

# Gauge-top unification

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based on Pierre Hosteins, RK, M. Ratz, K. Schmidt-Hoberg  
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June 18th 2009

# Outline

Motivation

GUTs in extra dimensions

String derived models

Phenomenological implications

Summary

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- Gauge-top unification in string derived models is **interesting**
- Consequences can give a chance to exclude models

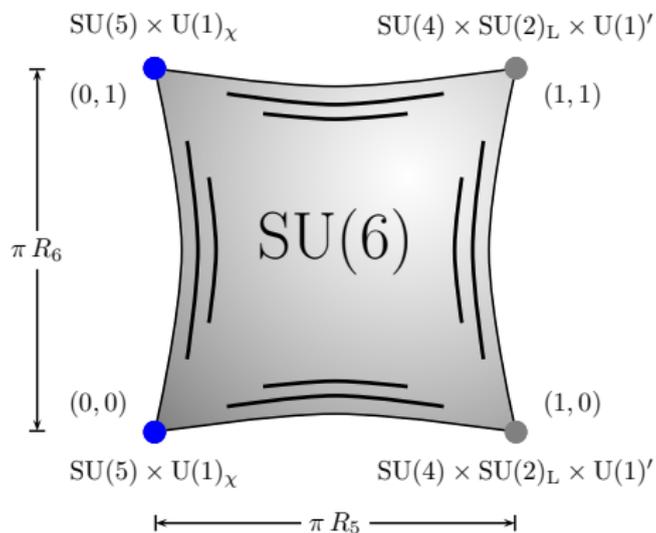
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We get the tree level relation  $y_t = g$  (large top Yukawa coupling)

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$$Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathcal{O}(g) \end{pmatrix} + \begin{pmatrix} s^{n_{11}} & s^{n_{12}} & s^{n_{13}} \\ s^{n_{21}} & s^{n_{22}} & s^{n_{23}} \\ s^{n_{31}} & s^{n_{32}} & s^{n_{33}} \end{pmatrix}$$

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Topic of this talk!

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- Effect even occurs when the effective FI term in 4D vanishes  $\Rightarrow$  **Local effect!** (compare to F-theory)

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- Zero mode profile:

$$\psi \simeq f \prod_I \left| \vartheta_1 \left( \frac{z - z_I}{2\pi} \middle| \tau \right) \right|^{\frac{1}{2\pi} g_6 q_\psi \xi_I} \exp \left( -\frac{1}{8\pi^2 \tau_2} g_6 q_\psi \xi_I (\text{Im}(z - z_I))^2 \right)$$

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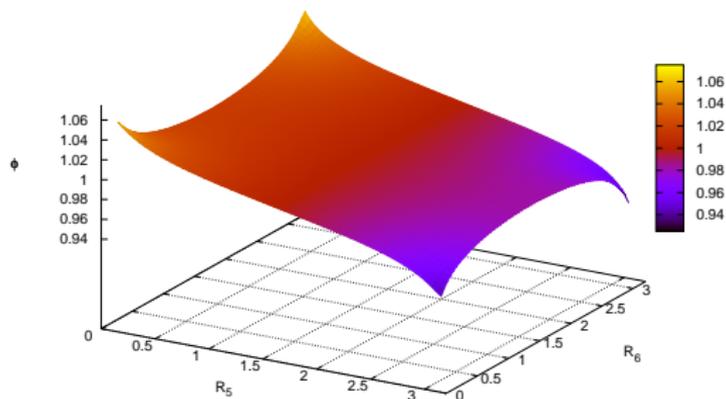
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- $\xi_I$  is the FI term:

$$\xi_I = \frac{1}{16\pi^2} g_6 \Lambda^2 \text{tr}(q_I), \quad \Lambda = \text{UV cutoff}$$

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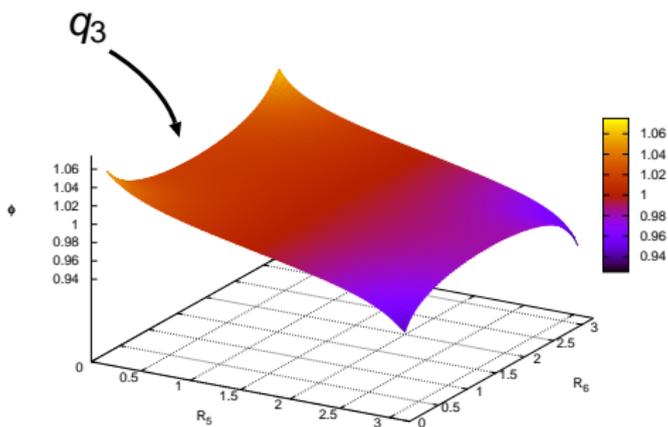
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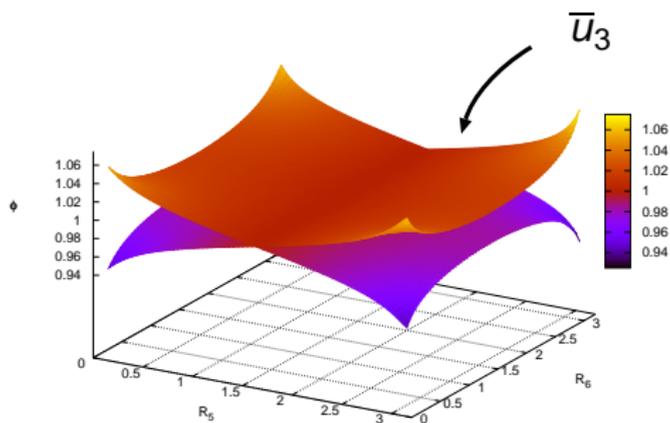
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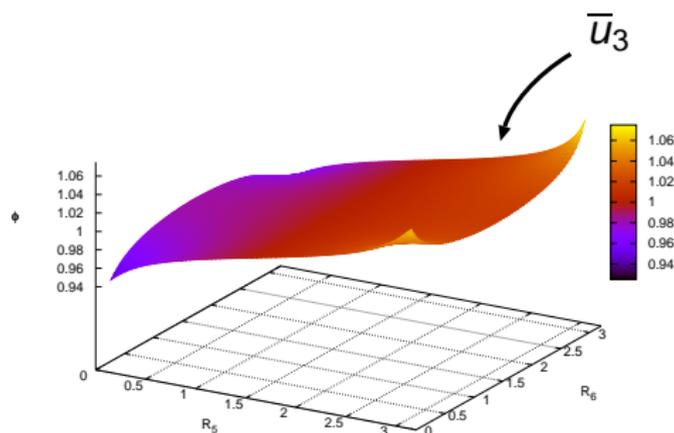


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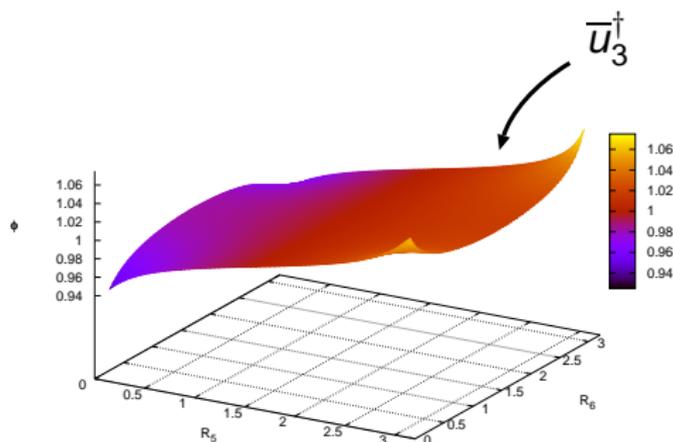
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Overlap integrals differ  $\Rightarrow y_t < g$

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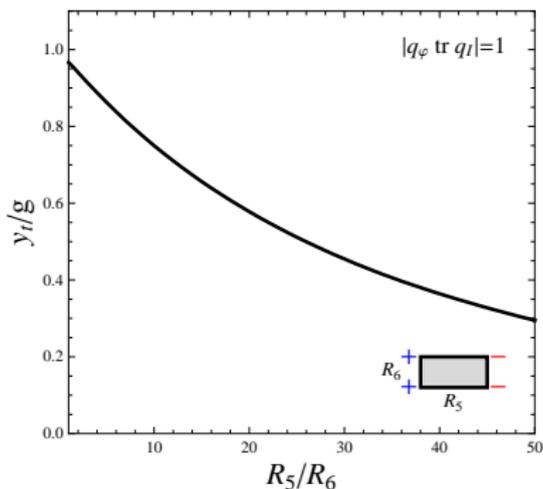
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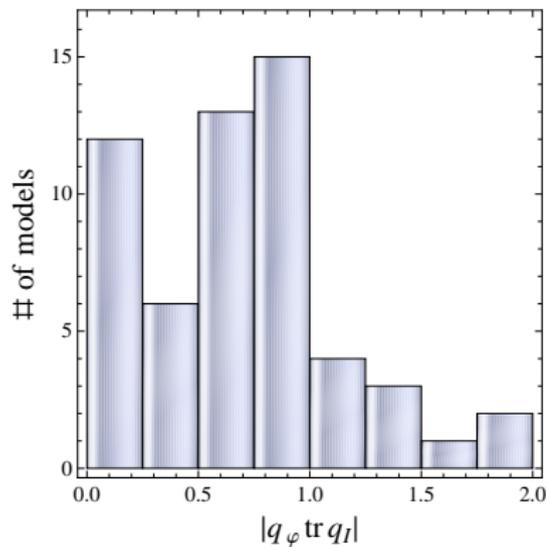
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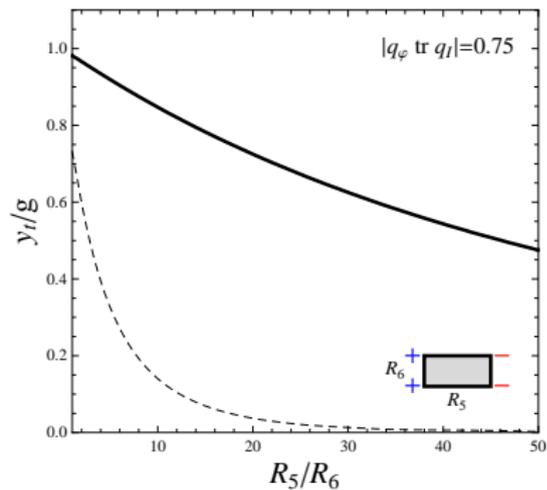
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- Similar effects should also occur in non heterotic GUTs

# Different models in heterotic string theory

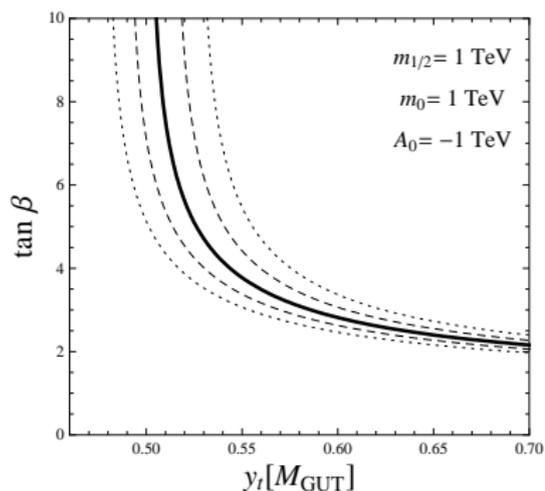


(a)  $|q_\varphi \text{tr} q_I|$  at  $l = (0, 0)$ .

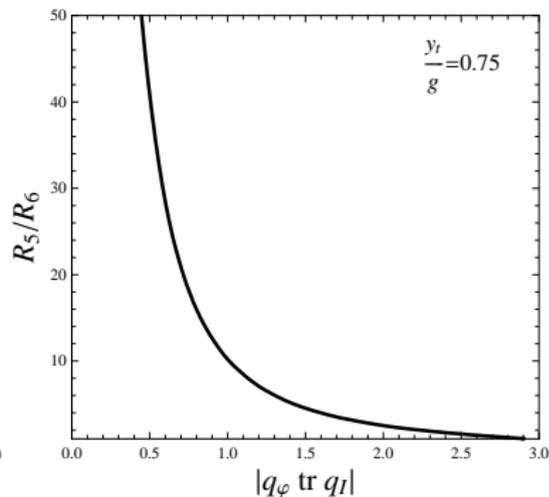


(b)  $y_t/g$ .

# $\tan \beta$ can be related to the extradimensions



(c)  $y_t$  vs.  $\tan \beta$ .



(d)  $R_5/R_6$  vs.  $|q_\varphi \text{ tr } q_I|$ .

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This has other important implications  $\Rightarrow$  Yukawa pattern, Gauge thresholds, etc.

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