Attraction to a radiation era and moduli stabilization in string cosmology

Hervé Partouche
Ecole Polytechnique

in collaboration with
F. Bourliot, J. Estes, T. Catelin-Jullien,
C. Kounnas and N. Toumbas

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Our approach

- Start from a static Universe, an "empty box".
- Fill it with a thermalized gas of states.
- The pressure will back-react on the walls of the box.
- A quasi-static evolution emerges, i.e. a succession of states in thermal equilibrium.

Plan

- Comparison : field vs. string theory.
- Determine the back-reaction.
- Attraction mechanisms to radiation eras.
- Space-time dimension dynamically stabilized.
- Stabilization of moduli.

Field theory versus String theory

- For a single bosonic degree of freedom of mass M :
- The quantum canonical ensemble can be studied with a Euclidean path integral on $S^1_\beta \times T^3$ with periodic B.C. along S^1_β (antiperiodic for a fermion) :

$$Z = \mathrm{Tr}\, e^{-\beta H} = \int \mathcal{D}\phi\, e^{-\int_0^\beta d\tau \int d^3x\, \phi(-\Box + M^2)\phi + \cancel{interactions}}$$

$$F = -\frac{\ln Z}{\beta} = \frac{V}{\beta}\int \frac{d^3k}{(2\pi)^3} \ln(1 - e^{-\beta\omega_k}) + V\int \frac{d^3k}{(2\pi)^3}\left(\frac{\omega_k}{2} + \infty\right)$$

$$\omega_k = \sqrt{\vec{k}^2 + M^2}$$

β-dependant

- In terms of Feynman diagrams,

$$F = -\frac{\ln Z}{\beta} = -\frac{1}{\beta}\left(Z^{(c)}_{1\text{-}loop} + high\cancel{e}r\ loops\right)$$

$$= -\int_0^{+\infty} \frac{dl}{2l}\,\frac{V}{(2\pi l)^{3/2}}\, e^{-\frac{M^2 l}{2}}\,\frac{1}{\sqrt{2\pi l}} \sum_{\tilde{m}_0} e^{-\frac{\beta^2 \tilde{m}_0^2}{2l}}$$

- The bosonic loop can be wrapped $\tilde{m}_0$ times along S^1_β .
- Finite in the I.F. as long as there is no tachyon.
- Divergent in the U.V. ($\tilde{m}_0 = 0\,, l \to 0$).
- The genus one computation in string theory should be finite in the U.V. because of the integration on the fundamental domain.

Examples

- The bosonic string has a tachyon even at T = 0. [F in bosonic string: Polchinski '86]
- Consider $\mathcal{N}$ = 1,2,4 susy heterotic models in 4D.

$\ast$ Compute $F = -\frac{Z_{genus\text{-}1}}{\beta}$ in the Euclidean background $S^1(R_0) \times T^3 \times \mathcal{M}_6$, where $\beta = 2\pi R_0$.

$\ast$ The string result regularizes the field theory one :

$$-\int_0^{+\infty} \frac{dl}{2l} \frac{V}{(2\pi l)^{3/2}} e^{-\frac{M^2 l}{2}} \frac{1}{\sqrt{2\pi l}} \sum_{\tilde{m}_0} e^{-\frac{\beta^2 \tilde{m}_0^2}{2l}} (-)^{a\tilde{m}_0}$$

(a=0 for bosons, a=1 for fermions)

$$-\int_{\mathcal{F}} \frac{d\tau_1 d\tau_2}{2\tau_2} \frac{V}{(2\pi)^3 \tau_2^{3/2}} \sum_{spectrum} e^{-\pi M^2 \tau_2} \frac{1}{2\pi\sqrt{\tau_2}} \sum_{\tilde{m}_0, n_0} e^{-\frac{\pi R_0^2}{\tau_2}|\tilde{m}_0 + n_0\tau|^2} (-)^{a\tilde{m}_0 + bn_0 + \tilde{m}_0 n_0}$$

Oscillators and internal lattice

$\ast$ Reversed GSO when the winding n_0 is odd :
A tachyon occurs for $$\frac{1}{R_H} < R_0 < R_H = \frac{1+\sqrt{2}}{2}$$

We restrict to $R_0 \gg 1$ to avoid the Hagedorn transition.

$\ast$ $e^{-\pi R_0^2 n_0^2 \tau_2}$ is exponentially small, except if $n_0 = 0$.

$\ast$ $(\tilde{m}_0, n_0)$ both even is susy : no contribution.
We are left with $(2\tilde{k}_0 + 1, 0)$.

$\ast$ Redef $\tau_2 = y\pi R_0^2(2\tilde{k}_0 + 1) \implies e^{-\pi M^2 \tau_2}$ is exponentially suppressed, except if $M = 0$.

$\ast$ Finally,

Number of massless boson/fermion pairs

$$P_{st} \equiv -\frac{\partial F}{\partial V} = n_T \, T_{st}^4 \, \frac{\pi^2}{48} \quad \text{where} \quad T_{st} = \frac{1}{2\pi R_0}$$

i.e. Stefan's law.

- For phenomenology: Consider a non-susy 4D model, Heterotic $\mathcal{N} = 1 \to 0$ spontaneously. And thermalize.

$\ast$ To implement the temperature, we have imposed

$$\varphi(t_E) = (-)^{a\tilde{m}_0}\varphi(t_E + 2\pi R_0 \tilde{m}_0)$$

[In field theory: Scherk, Schwarz '79]
[In string theory: Kounnas, Rostand '90]

Momentum $\dfrac{\tilde{m}_0 + \frac{a}{2}}{R_0}$ $\Rightarrow$ mass shift $\dfrac{1}{2\pi R_0} = T_{st}$

$\ast$ Compactify on $\dfrac{T^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$ and impose

$$\varphi(x^4) = (-)^{(a+Q)\tilde{m}_4}\varphi(x^4 + 2\pi R_4 \tilde{m}_4)$$

R-sym charge

$\Rightarrow$ mass shift $\dfrac{1}{2\pi R_4} = M_{st}$

susy breaking scale

$\ast$ To avoid Hagedorn-like transitions: $R_0, R_4 \gg 1,$

$$P_{st} = T_{st}^4 \left(n_T f_T(z) + n_T^{tw} \frac{\pi^2}{48} \right) + M_{st}^4 n_V f_V(z)$$

where $e^z = \dfrac{M_{st}}{T_{st}} = \dfrac{R_0}{R_4}$ is a "complex structure".

$$f_T(z) = \frac{\Gamma(5/2)}{\pi^{5/2}} \sum_{\tilde{k}_0, \tilde{k}_4} \frac{e^{4z}}{\left[e^{2z}(2\tilde{k}_0 + 1)^2 + (2\tilde{k}_4)^2 \right]^{5/2}}$$

We can rewrite : $P_{st} = T_{st}^4 \, p(z)$

Back-reaction:

✳ The pressure of the thermal gas of strings now pushes the walls of the "3D box".

$$S_{genus\text{-}0} = \int d^4x\,\sqrt{-G}\,e^{-2\phi_{dil}}\left[\frac{R}{2} + 2(\partial\phi_{dil})^2 + \frac{1}{2}(\partial \ln R_4)^2 + \cdots\right]$$

With originally constant dilaton, R_4 and metric :

back to Lorentzian

$$ds^2_{st} = -(2\pi R_0)^2 dt^2 + \underbrace{(2\pi R_{box})^2}_{a^2_{st}} \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right]$$

$$N^2_{st} = \frac{1}{T^2_{st}}$$: The laps function is the inverse temperature

$$S_{genus\text{-}1} = S_{genus\text{-}0} + \int d^4x\,\sqrt{-G}\,\frac{Z_{genus\text{-}1}}{V\beta}$$

The 1-loop correction to the orginally vanishing vacuum energy. It is our P_{st} .

NB: We don't need to compute the 1-loop corrections to the kinetic terms. (They can be absorbed by wave function redefinitions and translate into corrections to the 1-loop P at second order only.)

$\ast$ In Einstein frame:

$$S_{genus\text{-}1} = \int d^4x\,\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}(\partial\phi_\perp)^2 + \cdots + P\right]$$

where $\Phi, \phi_\perp$ are linear combinations of $\phi_{dil}, \ln R_4$,

$$P = T^4 p(z) \quad , \quad T = \frac{1}{2\pi R_0 e^{-\phi_{dil}}}$$

$$e^z = \frac{R_0}{R_4} = \frac{M}{T} \; , \; M = \frac{1}{2\pi R_4 e^{-\phi_{dil}}} \equiv \frac{e^{\sqrt{\frac{3}{2}}\Phi}}{2\pi}$$

$\ast$ For simplicity, we restrict to homogeneous and isotropic extrema of this action, which involve non-trivial backgrounds for $N, a, \Phi, \phi_\perp$ only (we partially relax this hypothesis later).

$$\frac{\delta S_{genus\text{-}1}}{\delta N} = 0 \quad \Longrightarrow \quad \frac{3}{N^2} H^2 = \rho + \frac{1}{2N^2}\dot{\Phi}^2 + \frac{1}{2N^2}\dot{\phi}_\perp^2$$

where $\rho = -P - N\dfrac{\partial P}{\partial N} \equiv -P + T\dfrac{\partial P}{\partial T} \equiv \dfrac{1}{V}\dfrac{\partial(\beta F)}{\partial \beta}$

NB: Variational principle and thermodynamics are consistent.

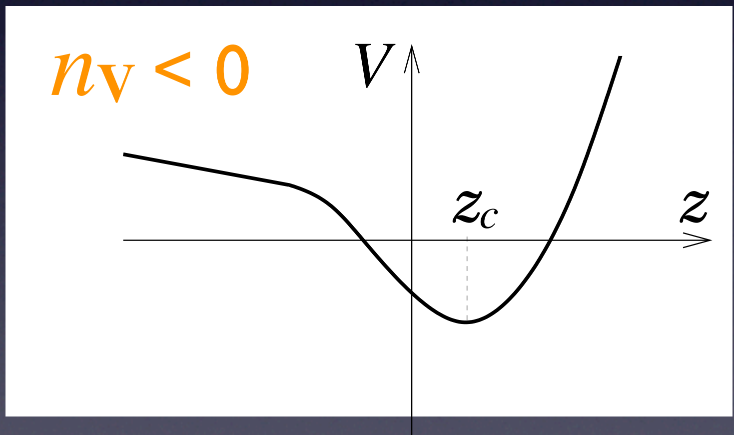
Attractors :

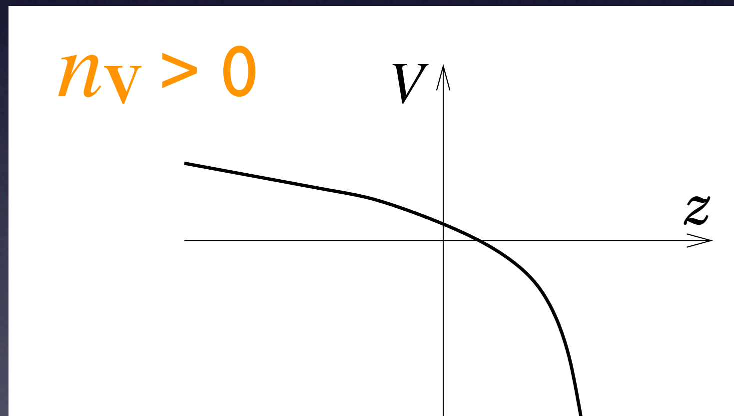
$\ast$ Write the Einstein equations coupled to $\Phi, \phi_\perp$ in presence of sources ρ, P .

z-equation: $$\mathcal{E}(z, \mathring{z}, \mathring{\mathring{z}}, \mathring{\phi}_\perp) + \frac{dV}{dz} = 0$$

$$r(z) - 4p(z)$$
$$(\rho = T^4 r(z), P = T^4 p(z))$$

where $e^z = \dfrac{M}{T}$ and $\circ \equiv \dfrac{d}{d \ln a}$.





$\ast$ Case $n_V < 0$: For generic I.B.C., there is an attraction to a particular solution, $z(t) \to z_c, \ \phi_\perp(t) \to cst.$, with eventually damped oscillations.

- After convergence,

$$M(t) \propto T(t) \propto \frac{1}{a(t)}$$ i.e. R_4, R_0, R_{box} proportional !

with Friedmann eq. $$3H^2 = c\,\frac{p(z_c)}{a^4}$$

$\Longrightarrow$ Attraction to an effective radiation era.

- A paradox: $r(z_c) = 4\,p(z_c) \Longrightarrow \rho = 4P$

$$\rho_{tot} \equiv \rho + \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}\dot{\phi}_\perp^2 = 3\left(P + \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}\dot{\phi}_\perp^2\right) \equiv 3P_{tot}$$

state equation for radiation in 4D

- Consistency : What perturbs the "static box" is

$$P_{tot}(t) \to 0 + 0 + 0$$

$\ast$ Case $n_V > 0$: Attraction to a run away solution $z(t) \to +\infty$ describing an era of contraction. (We should have started from a small box.)

$\ast$ In all Cases: If I.B.C. such that $z \ll -1$ i.e. $R_4 \gg R_0 \gg 1$, the KK in the direction 4 become continuous.

$\Longrightarrow$ The system is better understood in 5D, but anisotropic :

$$ds^2 = -\underbrace{N'^2}_{\frac{1}{T'^2}} dt^2 + a'^2 \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right] + \underbrace{(e^{-\frac{4}{3}\phi'_{dil}} 2\pi R_4)^2}_{b^2} (dx^4)^2$$

- Our fields are $\{T', a', b, \phi_{dil}\}$ and the pressure in 5D satisfies Stefan's law $P' = (n_T + n_T^{tw})\, T'^5\, \frac{93\zeta(5)}{64\pi^2}$.

- For arbitrary I.B.C., the solution converges to the effective radiation era $T'(t) \propto \frac{1}{a'(t)} \propto \frac{1}{b(t)}, \quad \phi'_{dil}(t) = cst$

Spontaneous enhancement of Lorentz group: $SO(1,3) \times U(1) \to SO(1,4)$

Moduli stabilization ?

$\ast$ We have considered the dynamics of 2 susy breaking moduli, R_0, R_4; supposing the others are frozen. Let us relax this hyp.

$\ast$ For one more dynamical susy-breaking modulus R_5 :

$$T = \frac{1}{2\pi R_0\, e^{-\phi_{dil}}} \qquad\qquad M = \frac{1}{2\pi\sqrt{R_4 R_5}\, e^{-\phi_{dil}}}$$

$$P = T^4\, p(z, Z)$$

$$e^z = \frac{M}{T} = \frac{R_0}{\sqrt{R_4 R_5}}$$ and $$e^Z = \frac{R_5}{R_4}$$ are complex structures

-There is an attraction to an effective radiation era, where $R_0(t) \propto R_4(t) \propto R_5(t)$ i.e. $(z, Z) \equiv (z_c, Z_c)$ are stabilized.

$\ast$ For one more dynamical non-susy-breaking modulus :

$$R_0 \gg 1, \qquad R_{box}, \qquad 0 < R_4 < +\infty, \qquad R_9 \gg 1, \qquad \phi_{dil}$$

$$R_0 \longrightarrow \frac{1}{T} \qquad R_4 \longrightarrow R_4 \leftrightarrow \frac{1}{R_4} \qquad R_9 \longrightarrow \frac{1}{M}$$

- The pressure is $P = T^4\, p\left(z, \frac{R_4}{R_9}, \frac{R_4}{\sqrt{\alpha'}}\right)$

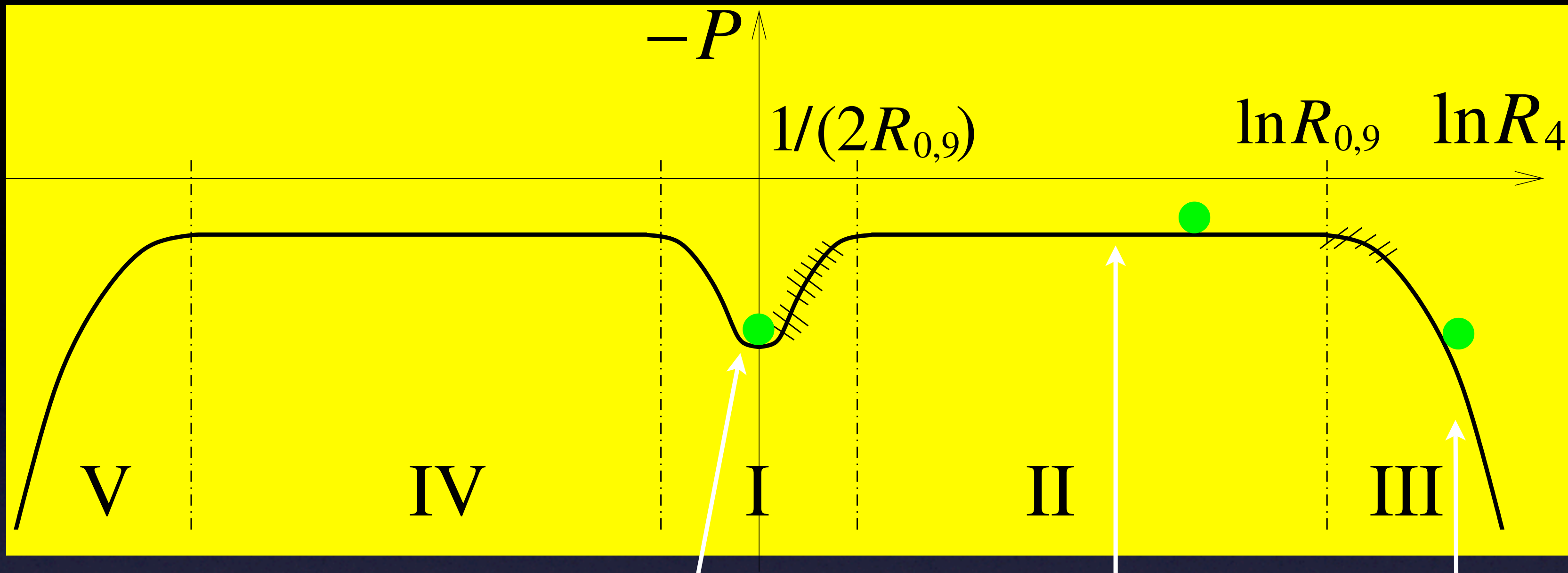
$\frac{R_4}{R_9}$: small / large $\Longrightarrow$ 4D or 5D

$\frac{R_4}{\sqrt{\alpha'}}$: If $\simeq 1$: enhanced symmetry point,

$U(1) \rightarrow SU(2)$

massless winding modes

- The potential for R_4 is $-P$. At fixed T, z, R_9 one has:



Up to $\mathcal{O}(e^{-R_{0,9}})$ terms:

4 dim

$P \simeq T^4 p(z)$

with n_T, n_V

4 dim

$U(1) \to SU(2)$

$\Longrightarrow n_T+2, n_V+2$

Connected by stringy effects

5 dim

$P' \simeq T'^5 p'(z)$

with n_T, n_V

Phase I : - The Kähler modulus R_4 is a Higgs field.

- Damped oscillations $\Longrightarrow$ Stabilized.
- The attractor is a radiation era in 4D.

Phase II : $\neq$ basins of attraction:

a) Friction dominates $\Longrightarrow (\ln R_4)^\circ \to 0$.

- gets stuck anywhere on the plateau.
- Not stabilized but frozen.
- The attractor is a radiation era in 4D.

b) Friction negligible $\Longrightarrow (\ln R_4)^\circ \to cst$.

- R_4 catches the moving edges and "falls".

Phase III : - Run away of the Kähler modulus R_4

i.e. spontaneous decompactification.

- The attractor is a radiation era in isotropic 5D.

Summary

- Consider a flat and static background. At 1-loop, it is cosmological, due to finite temperature effects.
- The free energy can be computed at the string level : It is free of singularity (not in field theory).
- We have not described in this talk the "Hagedorn era". However, for arbitrary I.B.C. at the time we exit this era, the evolution of the universe is attracted to a radiation era.
- The space-time dimension is dynamically stabilized.
- Kähler and complex structure moduli can be stabilized.
- Our approach is valid till $M \simeq Q_0$, the E.W. transmutation scale. There, large radiative corrections should induce the EW breaking and stabilize M. (work in progress)