Boost-invariant flow from string theory – near and far from equilibrium physics and AdS/CFT

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Idea

- heavy ion collisions @ RHIC - strongly coupled quark-gluon plasma (QGP)
- fully dynamical process - need for a new tool
- idea: exchange

  QCD in favor of $\mathcal{N} = 4$ SYM

  and use the gravity dual
- there are differences
  - SUSY
  - conformal symmetry at the quantum level
  - no confinement...
- ... but not very important at high temperature

Motivation

- RHIC suggests that QGP behaves as an **almost perfect fluid**
- there has been an enormous progress in understanding

  QGP hydrodynamics with the AdS/CFT

- can the AdS/CFT be used to shed light on

  far from equilibrium part of the QGP dynamics?

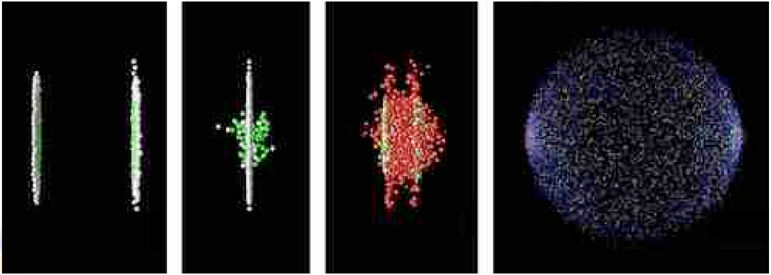
- maybe, but only at $\lambda \gg 1$!
- let's focus on

  the boost-invariant flow

  and use the AdS/CFT to learn about

  some near and far from equilibrium physics !

Boost-invariant dynamics



- one-dimensional expansion along the collision axis x^1
- natural coordinates
  - proper time τ and rapidity y
  - $x^0 = \tau \cosh y, \ x^1 = \tau \sinh y$
- boost invariance (no rapidity dependence)

AdS/CFT correspondence

Gauge-gravity duality is an equivalence between

$\mathcal{N} = 4$ **Supersymmetric Yang-Mills in** $\mathbb{R}^{1,3}$

- strong coupling
- non-perturbative results
- gauge theory operators

Superstrings in curved $AdS_5 \times S^5$ 10D spacetime

- (super)gravity regime
- classical behavior
- supergravity fields

AdS/CFT dictionary relates
energy-momentum tensor of $\mathcal{N} = 4$ SYM to 5D **AdS metric**

Gravity dual to the boost-invariant flow

- the energy-momentum tensor is specified by $\epsilon(\tau)$

$$T^{\mu\nu} = \mathrm{diag}\left\{\epsilon(\tau), -\frac{1}{\tau^2}\epsilon(\tau) - \frac{1}{\tau}\epsilon'(\tau), \epsilon(\tau) + \frac{1}{2}\tau\epsilon'(\tau)_{\perp}\right\}$$

- this suggests the metric Ansatz for the gravity dual

$$\mathrm{d}s^2 = \frac{-e^{a(\tau,z)}\mathrm{d}\tau^2 + \tau^2 e^{b(\tau,z)}\mathrm{d}y^2 + e^{c(\tau,z)}\mathrm{d}\mathbf{x}_{\perp}^2 + \mathrm{d}z^2}{z^2}$$

- Einstein equations

$$\mathcal{G}_{AB} = \mathcal{R}_{AB} - \frac{1}{2}\mathcal{R}\cdot g_{AB} - 6\, g_{AB} = 0$$

cannot be solved exactly ($\rightarrow$ numerics)

- however there are two regimes

$\tau \gg 1$ or $\tau \approx 0$

where analytic calculations can be done

$\tau \gg 1$ regime – hydrodynamics

Holographic reconstruction of space-time from $\epsilon(\tau) \sim \frac{1}{\tau^s}$

- Einstein eqns $\mathcal{G}_{AB}$ can be solved order by order in z^2, e.g.

$$a(\tau, z) = 0 + 0 \cdot z^2 + a_4(\tau) \cdot z^4 + a_6(\tau) \cdot z^6 + \ldots$$

where $a_4(\tau) = -\epsilon(\tau)$, $a_6(\tau) = -\frac{1}{4\tau}\epsilon'(\tau) - \frac{1}{12}\epsilon''(\tau)$, ...

- assuming $\epsilon(\tau) \sim \frac{1}{\tau^s}$ and choosing in each $a_{2k}(\tau)$ the leading contribution one ends up with $a(\tau, z) = a_{\mathrm{scaling}}\left(z \cdot \tau^{-s/4}\right)$, etc

- this reduces $\mathcal{G}_{AB}$ to solvable set of ODEs and then requiring regularity of $\mathcal{R}_{ABCD}\mathcal{R}^{ABCD}$ evaluated on $a, b, c_{\mathrm{scaling}}$

fixes s to be $\frac{4}{3}$ leading to $\epsilon(\tau) \sim \frac{1}{\tau^{4/3}}$

- $\epsilon(\tau) \sim \frac{1}{\tau^{4/3}}$ turns out to be the solution of hydrodynamics

Basics

- long-wavelength effective theory
- vast reduction of # degrees of freedom
  - velocity $\mathrm{u}^\mu(x)$ constrained by $\mathrm{u}^\mu \mathrm{u}_\mu = -1$
  - temperature $\mathrm{T}(x)$
- slow changes $\rightarrow$ gradient expansion
- expansion parameter $\frac{1}{\mathrm{L}\cdot\mathrm{T}}$
  (T is temperature, L is characteristic length-scale)

Gradient expansion

- definition of the energy-momentum tensor

$$T^{\mu\nu} = \epsilon\cdot\mathrm{u}^\mu\mathrm{u}^\nu + \mathrm{p}\cdot\Delta^{\mu\nu} - \eta\cdot\left(\Delta^{\mu\lambda}\nabla_\lambda\mathrm{u}^\nu + \Delta^{\nu\lambda}\nabla_\lambda\mathrm{u}^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla^\lambda\mathrm{u}_\lambda\right) + \ldots$$

- EOMs $\nabla_\mu T^{\mu\nu} = 0$ + equation of state (e.g. $\epsilon = 3p$)

Perfect hydrodynamics

- in conformal boost invariant hydrodynamics

$$\epsilon(\tau) \sim \mathrm{T}(\tau)^4, \quad \mathrm{u}^\mu = 1 \cdot [\partial_\tau]^\mu, \quad \eta_{\mu\nu} = \mathrm{diag}\left\{-1, \tau^2, 1, 1\right\}$$

- perfect hydro ($\nabla_\mu T^{\mu\nu} = 0$ for $T^{\mu\nu} = \epsilon \cdot \mathrm{u}^\mu \mathrm{u}^\nu + \mathrm{p} \cdot \Delta^{\mu\nu}$) gives

$$\partial_\tau \epsilon(\tau) = -\frac{\epsilon(\tau) + p(\tau)}{\tau}$$

- which together with $\epsilon = 3p$ leads to $\epsilon \sim \frac{1}{\tau^{4/3}}$

Gradient expansion

- *remainder:* in hydro the expansion parameter is $\frac{1}{\mathrm{L} \cdot \mathrm{T}}$
- in this setting $\mathrm{T} \sim \tau^{-1/3}$, $\mathrm{L}^{-1} \sim \nabla \mathrm{u} = \tau^{-1}$, so $\frac{1}{\mathrm{L} \cdot \mathrm{T}} \sim \frac{1}{\tau^{2/3}}$
- one should expect the general structure of $\epsilon(\tau)$ of the form

$$\epsilon(\tau) \sim \frac{1}{\tau^{4/3}} \left\{ \#_0 + \frac{1}{\tau^{2/3}} \#_1 + \frac{1}{\tau^{4/3}} \#_2 + \ldots \right\}$$

Boost-invariant flow and gradient expansion

Reminder:

$$\mathrm{d}s^2 = \frac{-e^{a(\tau,z)}\mathrm{d}\tau^2 + \tau^2 e^{b(\tau,z)}\mathrm{d}y^2 + e^{c(\tau,z)}\mathrm{d}x_\perp^2 + \mathrm{d}z^2}{z^2}$$

Gravitational gradient expansion:

$$a(\tau,z) = a_0\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{2/3}}a_1\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{4/3}}a_2\left(\frac{z}{\tau^{1/3}}\right) + \dots$$

$$b(\tau,z) = b_0\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{2/3}}b_1\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{4/3}}b_2\left(\frac{z}{\tau^{1/3}}\right) + \dots$$

$$c(\tau,z) = c_0\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{2/3}}c_1\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{4/3}}c_2\left(\frac{z}{\tau^{1/3}}\right) + \dots$$

$$\mathcal{R}^2(\tau,z) = \mathcal{R}_0^2(\frac{z}{\tau^{1/3}}) + \frac{1}{\tau^{2/3}}\mathcal{R}_1^2(\frac{z}{\tau^{1/3}}) + \frac{1}{\tau^{4/3}}\mathcal{R}_2^2(\frac{z}{\tau^{1/3}}) + \dots$$

This is AdS counterpart of hydrodynamics

$$\epsilon(\tau) = \left(\frac{N_c^2}{2\pi^2}\right)\frac{1}{\tau^{4/3}}\left\{1-2\eta_0\frac{1}{\tau^{2/3}}+[\frac{3}{2}\eta_0^2-\frac{2}{3}(\eta_0\tau_\Pi^0-\lambda_1^0)]\frac{1}{\tau^{4/3}}+\cdots\right\}$$

Further developments and why AdS/CFT is useful

Further developents

- corrections to the transport coefficients from finite λ and N_c

$$"S = \frac{1}{2l_p^3} \int \det g \, \left\{ \mathcal{R} + \frac{12}{L^2} + \gamma \cdot L^2 \, \mathrm{Weyl}^2 + \delta \cdot L^6 \, \mathrm{Weyl}^4 \right\}"$$

- apparent, event horizons and slow evolution

Why useful?

- transport properties at strong coupling
- inspired the correct formulation of second order hydro
- implications in GR as well (fluid/gravity correspondence)

$\tau \approx 0$ regime – dynamics far from equilibrium

Initial conditions and early times expansion of $\epsilon(\tau)$

- warp factors can be solved near the boundary given $\epsilon(\tau)$

$$a(\tau, z) = -\epsilon(\tau) \cdot z^4 + \left\{ -\frac{\epsilon'(\tau)}{4\tau} - \frac{\epsilon''(\tau)}{12} \right\} \cdot z^6 + \ldots$$

- for $\epsilon(\tau) = \epsilon_0 + \epsilon_1 \tau + \epsilon_2 \tau^2 + \epsilon_3 \tau^3 + \epsilon_4 \tau^4 + \epsilon_5 \tau^5 + \ldots$
  all ϵ_{2k+1} must vanish, otherwise $a(0, z) \to \infty$
- setting τ to zero in $a(\tau, z)$ for

$$\epsilon(\tau) = \epsilon_0 + \epsilon_2 \tau^2 + \epsilon_4 \tau^4 + \ldots$$

gives

$$a(0, z) = a_0(z) = \epsilon_0 \cdot z^4 + \frac{2}{3}\epsilon_2 \cdot z^6 + \left(\frac{\epsilon_4}{2} - \frac{\epsilon_0^2}{6} \right) \cdot z^8 + \ldots$$

- it defines map between initial profiles in the bulk and $\epsilon(\tau)$

Resummation of the energy density

- energy density power series @ $\tau = 0$

$$\epsilon(\tau) = \epsilon_0 + \epsilon_2 \tau^2 + \ldots + \epsilon_{2N_{cut}} \tau^{2N_{cut}} + \ldots$$

has a finite radius of convergence and thus

a resummation is needed

- presumably the simplest can be given by Pade approximation

$$\epsilon_{\text{approx}}(\tau)^3 = \frac{\epsilon_U^{(0)} + \epsilon_U^{(2)} \tau^2 + \ldots + \epsilon_U^{(N_{cut}-2)} \tau^{N_{cut}-2}}{\epsilon_D^{(0)} + \epsilon_D^{(2)} \tau^2 + \ldots + \epsilon_D^{(N_{cut}-2)} \tau^{N_{cut}+2}}$$

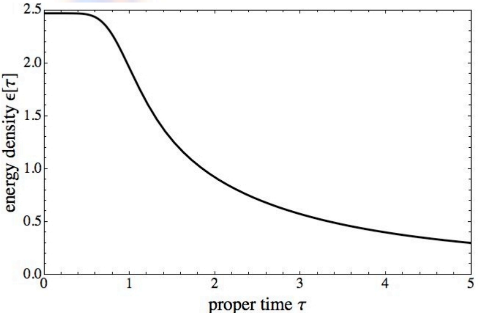
which uses the uniqueness of the asymptotic behavior

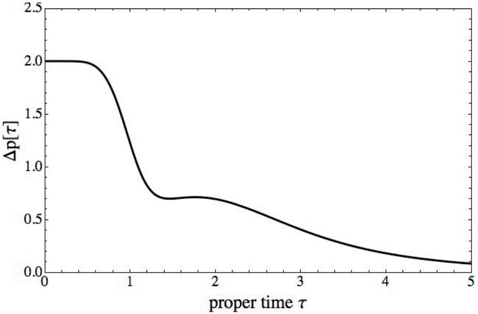
$$\epsilon \sim \frac{1}{\tau^{4/3}}$$

- nice example of initial data in the bulk is given by

$$a(0,z) = b(0,z) = 2\log\left\{\cos\frac{\pi}{2}z^2\right\} \quad \text{and} \quad c(\tau,z) = 2\log\left\{\cosh\frac{\pi}{2}z^2\right\}$$

leading to the following $\epsilon(\tau)$ and $\Delta p(\tau) = 1 - \frac{p_\parallel(\tau)}{p_\perp(\tau)}$





Summary

Results:

- AdS/CFT is indispensable not only near equilibrium
- Gauge/gravity duality may serve as a definition of

  strongly coupled non-equilibrium gauge theory

- transport properties of various plasmas at strong coupling
- estimates of thermalization time

Open questions:

- towards colliding shock-waves
- applications of non-conformal gauge/gravity dualities