Non-perturbative effects and wall crossing in 4d N=1,2 string vacua

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Based on I. Garcia-Etxebarria, A.U., arXiv:0711.1430
I. Garcia-Etxebarria, F. Marchesano, A.U. arXiv:0805.0713
A.U. arXiv:0808.2918
A. Collinucci, P. Soler, A.U., arXiv:09041133

Motivation

D-brane instantons as important ingredients in String Pheno

Moduli stabilization, violation of global U(1)'s,...

Need good knowledge of their properties, e.g.

- Understand D-brane instanton effects, globally in moduli space

    Solve certain puzzles involving wall crossing

    Provide powerful criteria for the generation of superpotentials

- Understand instanton sum, beyond individual contributions

    Multi-instantons (aka poly-inst)

    Multiply wrapped instantons

- Understand connection with topological string

    Wall crossing behaviour manifest

    New computational tools in 4d N=2,1

D-brane instantons and wall crossing

- Non-perturbative 4d effective action from D-brane instantons

    BPS instantons ⇒ 4d F-terms (“holomorphic”) e.g. superpotential

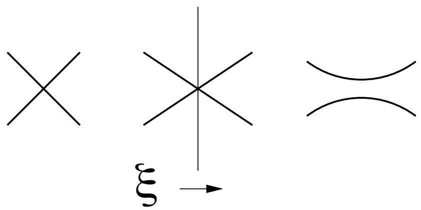
    Non-BPS instantons ⇒ 4d D-terms (“non-holomorphic”)

- List of BPS D-brane instantons can jump discontinuously by moving one real parameter closed string modulus: Walls of BPS stability

    Closed string modulus couples as FI-term
    Fayet model on worldvolume of D-brane instanton

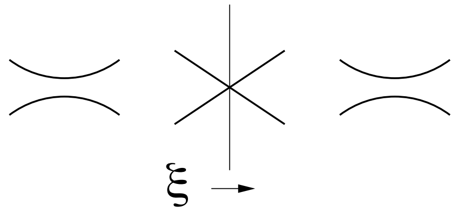
<u>Marginal stability</u>:
BPS ⇒ non-BPS

$$V_D = (|\phi|^2 - \xi)^2$$



<u>Threshold stability</u>:
BPS ⇒ split BPS

$$V_D = (|\phi_1|^2 - |\phi_2|^2 - \xi)^2$$



- Fate of non-perturbative terms upon wall crossing?

- Clue to the resolution:
  Number of (exact) fermion zero modes for D-brane instanton

  - Determines the kind of 4d superspace interaction

  - Is topological (continuous upon change of parameters)

  - Must include Goldstinos, at least 2 for BPS, 4 for non-BPS

Application to non-perturbative superpotentials

- Generated by BPS instantons with exactly 2 fermion zero modes

- Instantons contributing to superpotential can never become non-BPS

Not enough fermion zero modes to account for 4 required goldstinos

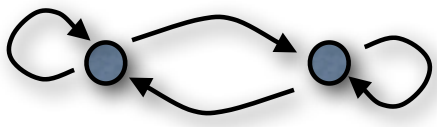
$\Rightarrow$ Safe against marginal wall crossing

$\Rightarrow$ Powerful criterion: Any instanton which can become non-BPS cannot contribute to superpotential

- What about threshold stability?

Threshold stability and non-pert. superpotentials

- BPS instanton splits into two BPS instantons



2-instanton process: 0-dim quiver theory

Ex: Translational Goldstones x_1,x_2;"Goldstinos" θ_1, $\tilde{\theta}_1$, θ_2;
bi-fundamental hyperm. Φ_{12}, Φ_{21} ie φ_{12}, φ_{21}, χ_{12}, χ_{21}

Contribution to superpotential localize onto $x_1=x_2$, couplings are

$$(\chi_{12}\,(\theta_1-\theta_2))\varphi_{12}^* - (\chi_{21}\,(\theta_1-\theta_2))\varphi_{21}^* + (\bar{\chi}_{12}\tilde{\theta})\varphi_{12} - (\bar{\chi}_{21}\tilde{\theta})\varphi_{21}$$

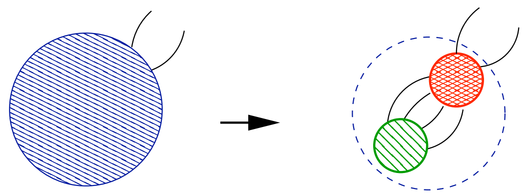
$$\chi_{12}\varphi_{21}\chi_{12}\varphi_{21} + 2\chi_{12}\chi_{21}\varphi_{12}\varphi_{21} + \varphi_{12}\chi_{21}\varphi_{12}\chi_{21} \quad + \mathrm{h.c.}$$

All fermions couple except for the overall Goldstinos $\theta_1+\theta_2$

Pull down interactions in exp(-S_{inst}) and soak up zero modes

We recover $$S_{4d} \simeq \int d^4\, d^2\theta\, e^{-(T_1+T_2)} = \int d^4\, d^2\theta\, e^{-T}$$

- Decay products combine into 2-instanton process reconstructing amplitude before decay



Marginal stability and 4d N=1 higher F-terms

- Global picture for instantons generating higher F-terms

- Non-perturbative higher F-terms are continuous across general lines of marginal stability (BPS ⇒ non-BPS)

- Consistent w/ standard wisdom of BPS=F-term, non-BPS=D-term

Inst. amplitude as 4d operator in non-trivial Beasley-Witten cohomology:
- Locally in moduli space, can be written as a D-term
- Obstruction (localized on BPS locus) to write as global D-term

Holomorphy of higher F-term throughout moduli space:
- Full instanton amplitude and expression at BPS locus differ by a D-term

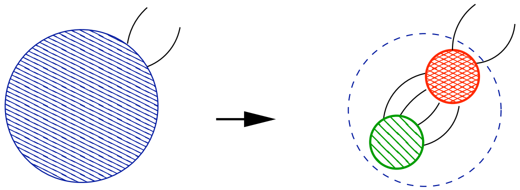
$$S_{4d} \simeq \int d^4x\, d^2\theta\, e^{-T}\, \bar{D}\bar{\Sigma}\, \bar{D}\bar{\Sigma} + (\mathrm{D-term}) \simeq \int d^4x\, d^4\theta\, e^{-T}\, f(\Sigma, \bar{\Sigma})$$

core F-term (BPS locus) — Away from BPS locus

- Subsequent beautiful analysis in 4d N=2 by Gaiotto, Moore, Neitzke

The topological string connection

- 4d non-perturbative terms are insensitive to the stability of D-brane instantons (decay products of unstable instanton reconstruct it)



- Reminiscent of D-branes in topological strings, defined by holomorphic conditions, without imposing stability condition (impose worldvolume F-term, but don't impose worldvolume D-term)

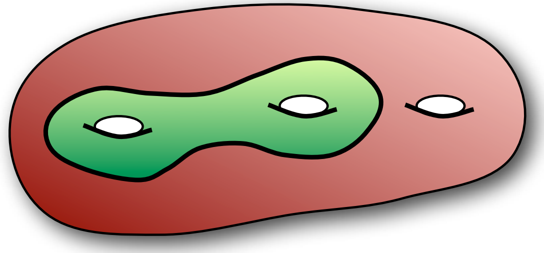
Non-perturbative F-terms computable in category of holomorphic branes

- Plan:

- Topological strings and non-perturbative effects in 4d, N=2
- Lessons for 4d, N=1

GV interpretation of topological strings [Gopakumar, Vafa '98]

- Topological string partition function



$$Z_{\rm top} = \sum_g \lambda^{2g-2} F_g(t)$$

- Computes F-terms for v.m. in 4d N=2 IIA on CY [Antoniadis, Gava, Narain, Taylor '93]

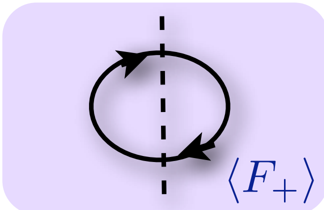
$$S_{4d} = \int d^4x \, d^4\theta \sum_g F_g(t) \, \mathcal{W}^{2g} = \int d^4x \, F_g(t) \, R_+^2 \, F_+^{2g-2} + \ldots$$

- Turn on graviphoton background $\langle F_+ \rangle = \lambda \to S_{4d} = \int d^4x \, Z_{\rm top}(\lambda) \, R_+^2$

  ⇒ Schwinger diagram for electric charges: D2/D0-branes
  Perturbative Fg from integrating out genus g D2/D0 particles

  ⇒ Non-pert. creation of 4d D2/D0-particles by Schwinger effect

- Interpretation in M-theory on CY x S^1 as wrapped genus g M2 particles with momentum on S^1



$$Z_{\rm top,n.p.} = \sum_{r,\, m,\, (n^r_{\mathbf{k}})} n^r_{\mathbf{k}} \int_\epsilon^\infty \frac{ds}{s} \left(2 \sin \frac{s}{2}\right)^{2r-2} \exp\left[-2\pi \frac{s}{\lambda} (k_i t_i + i\, m)\right]$$

D-brane instantons from M-theory on T2

- GV give non perturbative effects for v.m. from 4d D2/D0-particles
- Relate to D-brane instantons by S^1 compactification and T-duality

[Ooguri, Vafa '96]

Non-pert. effects on hyperm. moduli space of dual 4d IIB on CY

- IIA on CY x S^1 computed as M-theory on CY x T^2

(genus g M2 particles with momentum on T^2), then shrink T^2 for IIB

Manifest SL(2,Z) on type IIB side

Includes D1/D(-1)-brane instantons (general (p,q) string instantons)

- Not much known on higher F-terms for hm's, focus on genus 0

Corrections to hm metric

$$Z_{\rm M2} = \frac{1}{4\pi}\sum_{\mathbf{k}} n^{(0)}_{k_a} \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+\tau n|^{3/2}} \left(1 + 2\pi\,|m+\tau n|\, k_a t^a\right) e^{-S_{k_1,k_2}}$$

$\downarrow$

$$Z_{\rm D-inst} = \frac{1}{4\pi}\sum_{k_a} n_{k_a} \sum_{m\neq 0, n\in\mathbf{Z}} \frac{|z+q|^{1/2}}{|p|^{3/2}} \left[1 + \sum_{k=1}^{\infty} \frac{\Gamma(3/2+k)}{k!\,\Gamma(3/2-k)} (4\pi|p\tau_2||z+q|)^{-k}\right] e^{-S_{(p,q)}}$$

Reproduces [Robles-Llana, Rocek, Saueressig, Theis, Vandoren, '06]

Continuity under wall crossing

- GV includes only instantons with zero mutual "intersection number" (e.g. no D1, D(-1) but no D3, D5) i.e. no magnetic charges in Schwinger

  ⇒ Only threshold stability walls

  ⇒ Enough to discuss continuity of superpotential (in 4d N=1 setup)

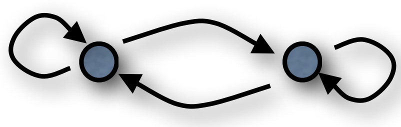
- Continuity across threshold walls from continuity of GV invariants

- More in detail, as a BPS D-brane particle splits into two BPS ones

  2-particle system: 1-dim quiver quantum mechanics [Denef '02]

  2-particle system has one bound state at threshold

  Bound state in Schwinger loop ensures continuity



- Interesting interpretation from the particle-instanton dictionary

  Particle bound state in Schwinger loop maps to 2-instanton process

Implications and tools for N=1

Need to reduce to N=1 to really discuss superpotentials

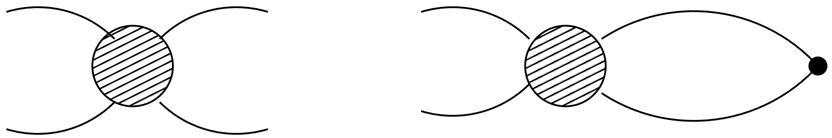
- Turn on N=2 → N=1 fluxes

  - Superpotential from flux lifting of fermion zero modes

    [Bergshoeff, Kallosh, Kashani-Poor, Sorkin, Tomasiello; ...; Billo, Ferro, Frau, Fucito, Lerda, Morales]

  - In effective theory, just dress non-pert. N=2 hm metric with flux superpotential to obtain non.pert superpotential of N=1 flux model

    [A.U.]



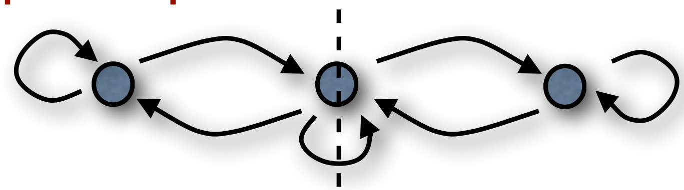
- Introduce orientifold planes

  - Topological string may produce useful tools [work in progress]

  - Partial lesson to study threshold walls: particle-instanton dictionary

    2 -particle system: orientifolded 1-dim quiver quantum mechanics

    Can check existence of bound state



Conclusions

We have studied certain formal properties of D-brane instantons

- Showed holomorphy across stability walls
- Criteria for the generation of superpotentials

Instantons which can become non-BPS cannot generate superp.

(can iff misalignment breaks spacetime susy)

- Described relation to topological string in 4d N=2

Resum of D-brane instanton corrections to hm metric

Wall crossing behaviour manifest

Particle-instanton T-duality: New computational tools in 4d N=2,1

[in progress]

Expect further applications to N=1 model building