Heterotic Orbifolds in Blowup

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and work in progress

in collaboration with

Michael Blaszczyk, Tae-Won Ha,Johannes Held, Dennis Klever, Hans-Peter Nilles, Filipe Paccetti, Felix Plöger, Michael Ratz, Fabian Rühle, Michele Trapletti, Patrick Vaudrevange, Martin Walter

1. Introduction and motivation
   - Calabi–Yau model building
   - Orbifold model building
2. Blowups: Connecting orbifolds with smooth CYs
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4. Blowing up $T^6/\mathbb{Z}_{6-\text{II}}$ MSSM orbifolds
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Introduction and motivation

One of the aims of String Phenomenology is to find the Standard Model of Particle Physics from String constructions :

- The $E_8 \times E_8$ Heterotic Strings naturally incorporates properties of GUT theories ,
- and could lead to the Supersymmetric Standard Model (MSSM) .

Two approaches are often considered to achieve this goal:

- smooth Calabi–Yau compactifications with gauge bundles
  Candelas,Horowitz,Strominger,Witten'85
- *singular* Orbifold constructions Dixon,Harvey,Vafa,Witten'85

Calabi–Yau model building

Calabi–Yau manifolds can be constructed as:

- a bundle $\mathcal{M} \to B$ with an elliptically fibered torus ,
- complete Intersections CY : hypersurfaces in projective spaces.

Gauge backgrounds that satisfy the Hermitean Yang–Mills equations , i.e. stable bundles , can be obtained by

- spectral covers over elliptically fibered CYs

  Friedman,Morgan,Witten'97, Donagi'97, Donagi,Lukas,Ovrut,Waldram'99
- monad constructions

  Blumenhagen,Schimmrigk,Wisskirchen'96, Anderson,He,Lukas'07
- method of extensions Donagi,Ovrut,Pantev,Waldram'00, Andreas,Curio'06

Stability depends on the Kähler moduli and only recently there has been some physical understanding of this condition

Blumenhagen,Honecker,Weigand'05 Anderson,Grey,Lukas,Ovrut'09

Pros and cons of Calabi-Yau model building

Heterotic supergravities on CY s with bundles have as advantages:

- generic point in moduli space,
- chiral spectrum determined by topological data ,
- fixing of some (Kähler) moduli,
- some MSSMs have been constructed in this way.

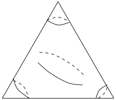
  Braun,He,Ovrut,Pantev'05, Bouchard,Donagi'07

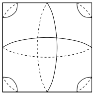
Their disadvantages are:

- construction CYs is difficult ,
- classification of their gauge bundles is complicated ,
- SUGRA approximation only; not full string theory.

Orbifold model building

Orbifolds look like pillows :

$T^2/\mathbb{Z}_3$: 

$T^2/\mathbb{Z}_2$: 

- Orbifolds are flat spaces except for the orbifold fixed points that have curvature singularities .
- These singularities break a certain amount of supersymmetry and allow for a chiral spectrum .

A heterotic orbifold model is fully specified by defining the action of the space group on the gauge degrees of freedom using

- Orbifold gauge shift vector (related to the orbifold rotation)
- Wilson lines (related to torus translations)

Pros and cons of orbifold model building

Heterotic strings on orbifolds have as advantages:

- they described by free CFTs , that can be systematic classified ,
  Ibanez,Mas,Nilles,Quevedo'88
- full spectrum (not only massless states) computable,
- give rise to a large pool of possible MSSMs on $T^6/\mathbb{Z}_{6\text{–II}}$.
  Buchmuller,Hamaguchi,Lebedev,Ratz'04,
  Lebedev,Nilles,Raby,Ramos-Sanchez,Ratz,Vaudrevange,Wingerter'07

Their disadvantages are:

- singular spaces with curvature singularities ,
- field theory on them ill–defined ,
- have only a restricted number of combinations of Hodge numbers,
- they define a special corner in full moduli space.

Connecting orbifolds with smooth CYs

Our major aim is to connect both approaches, therefore

1. we start form a heterotic $T^6/\mathbb{Z}_N$ orbifold model,
2. We resolve the compact orbifold by
   - using toric geometry to blow up the individual fixed points (sometime explicit blowups can be constructed),
   - and subsequently gluing their local resolutions together.
3. Next describe possible (Abelian) gauge backgrounds .
4. Interpret and investigated the resulting models as Calabi-Yau compactifications.

Heterotic $T^6/\mathbb{Z}_N$ orbifold models

A heterotic orbifold model is specified by the action of the space symmetry group on the gauge group:

- Orbifold gauge shift vector (related to the orbifold rotation):

$$A(\theta z) = U\, A(z)\, U^{-1}\,, \quad U = e^{2\pi i\, v^I H_I}$$

- Wilson lines (related to torus translations):

$$A(z + e_i) = T_i\, A(z)\, T_i^{-1}\,, \quad T_i = e^{2\pi i\, w_i^I H_I}$$

In string theory the gauge shift and Wilson lines have to satisfy the modular invariance constraints, for example

Dixon,Harvey,Vafa,Witten'85, Ibanez,Nilles,Quevedo'87

$$N\,\phi^2 - N\,v^2 \approx 0\,, \qquad \theta = \left(e^{2\pi\phi_1}, e^{2\pi\phi_2}, e^{2\pi\phi_3}\right)$$

Towards orbifolds in blowup

What do we need to know of a resolution of an orbifold to investigate a blowup model ?

- A basis of the divisors corresponding to (1,1)–forms,
- and all the (self–)intersections of these divisors .

For example for the resolution of $T^6/\mathbb{Z}_{6\text{–II}}$ this means

- identify a basis for $h_{11} = 35$ divisors ,
- and calculate $35 \cdot 34 \cdot 33/3! + 35 \cdot 34/2 + 35 = 7\,175$ intersections ,

and for $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$

- a basis of $h_{11} = 51$ divisors
- $51 \cdot 50 \cdot 49/3! + 51 * 50/2 + 51 = 22\,151$ intersections .

Toric resolutions of non–compact orbifolds

For an introduction to toric geometry see: Fulton, Oda, Hori et al.
Orbifolds can be resolved using toric geometry Erler,Klemm'92, Lust,Reffert,Scheidegger,Stieberger'06, SGN,Ha,Trapletti'07

The basic idea of toric resolutions is to replace the $\mathbb{Z}_n$ action:

$$\theta: (z_1, z_2, z_3) \to (e^{2\pi i \phi_1} z_1, e^{2\pi i \phi_2} z_2, e^{2\pi i \phi_3} z_3)$$

by one or more $\lambda \in \mathbb{C}^* = \mathbb{C} - 0$ complex scaling(s)

$$(z_1, z_2, z_3; x_1, \ldots) \sim (\lambda^{p_1} z_1, \lambda^{p_2} z_2, \lambda^{p_3} z_3; \lambda^{q_1} x_1, \ldots).$$

- The additional coordinates $x_1, \ldots$ keep the dimensionality the same as the orbifold.
- Setting all $x_1 = \ldots = 1$ does not determine scaling(s) uniquely, and gives precisely $\mathbb{Z}_n$ phases back.

Divisors and intersections

Divisors are defined as complex co-dimension one hyper surfaces :

$$\text{ordinary} \;:\; D_i \;=\; \{z_i = 0\}\;,$$
$$\text{exceptional} \;:\; E_r \;=\; \{x_r = 0\}\;.$$

By Poincaré duality divisors can be interpreted as $(1,1)$–forms, i.e. their first Chern class $c_1(D)$:

We identify integrals over (1,1)–forms and intersections of divisors:

$$\int c_1(D_1)c_1(D_2)c_1(E_2) \;=\; D_1 \cdot D_2 \cdot E_2$$

These intersection number are not always unique.

Explicit blowups of non–compact orbifolds

In some specific cases it is possible to obtain the explicit form of the non–compact Calabi-Yau metric of orbifold blowups:

$\mathbb{C}^2/\mathbb{Z}_N$ orbifolds: The Eguchi-Hanson spaces or multi-centerred gravitational instantons Eguchi,Hanson'78, Gibbons,Hawking'78

$$\mathrm{d}s^2 = V^{-1}\big(\mathrm{d}x_4 + \vec{\omega}\cdot \mathrm{d}\vec{x}\big)^2 + V\, d\vec{x}^2 , \quad V = \sum_{r=1}^{N} V_r , \quad V_r = \frac{R/2}{|\vec{x}-\vec{x}_r|} ,$$

Bringing N centers together results in the $\mathbb{Z}_N$ singularity

The harmonic (1,1) forms are given as

$$E_r = (\frac{V_r}{V})_{,i}\,(e_4\, e_i + \frac{1}{2}\epsilon_{ijk} e_j\, e_k) ,$$

Resolved $T^6/\mathbb{Z}_N$ orbifold

The resolution procedure for the $T^6/\mathbb{Z}_N$ orbifold involves the following gluing ingredients:

1. bookkeeping for the exceptional divisors E coming from the local singularities
2. 3 new inherited divisors R from the torus T^6
3. linear equivalence relations for ordinary divisors D
4. computation of intersection numbers

These intersection numbers are often highly non-unique:

$\Rightarrow$ a single orbifold may correspond to many topological distinct Calabi-Yau geometries .

General gauge background

General gauge background have to satisfy the Hermitean Yang-Mills equations (at lowest order in α'):

$$\mathcal{F}_{\alpha\beta} = \mathcal{F}_{\underline{\alpha}\underline{\beta}} = 0 \ \ (F - term), \qquad G^{\underline{\alpha}\alpha}\mathcal{F}_{\alpha\underline{\alpha}} = 0 \ \ (D - term)$$

These condition can be solved when the condition of the Donaldson, Uhlenbeck,Yau theorem are fulfilled:

- $\mathcal{F}$ is a (1,1) form
- Stable vector V bundle with

$$\mu_J(W) \leq \mu_J(V) = \int J^2\mathcal{F} = 0$$

for all proper sub-bundles $W \subset V$.

Bianchi identities

On compact spaces the Bianchi identities need to be satisfied on all divisors $S = R_i, E_r$: Witten'84, Candelas,Horowitz,Strominger,Witten'85

$$\int_S (\text{tr}\mathcal{R}^2 - \text{tr}\mathcal{F}^2) = \int_S \text{d}H = 0\,.$$

Using the splitting principle the second Chern class,

$$c_2(\mathcal{R}) = -\frac{1}{2}\,\text{tr}\mathcal{R}^2,$$

can be expanded in terms of divisors.

Therefore we can determine the set of Bianchi identities provided that we know all intersection numbers.

Spectrum computation

The spectrum of chiral states can be computed using a representation sensitive index theorem encoded in the following multiplicity operator:
SGN,Trapletti,Walter'06

$$N = \int_X \left\{ \frac{1}{6} \left(\frac{\mathcal{F}}{2\pi} \right)^3 - \frac{1}{24} \operatorname{tr} \left(\frac{\mathcal{R}}{2\pi} \right)^2 \frac{\mathcal{F}}{2\pi} \right\}$$

On each of the 248×248 states of the $E_8 \times E_8$ theory it gives the multiplicity of states.

This method can find states even if they are vector–like pairs as far as non-Abelian gauge symmetries are concerned.

Abelian gauge background

To be able to do heterotic model building on such a resolution we need to represent

- the gauge shift vector v,
- and the Wilson lines w_i's.

These define only boundary conditions, but not to explicit flux on the torus, therefore Abelian gauge backgrounds can be expanded in exceptional divisors only: SGN,Ha,Trapletti'07

$$\frac{\mathcal{F}}{2\pi} = V_r^I \, E_r \, H_I \ ,$$

To each exceptional divisor E_r we associate a line bundle vectors V_r .

Non-Abelian gauge background

Non-Abelian gauge backgrounds are much more difficult to describe, but on Eguchi-Hanson resolutions of $\mathbb{C}^2/\mathbb{Z}_N$ more systematic results are known:

- An SU(2) instanton supported on p of the N centers has instanton number: Bianchi,Fucito,Rossi,Martellini'96

$$\int c_2(\mathcal{F}) = p - \frac{1}{N}$$

- such instantons can be embedded multiple times and be combined with Abelian back grounds, giving a large number of stable bundles

In orbifold models this corresponds to changing the VEVs of the (twisted) states: SGN,Paccetti,Trapletti'08

- Single VEV at a fixed point $\Rightarrow$ Abelian bundles
- Multiple VEVs at a fixed point $\Rightarrow$ Non-Abelian bundles

Abelian and Non-Abelian backgrounds on $\text{Res}(\mathbb{C}^2/\mathbb{Z}_3)$

Solutions to the Bianchi identity: $n * \frac{2}{3} + 3V^2 = \frac{8}{3}$

n SU(2) emb	U(1) emb V	Unbroken gauge group
4	(0^{16})	SO(16) $\times$ Sp(8)
3	$\frac{1}{3}(1^2, 0^{14})$	SO(16) $\times$ Sp(6) $\times$ SU(2)
2	$\frac{1}{3}(1^4, 0^{12})$	SO(16) $\times$ Sp(4) $\times$ SU(4)
1	$\frac{1}{3}(1^6, 0^{10})$	SO(16) $\times$ SU(2) $\times$ SU(6)
0	$\frac{1}{3}(1^8, 0^8)$	SO(16) $\times$ SU(8)

All obtained from a single $\mathbb{C}^2/\mathbb{Z}_3$ orbifold model with gauge group $SO(16) \times SU(8) \times U(1)$ and spectrum: SGN,Paccetti,Trapletti'08

$$\frac{1}{9}\left[(\mathbf{16},\mathbf{8})_1 + (\mathbf{1},\mathbf{28})_2 + 2(\mathbf{1},\mathbf{1})_0\right] \qquad \mathbf{28} = \begin{pmatrix} V_1\epsilon & & & \\ & V_2\epsilon & & \\ & & V_3\epsilon & \\ & & & V_4\epsilon \end{pmatrix},$$

$$+(\mathbf{1},\mathbf{28})_{-2/3} + 2(\mathbf{1},\mathbf{1})_{8/3}$$

Axions, Kähler moduli and twisted modes

On Calabi-Yaus there can be multiple anomalous U(1)s; contrary to orbifolds where there is at most a single one.

Anomalous U(1) 's are canceled by axions β_r appearing in the expansion Blumenhagen,Honecker,Weigand'05

$$B_2 = b_2 + \beta_r \, E_r \,, \qquad \delta \beta_r = V_r^I \, \Lambda^I \,.$$

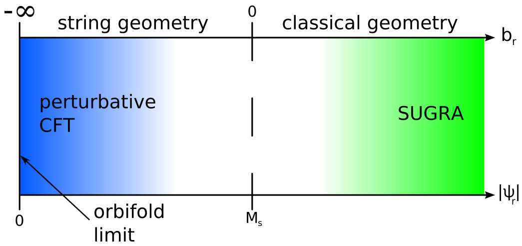
This can be identified with twisted blowup modes SGN,Nilles,Trapletti'07

$$\Psi_r \; = \; M_s \, e^{2\pi (b_r + i \beta_r)} \,, \qquad \delta \Psi_r \; = \; e^{2\pi i \, V_r^I \Lambda^I} \, \Psi_r \,,$$

whose VEV's generate the blowup . The b_r are related to the volumes of the blown up divisors E_r.

Axions, Kähler moduli and twisted blowup modes

The Kähler form: $J = a_i R_i - b_r E_r$ measures the volumes of the divisors R_i ,E_r .



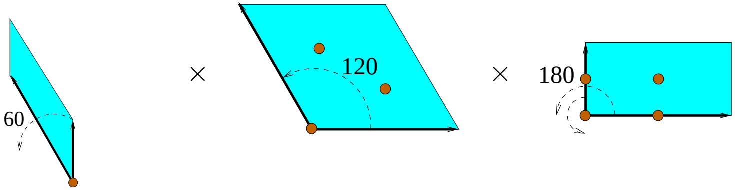
$$\Psi_r = M_s \, e^{2\pi(b_r + i\beta_r)} ,$$

The supergravity description start breaking down when $b_r \approx 0$ because then some volumes tend to become zero, but the orbifold regime is only reached when $b_r \to -\infty$.

$T^6/\mathbb{Z}_{6-\mathrm{II}}$ orbifold geometry

(For details see Michele Trapletti's talk)

T^2		T^2		T^2
(G_2 lattice)		($SU(3)$ lattice)		($SO(4)$ lattice)



$\theta \;:\; z_1 \to e^{2\pi i/6} z_1 \qquad z_2 \to e^{2\pi i/3} z_2 \qquad z_3 \to e^{\pi i} z_3$

The dots indicate the locations of the $\mathbb{Z}_{6-\mathrm{II}}$ orbifold fixed points in the three torus planes.

Example of a $T^6/\mathbb{Z}_{6-II}$ MSSM: Benchmark model 2

Gauge group:

$$SU(3) \times SU(2) \times U(1)_Y \quad \times \quad SO(8) \times SU(2)$$

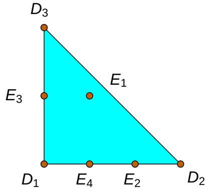
Spectrum: Lebedev,Nilles,Raby,Ratz,Ramos-Sanchez,Vaudrevange,Wingerter'06

#	irrep.	label	#	irrep.	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{1/6}$	q_i	3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{-2/3}$	$\bar{u}_i$
7	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{1/3}$	$\bar{d}_i$	4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/3}$	d_i
8	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{-1/2}$	ℓ_i	5	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{1/2}$	$\bar{\ell}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1}$	$\bar{e}_i$			
47	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{0}$	s_i^0	26	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{0}$	h_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1/2}$	s_i^+	20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/2}$	s_i^-
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{1/2}$	x_i^+	2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{-1/2}$	x_i^-
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/6}$	$\bar{\varphi}_i$	4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{1/6}$	φ_i
2	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})_{0}$	y_i	9	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{0}$	w_i
4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{0}$	m_i			

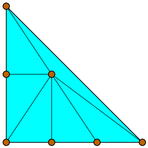
- vector-like exotics
- hidden sector matter
- states charged under both E_8's

The 5 resolutions of $\mathbb{C}^3/\mathbb{Z}_{6-\mathrm{II}}$

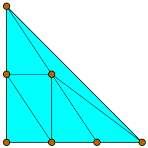
There are about 5^{12} resolutions of the $T^6/\mathbb{Z}_{6-\mathrm{II}}$ because the triangulation of the 12 $\mathbb{Z}_{6-\mathrm{II}}$ singularities are not unique :



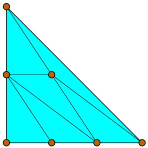
no triangulation



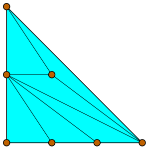
triangulation 1



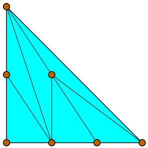
triangulation 2



triangulation 3



triangulation 4



triangulation 5

Bianchi identities

The Bianchi identities form a large set of 24 quadratic equations :

SGN,Held,Rühle,Trappletti,Vaudrevange'08

$$\sum_\gamma V_{3,1\gamma}^2 + 3\sum_\gamma V_{3,2\gamma}^2 = 24\,, \qquad \sum_\beta (V_{2,1\beta}; V_{4,1\beta}) + 2\sum_\beta (V_{2,3\beta}; V_{4,3\beta}) = 24\,,$$

$$3V_{2,1\beta}^2 + 4(V_{2,1\beta}; V_{4,1\beta}) = 12 + 3V_{2,1\beta}\cdot\sum_\gamma V_{1,\beta\gamma}\,,$$

$$6V_{4,1\beta}^2 + 2(V_{2,1\beta}; V_{4,1\beta}) = 12 + 3V_{4,1\beta}\cdot\sum_\gamma V_{1,\beta\gamma}\,,$$

$$2V_{3,1\gamma}^2 = 2 + V_{3,1\gamma}\cdot\sum_\beta V_{1,\beta\gamma}\,, \qquad 3V_{1,\beta\gamma}^2 = V_{3,1\gamma}^2 + (V_{2,1\beta}; V_{4,1\beta})\,,$$

where $(V_1; V_2) = V_1^2 + V_2^2 - V_1\cdot V_2$.

These Bianchi identities are very sensitive to the triangulations.
(Here only triangulation 1 has been used.)

Chiral spectrum

This resolution of the Benchmark model 2 has gauge group

$$U(1)^4 \times U(1)_Y \times SU(3) \times SU(2) \quad \times \quad U(1)^6 \times SU(4)$$

and the chiral spectrum reads:

visible E_8			
#	irrep	#	irrep
3 Q	$(\mathbf{3},\mathbf{2})_{1/6}$	3 U	$(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$
5 D	$(\overline{\mathbf{3}},\mathbf{1})_{1/3}$	2 $\bar{D}$	$(\mathbf{3},\mathbf{1})_{-1/3}$
5 L	$(\mathbf{1},\mathbf{2})_{-1/2}$	2 $\bar{L}$	$(\mathbf{1},\mathbf{2})_{1/2}$
6 E	$(\mathbf{1},\mathbf{1})_{1}$	1 $\bar{E}$	$(\mathbf{1},\mathbf{1})_{-1}$
17	$(\mathbf{1},\mathbf{1})_{0}$		

hidden E_8			
#	irrep	#	irrep
4	$(\mathbf{4})$	3	$(\mathbf{6})$
44	$(\mathbf{1})$		

Hence the spectrum is identical to that of three generations of Quarks and Leptons , except that we find two additional right-handed electrons

SGN,Held,Rühle,Trappletti,Vaudrevange'08

Hyper charge in blow up

The hyper charge is broken SGN,Held,Rühle,Trappletti,Vaudrevange'08

- from the smooth Calabi-Yau perspective , because it is not perpendicular to all the bundle vectors;

  ($U(1)_Y$ is of type i, i.e. part of the structure group Distler,Greene'88)
- from the orbifold perspective in full blowup , because there are fixed points with only MSSM charged twisted states .

This seems to be generic result, because all heterotic MSSM orbifolds in the "mini-landscape" have

- a Wilson line which is not orthogonal to the hypercharge (unless there is a Wilson line that in blowup leads to breaking of SU(2));
- fixed points with only MSSM charged matter .

Blowing up $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ GUT Orbifolds

(Work in progress. For details see Patrick Vaudrevange's talk)

The major reason why the hyper charge got broken was that in blow up it corresponds to a gauge flux that breaks the GUT to the SM is localized on a finite size divisor.

If the GUT breaking is realized via fully non-local effects on the blowup, then no gauge flux can be associated with the gauge symmetry breaking and so the hyper charge remains massless.

- this idea has put forward by Donagi,Ovrut, et al'99,'05 Hebecker,Trapletti'05

We explore this idea in the context of orbifold GUT model building and their subsequent blowup.

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ with freely acting $\mathbb{Z}_2$

We consider a $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold where the $\mathbb{Z}_2$ act as pure reflections

$$\theta_1(z_1, z_2, z_3) = (z_1, -z_2, -z_3) \quad \theta_2(z_1, z_2, z_3) = (-z_1, z_2, -z_3)$$

on the complex coordinates with unit radii.

- 64 $\mathbb{Z}_2 \times \mathbb{Z}_2'$ fixed points, each with 4 inequivalent resolutions
- 48 exceptional divisors in blowup

Modding out an additional freely acting $\mathbb{Z}_2$: Donagi,Wendland'08

$$t(z_1, z_2, z_3) = (z_1 + \frac{1}{2}, z_2 + \frac{1}{2}, z_3 + \frac{1}{2})$$

reduces the number of fixed points and exceptional divisors by half.

At the self-dual point should correspond to a Free Fermionic Model Faraggi et al

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ SU(5) GUT

We choose shifts and Wilson lines such that we arrive at a 4D model:

- SU(5) GUT
- chiral matter field come from twisted sectors only
- six generations of $\mathbf{10} + \overline{\mathbf{5}}$

In order that we can mod out the freely acting $\mathbb{Z}_2$ Wilson line W we find various consistency requirements

$$2\,V_i \equiv 2\,W_i \equiv 2\,W_i' \equiv 0 \qquad W_i' \equiv 2\,W$$

Modular invariance gives additional constraints on the freely acting Wilson line

$$2\,(a_i W_i + a_i' W_i' + W)^2 \equiv 0$$

Such conditions do not seem to be visible using the supergravity compactifications on Calabi-Yaus.

Conclusions

1. We have obtained compact orbifold resolutions
   - using toric geometry to describe the local resolutions,
   - which we after that glue together .
2. Abelian and non-Abelian gauge backgrounds can be related by changing VEVs in orbifold models.
3. We have a heterotic MSSM orbifold in full resolution .
   - But in any $T^6/\mathbb{Z}_{6-\mathrm{II}}$ mini-landscape MSSM the hyper charge is broken unless the orbifold is not completely blown up .
4. We are currently investigating $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ GUT models in which the SM is obtained by modding out a freely acting $\mathbb{Z}_2$ Wilson line.