometry, if any, corresponding to FI terms and kinetic mixing?

One possible application

The cosmological constant problem.

Many recent proposals relate a small Λ to a large number of vacua

- **Bousso-Polchinski** (towers of metastable vacua, universe cascades down to a state with very small Λ)
- **Kane-Perry-Zytkow** (Bloch-wave ground state with bottom of band approaching zero energy)

However, prior to our work, no concrete models realizing these vacuum structures have ever been proposed.

Our models thus provide the first explicit and calculable field-theoretic scenarios in which these ideas can be modeled and tested.

Another possible application

The string landscape.

One method that has been proposed for extracting phenomenological predictions from string theory --- even without a way to determine the “correct” string compactification --- is through statistical studies of the string landscape.

But moose theories of the sort we have been analyzing are natural low-energy limits of string theories, particularly flux compactifications. Moreover, because they give rise to multiple vacua, the traditional one-to-one connection between string *models* and string *vacua* need not apply.

Thus, the full landscape of string theory may be even richer than previously imagined, since all long-lived metastable vacua must be included in the analysis.

In particular, even if vacuum structures of this sort we are discussing are relatively rare across the landscape, ***the fact that they give rise to infinite numbers of vacuum states may mean that they completely dominate the statistical properties of the landscape as a whole!***

Needless to say, there are many other potential applications as well...

- New possibilities for SUSY-breaking scenarios
- Cosmological applications/implications: phase transitions, defects, cosmic string networks, domain walls...
- New possibilities for Z' phenomenology
- Effects of the “vacuum oscillations” that result from the Bloch-wave ground states, assuming we are originally placed in one of the individual n -vacua.

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Undoubtedly, this is only the tip of the iceberg. Much more work is needed, but the prospects are clearly exciting.